Pensieve header: Hour 30: Perturbing the Heisenberg-algebra knot invariant (2).

Recall,

$$m_k^{ij} = e^{(\xi_i+\xi_j)\,x_k+(\pi_i+\pi_j)\,p_k-\xi_i\,\pi_j},$$

$$R_\epsilon = e^{(T-1)\,(p_i-p_j)\,x_j+R'};\;\; R' = \textstyle\sum_{k=1}^{\$k} \epsilon^k R^{(k)},$$

$$_A\mathcal{L}_B \,//\, _B\mathcal{M}_C = e^{\sum_{i\in B}\partial_{z_i}\partial_{\zeta_i}}(\mathcal{L}\cdot\mathcal{M}),$$

$$\langle F:\mathcal{E}\rangle_B = e^{\frac{1}{2}\sum_{u,v\in B}F_{uv}\,\partial^u\partial^v}\,\mathcal{E}\;|_{z_B=0} \;\text{ and }\; [F:\mathcal{E}]_B = e^{\frac{1}{2}\sum_{u,v\in B}F_{uv}\,\partial^u\partial^v}\,\mathcal{E}$$

(Note, the two are equi-computable: clearly if we know how to compute $[F:\mathcal{E}]$ we also know how to compute $\langle F:\mathcal{E}\rangle$, and also $[F:\mathcal{E}]\;|_{z_B\to\bar{z}_B} = \langle F:\mathcal{E}\;|_{z_B\to z_B+\bar{z}_B}\rangle$).

$Z_\lambda := \log[\lambda F: e^E]$ satisfies $Z_0 = E$ and the "synthesis equation",

$$\partial_\lambda Z_\lambda = \tfrac{1}{2}\,F_{uv}(\partial_u\,\partial_v\,Z_\lambda + (\partial_u\,Z_\lambda)\,(\partial_v\,Z_\lambda)).$$

Lemma 1. $\left\langle F:\mathcal{E}\,e^{\frac{1}{2}\sum_{i,j\in B}G_{ij}\,z_i\,z_j}\right\rangle_B = \det(I-\mathrm{GF})^{-1/2}\left\langle F(I-\mathrm{GF})^{-1}:\mathcal{E}\right\rangle_B$.

Lemma 2. $\langle F:\mathcal{E}\,e^{\sum_{i\in B}y_i\,z_i}\rangle_B = e^{\frac{1}{2}\sum_{i,j\in B}F_{ij}\,z_i\,z_j}\left\langle F:\mathcal{E}\;|_{Z_B\to Z_B+\mathrm{F}y_B}\right\rangle_B$.

To solve the synthesis equation in general, we write $Z_\lambda = \sum Z[m]\,\lambda^m$ and then solve iteratively $Z[0] = Z_0 = E$ and

$$(m+1)\,Z[m+1] = \tfrac{1}{2}\,F_{uv}\big(\partial_u\,\partial_v\,Z[m] + \textstyle\sum_j(\partial_u\,Z[j])\cdot(\partial_v\,Z[m-j])\big).$$

Definition. A power series f in an auxiliary variable ϵ and in the z_i's, including $i\notin B$, is called *docile* if every monomial μ in it satisfies $\deg_z\mu \le 2\deg_\epsilon\mu + 2$; we will short that to $\deg_z f \le 2\deg_\epsilon f + 2$.

Claim 1. The synthesis equation preserves docility: if E is docile then so is Z_λ, and in particular, so is $\log\langle F: e^E\rangle$.

(And so it makes sense to restrict our attention to docile perturbations!)

Claim 2. Restricting attention to $\{z_i : i\in B\}$, if $\deg_{z_B} E \le 4\deg_\epsilon E$ then $\deg_{z_B} Z_\lambda \le 4\deg_\epsilon Z_\lambda - 2\deg_\lambda Z_\lambda$ and thus $\deg_\lambda Z_\lambda \le 2\deg_\epsilon Z_\lambda$.

Claim 2 implies that if $E\;|_{\epsilon=0}$ is independent of z_B and if we only care about Z_λ up to ϵ^k, then the iterative process for finding Z_λ terminates at $Z[2\,k]$.

Conclusion. We can compute efficiently (in polynomial time!) if all of our generating functions are of the form $\omega\,e^{Q+P}$, where ω is a scalar, Q is an ϵ-free quadratic, and $P = \sum_{k=1}^{\$k} P^{(k)}\,\epsilon^k$, where $\deg P^{(k)} \le 2\,k+2$.

On to the implementation...

$\mathbb{E}[\omega, Q, \text{P\_}\epsilon\text{Series}]$ represents $\omega\,e^{Q+P}$, where ω is a scalar, Q is an ϵ-free quadratic, and $P = \sum_{k=0}^{\$k} P[\![k]\!]\,\epsilon^k$ is a docile perturbation (it is ill-advised to include ω in P because then it will have log terms, so always, $P[\![0]\!] = 0$).

Initialization and minor utilities

(Alt) In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Classes\\21-1350-KnotTheory"];
Once[<< "Common.m"];
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

(Alt) In[∘]:=
```
$k=1;
```

(Alt) In[∘]:=
```
CCF[ℰ_] := ExpandDenominator@ExpandNumerator@Together[ℰ];
```

(Alt) In[∘]:= ComposeList$\left[\{\text{Together}, \text{ExpandNumerator}, \text{ExpandDenominator}\}, \frac{1}{T-1}+\frac{1}{T-2}\right]$

(Alt) Out[∘]= $\left\{\frac{1}{-2+T}+\frac{1}{-1+T}, \frac{-3+2T}{(-2+T)\times(-1+T)}, \frac{-3+2T}{(-2+T)\times(-1+T)}, \frac{-3+2T}{2-3T+T^2}\right\}$

(Alt) In[∘]:= CCF$\left[\frac{1}{T-1}+\frac{1}{T-2}\right]$

(Alt) Out[∘]= $\frac{-3+2T}{2-3T+T^2}$

(Alt) In[∘]:=
```
CF[ℰ_List] := CF /@ ℰ;
CF[ℰ_𝔢Series] := CF /@ ℰ;
CF[ℰ_] := Module[
    {vs = Cases[ℰ, (p | x | π | ξ)_, ∞] ⋃ {p | x | π | ξ}},
    Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) ⧴ CCF[c] (Times @@ vs^ps)]
   ];
CF[ℰ_𝔼] := CF /@ ℰ;
CF[𝔼_sp___[ℰs___]] := CF /@ 𝔼_sp[ℰs];
```

(Alt) In[∘]:= CF$\left[(T\,x_1+(T-1)\,x_2)\left(\frac{p_1}{T-2}+p_2\right)\right]$

(Alt) Out[∘]= $\frac{T\,p_1\,x_1}{-2+T}+T\,p_2\,x_1+\frac{(-1+T)\,p_1\,x_2}{-2+T}+(-1+T)\,p_2\,x_2$

(Alt) In[∘]:=
```
ϵSeries /: S1_ϵSeries ≡ S2_ϵSeries :=
  Length[S1] == Length[S2] ∧ Inner[CF[#1] == CF[#2] &, S1, S2, And];
ϵSeries[0] := ϵSeries @@ Table[0, $k + 1];
ϵSeries /: S1_ϵSeries + S2_ϵSeries :=
  ϵSeries @@ Table[S1⟦k⟧ + S2⟦k⟧, {k, Min[Length@S1, Length@S2]}];
ϵSeries /: S1_ϵSeries * S2_ϵSeries := ϵSeries @@
   Table[Sum[S1⟦j + 1⟧ * S2⟦k - j + 1⟧, {j, 0, k}], {k, 0, Min[Length@S1, Length@S2] - 1}];
ϵSeries /: c_ * S_ϵSeries := (c #) & /@ S;
ϵSeries /: ∂_{vs___} S_ϵSeries := (s ↦ ∂_{vs} s) /@ S;
```

(Alt) In[∘]:=
```
ϵSeries[0, 2, 7, -1] ϵSeries[0, 3, 5, 19, 1350]
```

(Alt) Out[∘]=
```
ϵSeries[0, 0, 6, 31]
```

The Main Program

Variables and their duals:

(Alt) In[∘]:=
```
{p*, x*, π*, ξ*} = {π, ξ, p, x};
(vs_List)* := (v ↦ v*) /@ vs;
(u_{i_})* := (u*)_i;
```

𝔼 operations:

(Alt) In[∘]:=
```
𝔼 /: 𝔼[ω1_, Q1_, P1_] ≡ 𝔼[ω2_, Q2_, P2_] := CF[ω1 == ω2] ∧ CF[Q1 == Q2] ∧ (P1 ≡ P2);
𝔼 /: 𝔼[ω1_, Q1_, P1_] 𝔼[ω2_, Q2_, P2_] := 𝔼[ω1 ω2, Q1 + Q2, P1 + P2];
𝔼_{d1_→r1_}[ℰ1s___] ≡ 𝔼_{d2_→r2_}[ℰ2s___] ^:= (d1 == d2) ∧ (r1 == r2) ∧ (𝔼[ℰ1s] ≡ 𝔼[ℰ2s]);
𝔼_{d1_→r1_}[ℰ1s___] 𝔼_{d2_→r2_}[ℰ2s___] ^:= 𝔼_{(d1⋃d2)→(r1⋃r2)} @@ (𝔼[ℰ1s] 𝔼[ℰ2s]);
```

Getting rid of the quadratic using lemma 1.

Lemma 1. $\left\langle F:\mathcal{E}\, e^{\frac{1}{2}\sum_{i,j\in B} G_{ij} z_i z_j}\right\rangle_B = \det(I-GF)^{-1/2}\left\langle F(I-GF)^{-1}:\mathcal{E}\right\rangle_B.$

```
Zip1_{} = Identity;
Zip1_{vs_}@⟨ℱ_, 𝔼[ω_, Q_, P_]⟩ := Module[{I, F, G, u, v},
  I = IdentityMatrix@Length@vs;
  F = Table[∂_{u,v} ℱ, {u, vs*}, {v, vs*}];
  G = Table[∂_{u,v} Q, {u, vs}, {v, vs}];
  CF /@ ⟨
    vs*.F.Inverse[I - G.F].vs* / 2,
    𝔼[PowerExpand@Factor[ω Det[I - G.F]^(-1/2)], Q - vs.G.vs / 2, P]
   ⟩
 ]
```

(Alt) In[∘]:=
```
Ex1 = CF /@ ⟨2/2 ξ₁², 𝔼[19, 1/2 {x₁, x₂}.(-3 2; 2 7).{x₁, x₂}, ϵSeries[0, x₁]]⟩
```

(Alt) Out[∘]=
$$\left\langle \xi_1^2, \mathbb{E}\left[19, -\frac{3 x_1^2}{2} + 2 x_1 x_2 + \frac{7 x_2^2}{2}, \epsilon\text{Series}[0, x_1]\right]\right\rangle$$

(Alt) In[∘]:=
```
Ex1 // Zip1_{x₁}
```

(Alt) Out[∘]=
$$\left\langle \frac{\xi_1^2}{7}, \mathbb{E}\left[\frac{19}{\sqrt{7}}, 2 x_1 x_2 + \frac{7 x_2^2}{2}, \epsilon\text{Series}[0, x_1]\right]\right\rangle$$

Getting rid of linear terms using Lemma 2.

Lemma 2. $\langle F : \mathcal{E}\, e^{\sum_{i\in B} y_i z_i}\rangle_B = e^{\frac{1}{2}\sum_{i,j\in B} F_{ij} z_i z_j} \left\langle F : \mathcal{E} \mid_{z_B \to z_B + F y_B}\right\rangle_B$.

(Alt) In[∘]:=
```
Zip2_{} = Identity;
Zip2_{vs_} @⟨ℱ_, 𝔼[ω_, Q_, P_]⟩ := Module[{F, Y, u, v},
  F = Table[∂_{u,v} ℱ, {u, vs*}, {v, vs*}];
  Y = Table[∂_v Q, {v, vs}];
  CF /@ ⟨ℱ, 𝔼[ω, Q - Y.vs + Y.F.Y / 2, P /. Thread[vs → vs + F.Y]]⟩
 ]
```

Dealing with Feynman diagrams without ever seeing them, using the synthesis equation and iteration. Write $Z_\lambda = \sum Z[m]\,\lambda^m$ and then $Z[0] = Z_0 = E$ and

$$Z[m+1] = \frac{1}{2\,(m+1)}\, F_{uv}\Big(\partial_u \partial_v Z[m] + \textstyle\sum_j (\partial_u Z[j]) \cdot (\partial_v Z[m-j])\Big),$$

and we only care to compute up to $Z[\![2\,\$k]\!]$:

(Alt) In[∘]:=
```
Zip3_{vs_} @⟨ℱ_, 𝔼[ω_, Q_, P_]⟩ := Module[{Z, u, v, m, j},
  Z[0] = P;
  For[m = 0, m < 2 $k, ++m,
   Z[m + 1] = CF[1/(2 (m + 1))
       Sum[∂_{u*,v*} ℱ (∂_{u,v} Z[m] + Sum[(∂_u Z[j]) (∂_v Z[m - j]), {j, 0, m}]), {u, vs}, {v, vs}]]
  ];
  𝔼[ω, Q, CF[Sum[Z[m], {m, 0, 2 $k}] /. Table[v → 0, {v, vs}]]]
 ]
```

(Alt) In[∘]:=
```
Block[{$k = 2}, ⟨1/2 ξ², 𝔼[1, 0, ϵSeries[0, 1/6 x³, 0]]⟩ // Zip3_{x}]
```

(Alt) Out[∘]=
$$\mathbb{E}\left[1, 0, \epsilon\text{Series}\left[0, 0, \frac{5}{24}\right]\right]$$

(Alt) In[∘]:= $\frac{1}{2^3} + \frac{1}{3!\,2}$

(Alt) Out[∘]= $\frac{5}{24}$

(Alt) In[∘]:= Block$\left[\{\$k = 4\},\ \left\langle \frac{1}{2}\,\xi^2,\ \mathbb{E}\left[1, 0, \epsilon\text{Series}\left[0, \frac{1}{6}\,x^3, 0, 0, 0\right]\right]\right\rangle\ //\ \text{Zip3}_{\{x\}}\right]$

(Alt) Out[∘]= $\mathbb{E}\left[1, 0, \epsilon\text{Series}\left[0, 0, \frac{5}{24}, 0, \frac{5}{16}\right]\right]$

From “Cubic Multigraphs A005967” by Richard J. Mathar, https://oeis.org/A005967/a005967.pdf:

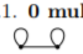
CUBIC MULTIGRAPHS A005967

RICHARD J. MATHAR

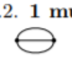
Abstract. These are illustrations of the undirected connected cubic (3-regular) multigraphs up to 10 vertices as counted in [1, A005967].

1. 2 vertices

1.1. **0 multi-edges 2 loops.** (1 graphs)



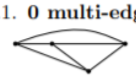
1.2. **1 multi-edges 0 loops.** (1 graphs)



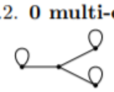
Total: 2 graphs, 1 without loops.

2. 4 vertices

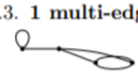
2.1. **0 multi-edges 0 loops.** (1 graphs)



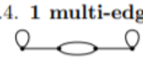
2.2. **0 multi-edges 3 loops.** (1 graphs)



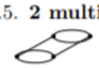
2.3. **1 multi-edges 1 loops.** (1 graphs)



2.4. **1 multi-edges 2 loops.** (1 graphs)



2.5. **2 multi-edges 0 loops.** (1 graphs)



Total: 5 graphs, 2 without loops.

Date: November 12, 2018.
2010 *Mathematics Subject Classification*. Primary 05C30; Secondary 05C75, 81Q30.
Key words and phrases. Graph Enumeration, Combinatorics.

1

(Alt) In[∘]:= $\frac{1}{4!}+\frac{1}{3!\,2^3}+\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^4}$

(Alt) Out[∘]= $\frac{5}{16}$

(Alt) In[∘]:=
```
Block[{$k = 8}, ⟨1/2 ξ^2, 𝔼[1, 0, ϵSeries[0, 1/6 x^3, Sequence @@ Table[0, $k - 1]]]⟩ // Zip3_{x}]
```

(Alt) Out[∘]= $\mathbb{E}\left[1, 0, \epsilon\text{Series}\left[0, 0, \frac{5}{24}, 0, \frac{5}{16}, 0, \frac{1105}{1152}, 0, \frac{565}{128}\right]\right]$

Check out https://oeis.org/!

```
Module[{l, m}, Log[Sum[l = 3 m / 2; ((3 m)! ϵ^m)/(2^l l! 6^m m!), {m, 0, 30, 2}] + O[ϵ]^31]]
```

(Alt) Out[∘]= $\frac{5\,\epsilon^2}{24}+\frac{5\,\epsilon^4}{16}+\frac{1105\,\epsilon^6}{1152}+\frac{565\,\epsilon^8}{128}+\frac{82825\,\epsilon^{10}}{3072}+\frac{19675\,\epsilon^{12}}{96}+\frac{1282031525\,\epsilon^{14}}{688128}+\frac{80727925\,\epsilon^{16}}{4096}+\frac{1683480621875\,\epsilon^{18}}{7077888}+\frac{13209845125\,\epsilon^{20}}{4096}+\frac{2239646759308375\,\epsilon^{22}}{46137344}+\frac{19739117098375\,\epsilon^{24}}{24576}+\frac{6320791709083309375\,\epsilon^{26}}{436207616}+\frac{32468078556378125\,\epsilon^{28}}{114688}+\frac{38362676768845045751875\,\epsilon^{30}}{6442450944}+O[\epsilon]^{31}$

Check out https://oeis.org/!

(Alt) In[∘]:=
```
Zip_{vs_}[ℱ_, ℰ_] := ⟨ℱ, ℰ⟩ // Zip1_{vs} // Zip2_{vs} // Zip3_{vs}
```

(Alt) In[∘]:=
```
𝔼_{d1_→r1_}[ℰ1s___] // 𝔼_{d2_→r2_}[ℰ2s___] := Module[{is = r1 ⋂ d2, lvs},
  lvs = Flatten@Table[{x_{$@i}, p_{$@i}}, {i, is}];
  𝔼_{(d1⋃Complement[d2,is])→(r2⋃Complement[r1,is])} @@ (Zip_{lvs⋃lvs*}[lvs*.lvs, Times[
      𝔼[ℰ1s] /. Table[(v : x | p)_i → v_{$@i}, {i, is}],
      𝔼[ℰ2s] /. Table[(v : ξ | π)_i → v_{$@i}, {i, is}]
     ]])
 ]
```

The Basic Tensors

(Alt) In[∘]:=
```
η_{i_} := 𝔼_{{}→{i}}[1, 0, ϵSeries[0]];
m_{i_,j_→k_} := 𝔼_{{i,j}→{k}}[1, -ξ_i π_j + (π_i + π_j) p_k + (ξ_i + ξ_j) x_k, ϵSeries[0]]
```

(Alt) In[∘]:=
```
AllMonomials[{}, 0] = {1};
AllMonomials[{}, d_Integer] /; d > 0 := {};
AllMonomials[{v_, vs___}, d_Integer] :=
  Join @@ Table[v^(d-k) AllMonomials[{vs}, k], {k, 0, d}];
AllMonomials[vs_List, {d_}] := Join @@ Table[AllMonomials[vs, k], {k, 0, d}];
```

(Alt) In[∘]:=
```
Basis[js_List, m_] := Flatten@Outer[Times,
    AllMonomials[Table[p_j, {j, js}], m], AllMonomials[Table[x_j, {j, js}], m]];
Basis[js_List, {m_}] := Flatten@Table[Basis[js, k], {k, 0, m}]
```

In[∘]:=
```
Basis[{i, j}, {2}]
```

Out[∘]= $\{1, p_i x_i, p_i x_j, p_j x_i, p_j x_j, p_i^2 x_i^2, p_i^2 x_i x_j, p_i^2 x_j^2, p_i p_j x_i^2, p_i p_j x_i x_j, p_i p_j x_j^2, p_j^2 x_i^2, p_j^2 x_i x_j, p_j^2 x_j^2\}$

(Alt) In[∘]:=
```
GenericCombination[bas_, c_] := bas.Table[c_j, {j, Length@bas}];
GenericCombination[bas_, c_k_] := bas.Table[c_{k,j}, {j, Length@bas}];
```

In[∘]:=
```
GenericCombination[Basis[{i, j}, {2}], c_1]
```

Out[∘]= $c_{1,1} + p_i x_i c_{1,2} + p_i x_j c_{1,3} + p_j x_i c_{1,4} + p_j x_j c_{1,5} + p_i^2 x_i^2 c_{1,6} + p_i^2 x_i x_j c_{1,7} + p_i^2 x_j^2 c_{1,8} + p_i p_j x_i^2 c_{1,9} + p_i p_j x_i x_j c_{1,10} + p_i p_j x_j^2 c_{1,11} + p_j^2 x_i^2 c_{1,12} + p_j^2 x_i x_j c_{1,13} + p_j^2 x_j^2 c_{1,14}$

(Alt) In[∘]:=
```
R_{i_,j_} := 𝔼_{{}→{i,j}}[T^{1/2}, (T - 1) (p_i - p_j) x_j , ϵSeries @@
    Prepend[0]@Table[GenericCombination[Basis[{i, j}, {k + 1}], c_k], {k, $k}]];
R̄_{i_,j_} := 𝔼_{{}→{i,j}}[T^{-1/2}, (T^{-1} - 1) (p_i - p_j) x_j, ϵSeries @@
    Prepend[0]@Table[GenericCombination[Basis[{i, j}, {k + 1}], d_k], {k, $k}]];
C_{i_} := 𝔼_{{}→{i}}[T^{1/2}, 0, ϵSeries @@ Prepend[0]@
     Table[GenericCombination[Basis[{i}, {k + 1}], e_k], {k, $k}]];
C̄_{i_} := 𝔼_{{}→{i}}[T^{-1/2}, 0, ϵSeries @@ Prepend[0]@
     Table[GenericCombination[Basis[{i}, {k + 1}], f_k], {k, $k}]];
```

In[∘]:=
```
{R_{1,2}, R̄_{1,2}, C_1, C̄_1}
```

Out[∘]= $\{\mathbb{E}_{\{\}\to\{1,2\}}[\sqrt{T}, (-1+T)(p_1-p_2)x_2,$
$\epsilon\text{Series}[0, c_{1,1} + p_1 x_1 c_{1,2} + p_1 x_2 c_{1,3} + p_2 x_1 c_{1,4} + p_2 x_2 c_{1,5} + p_1^2 x_1^2 c_{1,6} + p_1^2 x_1 x_2 c_{1,7} + p_1^2 x_2^2 c_{1,8} + p_1 p_2 x_1^2 c_{1,9} + p_1 p_2 x_1 x_2 c_{1,10} + p_1 p_2 x_2^2 c_{1,11} + p_2^2 x_1^2 c_{1,12} + p_2^2 x_1 x_2 c_{1,13} + p_2^2 x_2^2 c_{1,14}]],$
$\mathbb{E}_{\{\}\to\{1,2\}}[\frac{1}{\sqrt{T}}, \left(-1+\frac{1}{T}\right)(p_1-p_2)x_2, \epsilon\text{Series}[0,$
$d_{1,1} + p_1 x_1 d_{1,2} + p_1 x_2 d_{1,3} + p_2 x_1 d_{1,4} + p_2 x_2 d_{1,5} + p_1^2 x_1^2 d_{1,6} + p_1^2 x_1 x_2 d_{1,7} + p_1^2 x_2^2 d_{1,8} + p_1 p_2 x_1^2 d_{1,9} + p_1 p_2 x_1 x_2 d_{1,10} + p_1 p_2 x_2^2 d_{1,11} + p_2^2 x_1^2 d_{1,12} + p_2^2 x_1 x_2 d_{1,13} + p_2^2 x_2^2 d_{1,14}]],$
$\mathbb{E}_{\{\}\to\{1\}}[\sqrt{T}, 0, \epsilon\text{Series}[0, e_{1,1} + p_1 x_1 e_{1,2} + p_1^2 x_1^2 e_{1,3}]],$
$\mathbb{E}_{\{\}\to\{1\}}[\frac{1}{\sqrt{T}}, 0, \epsilon\text{Series}[0, f_{1,1} + p_1 x_1 f_{1,2} + p_1^2 x_1^2 f_{1,3}]]\}$

(Alt) In[∘]:=
```
RMoves := {
  (R1,2 R4,3 R5,6 // m1,4→1 // m2,5→2 // m3,6→3) ≡ (R2,3 R4,5 R1,6 // m1,4→1 // m2,5→2 // m3,6→3),
  (R1,2 R̄3,4 // m1,3→1 // m2,4→2) ≡ (η1 η2),
  (C1 C̄2 // m1,2→1 ) ≡ η1,
  (R1,4 R̄5,2 C̄3 // m2,4→2 // m1,3→1 // m1,5→1) ≡ C̄1 η2,
  (C3 R1,2 // m2,3→2 // m2,1→1) ≡ (C̄3 R1,2 // m1,3→1 // m1,2→1),
  (C̄2 R1,3 // m1,2→1 // m1,3→1) ≡ η1, (C̄2 R̄3,1 // m1,2→1 // m1,3→1) ≡ η1,
  (C2 R̄1,3 // m1,2→1 // m1,3→1) ≡ η1, (C2 R3,1 // m1,2→1 // m1,3→1) ≡ η1
}
```

Solving for *R*, *C*, \$k = 1

(Alt) In[∘]:=
```
$k = 1;
{R1,2, C1}
unknowns = Cases[{R1,2, R̄1,2, C1, C̄1}, (c | d | e | f)$k,_, ∞] // Union
```

(Alt) Out[∘]=
$$\{\mathbb{E}_{\{\}\to\{1,2\}}[\sqrt{T}, (-1+T)(p_1-p_2)x_2, \epsilon\text{Series}[0, c_{1,1}+p_1x_1c_{1,2}+p_1x_2c_{1,3}+p_2x_1c_{1,4}+p_2x_2c_{1,5}+p_1^2x_1^2c_{1,6}+p_1^2x_1x_2c_{1,7}+p_1^2x_2^2c_{1,8}+p_1p_2x_1^2c_{1,9}+p_1p_2x_1x_2c_{1,10}+p_1p_2x_2^2c_{1,11}+p_2^2x_1^2c_{1,12}+p_2^2x_1x_2c_{1,13}+p_2^2x_2^2c_{1,14}]], \mathbb{E}_{\{\}\to\{1\}}[\sqrt{T}, 0, \epsilon\text{Series}[0, e_{1,1}+p_1x_1e_{1,2}+p_1^2x_1^2e_{1,3}]]\}$$

(Alt) Out[∘]=
$$\{c_{1,1}, c_{1,2}, c_{1,3}, c_{1,4}, c_{1,5}, c_{1,6}, c_{1,7}, c_{1,8}, c_{1,9}, c_{1,10}, c_{1,11}, c_{1,12}, c_{1,13}, c_{1,14}, d_{1,1}, d_{1,2}, d_{1,3}, d_{1,4}, d_{1,5}, d_{1,6}, d_{1,7}, d_{1,8}, d_{1,9}, d_{1,10}, d_{1,11}, d_{1,12}, d_{1,13}, d_{1,14}, e_{1,1}, e_{1,2}, e_{1,3}, f_{1,1}, f_{1,2}, f_{1,3}\}$$

(Alt) In[∘]:=
```
Short[errors = CCF /@ Cases[RMoves, a_ == b_ :→ a - b], 25]
```

(Alt) Out[∘]//Short=
$$\{T p_1 x_3 c_{1,2} - T^2 p_1 x_3 c_{1,2} + p_1 x_3 c_{1,3} - T p_1 x_3 c_{1,3} - p_2 x_1 c_{1,4} + T p_2 x_1 c_{1,4} + p_3 x_1 c_{1,4} - T p_3 x_1 c_{1,4} + p_1 x_2 c_{1,4} - T p_1 x_2 c_{1,4} - p_2 x_2 c_{1,4} + 2 T p_2 x_2 c_{1,4} - T^2 p_2 x_2 c_{1,4} - T p_3 x_2 c_{1,4} + T^2 p_3 x_2 c_{1,4} + T p_2 x_3 c_{1,4} - T^2 p_2 x_3 c_{1,4} + p_1 x_3 c_{1,5} - T p_1 x_3 c_{1,5} - 2 p_1^2 x_1 x_2 c_{1,6} + 2 T p_1^2 x_1 x_2 c_{1,6} + \ll 154 \gg + 4 T p_2^2 x_2 x_3 c_{1,13} - 4 T^2 p_2^2 x_2 x_3 c_{1,13} + T^3 p_2^2 x_2 x_3 c_{1,13} + 2 T p_1 p_3 x_2 x_3 c_{1,13} - 2 T^2 p_1 p_3 x_2 x_3 c_{1,13} - 2 T p_2 p_3 x_2 x_3 c_{1,13} + 4 T^2 p_2 p_3 x_2 x_3 c_{1,13} - 2 T^3 p_2 p_3 x_2 x_3 c_{1,13} - T^2 p_3^2 x_2 x_3 c_{1,13} + T^3 p_3^2 x_2 x_3 c_{1,13} + T p_2^2 x_3^2 c_{1,13} - 2 T^2 p_2^2 x_3^2 c_{1,13} + T^3 p_2^2 x_3^2 c_{1,13} + 2 p_2^2 x_2 x_3 c_{1,14} - 2 T p_2^2 x_2 x_3 c_{1,14} + p_1^2 x_3^2 c_{1,14} - 2 T p_1^2 x_3^2 c_{1,14} + T^2 p_1^2 x_3^2 c_{1,14} + 2 T p_1 p_3 x_3^2 c_{1,14} - 2 T^2 p_1 p_3 x_3^2 c_{1,14} - 2 T p_2 p_3 x_3^2 c_{1,14} + 2 T^2 p_2 p_3 x_3^2 c_{1,14}, \frac{\ll 1 \gg}{T^2}, \ll 1 \gg, \ll 3 \gg, \ll 1 \gg, \frac{\ll 1 \gg}{T^2}, \frac{\ll 1 \gg}{T^2}\}$$

(Alt) In[∘]:=
```
eqns = Thread[0 == Union @@ (CoefficientRules[#, {x1, x2, x3, p1, p2, p3}][[;; , 2]] & /@ errors)]
```

(Alt) Out[∘]=
$$\{0 == c_{1,4} - T c_{1,4}, 0 == -c_{1,4} + T c_{1,4}, 0 == T c_{1,4} - T^2 c_{1,4}, 0 == -c_{1,4} + 2 T c_{1,4} - T^2 c_{1,4}, 0 == -T c_{1,4} + T^2 c_{1,4}, 0 == T c_{1,2} - T^2 c_{1,2} + c_{1,3} - T c_{1,3} + c_{1,5} - T c_{1,5}, 0 == -2 c_{1,6} + 2 T c_{1,6}, 0 == 2 T c_{1,6} - 2 T^2 c_{1,6}, 0 == c_{1,9} - T c_{1,9}, 0 == -c_{1,9} + T c_{1,9}, 0 == 2 T c_{1,9} - 2 T^2 c_{1,9}, 0 == -2 c_{1,9} + 4 T c_{1,9} - 2 T^2 c_{1,9}, 0 == -2 T c_{1,9} + 2 T^2 c_{1,9}, 0 == 2 T c_{1,6} - 2 T^2 c_{1,6} - c_{1,9} + 4 T c_{1,9} - 4 T^2 c_{1,9} + T^3 c_{1,9}, 0 == 2 T c_{1,8} - 2 T^2 c_{1,8} + T^2 c_{1,9} - 2 T^3 c_{1,9} + T^4 c_{1,9} + T c_{1,10} - 2 T^2 c_{1,10} + T^3 c_{1,10}, 0 == 2 T c_{1,7} - 2 T^2 c_{1,7} - c_{1,10} + 4 T c_{1,10} - 3 T^2 c_{1,10} + 2 c_{1,11} - 2 T c_{1,11},$$

$$0 == T^2 c_{1,9} - T^3 c_{1,9} + 2 T c_{1,12} - 2 T^2 c_{1,12},\ 0 == c_{1,12} - T^2 c_{1,12},\ 0 == -c_{1,12} + 2 T c_{1,12} - T^2 c_{1,12},$$
$$0 == c_{1,9} - 2 T c_{1,9} + T^2 c_{1,9} + c_{1,12} - 2 T c_{1,12} + T^2 c_{1,12},\ 0 == -2 T c_{1,12} + 2 T^2 c_{1,12},$$
$$0 == -4 T c_{1,12} + 8 T^2 c_{1,12} - 4 T^3 c_{1,12},\ 0 == -2 c_{1,12} + 6 T c_{1,12} - 6 T^2 c_{1,12} + 2 T^3 c_{1,12},$$
$$0 == -2 T^2 c_{1,12} + 2 T^3 c_{1,12},\ 0 == -T^2 c_{1,12} + 2 T^3 c_{1,12} - T^4 c_{1,12},$$
$$0 == -c_{1,12} + 4 T c_{1,12} - 6 T^2 c_{1,12} + 4 T^3 c_{1,12} - T^4 c_{1,12},\ 0 == -2 T c_{1,12} + 6 T^2 c_{1,12} - 6 T^3 c_{1,12} + 2 T^4 c_{1,12},$$
$$0 == 2 T c_{1,13} - 2 T^2 c_{1,13},\ 0 == T c_{1,13} - T^2 c_{1,13},\ 0 == 2 T c_{1,12} - 2 T^2 c_{1,12} + T c_{1,13} - T^2 c_{1,13},$$
$$0 == 2 c_{1,8} - 2 T c_{1,8} + c_{1,10} - 2 T c_{1,10} + T^2 c_{1,10} + c_{1,13} - 2 T c_{1,13} + T^2 c_{1,13},\ 0 == -2 T c_{1,13} + 2 T^2 c_{1,13},$$
$$0 == -2 T c_{1,13} + 4 T^2 c_{1,13} - 2 T^3 c_{1,13},\ 0 == T^2 c_{1,12} - 2 T^3 c_{1,12} + T^4 c_{1,12} + T c_{1,13} - 2 T^2 c_{1,13} + T^3 c_{1,13},$$
$$0 == -T^2 c_{1,13} + T^3 c_{1,13},\ 0 == -c_{1,13} + 4 T c_{1,13} - 4 T^2 c_{1,13} + T^3 c_{1,13} + 2 c_{1,14} - 2 T c_{1,14},$$
$$0 == 2 T c_{1,14} - 2 T^2 c_{1,14},\ 0 == T^2 c_{1,6} - 2 T^3 c_{1,6} + T^4 c_{1,6} + T c_{1,7} - 2 T^2 c_{1,7} + T^3 c_{1,7} +$$
$$c_{1,8} - 4 T c_{1,8} + 3 T^2 c_{1,8} + c_{1,11} - 2 T c_{1,11} + T^2 c_{1,11} + c_{1,14} - 2 T c_{1,14} + T^2 c_{1,14},$$
$$0 == -2 T c_{1,14} + 2 T^2 c_{1,14},\ 0 == c_{1,1} + d_{1,1},\ 0 == c_{1,1} - c_{1,4} + \frac{c_{1,4}}{T} + 2 c_{1,12} + \frac{2 c_{1,12}}{T^2} - \frac{4 c_{1,12}}{T} + d_{1,1},$$
$$0 == c_{1,2} + c_{1,4} - \frac{c_{1,4}}{T} - 2 c_{1,9} + \frac{2 c_{1,9}}{T} - 4 c_{1,12} - \frac{4 c_{1,12}}{T^2} + \frac{8 c_{1,12}}{T} + d_{1,2},$$
$$0 == \frac{c_{1,4}}{T} + \frac{4 c_{1,12}}{T^2} - \frac{4 c_{1,12}}{T} + d_{1,4},\ 0 == c_{1,2} + d_{1,2} + d_{1,4} - T d_{1,4},\ 0 == c_{1,4} + T d_{1,4},$$
$$0 == c_{1,2} - \frac{c_{1,2}}{T} + \frac{c_{1,3}}{T} + d_{1,3} + d_{1,5} - T d_{1,5},\ 0 == c_{1,4} - \frac{c_{1,4}}{T} + \frac{c_{1,5}}{T} + T d_{1,5},$$
$$0 == c_{1,4} - \frac{c_{1,4}}{T} + \frac{c_{1,5}}{T} - 4 c_{1,12} - \frac{4 c_{1,12}}{T^2} + \frac{8 c_{1,12}}{T} + \frac{2 c_{1,13}}{T^2} - \frac{2 c_{1,13}}{T} + T d_{1,5},$$
$$0 == c_{1,6} + c_{1,9} - \frac{c_{1,9}}{T} + c_{1,12} + \frac{c_{1,12}}{T^2} - \frac{2 c_{1,12}}{T} + d_{1,6},\ 0 == \frac{c_{1,9}}{T} - \frac{2 c_{1,12}}{T^2} + \frac{2 c_{1,12}}{T} + d_{1,9},$$
$$0 == 2 c_{1,9} - \frac{2 c_{1,9}}{T} + \frac{c_{1,10}}{T} + 4 c_{1,12} + \frac{4 c_{1,12}}{T^2} - \frac{8 c_{1,12}}{T} - \frac{2 c_{1,13}}{T^2} + \frac{2 c_{1,13}}{T} + T d_{1,10},$$
$$0 == -2 c_{1,9} + \frac{c_{1,9}}{T} + T c_{1,9} + c_{1,10} - \frac{c_{1,10}}{T} + \frac{c_{1,11}}{T} - 6 c_{1,12} - \frac{2 c_{1,12}}{T^2} +$$
$$\frac{6 c_{1,12}}{T} + 2 T c_{1,12} + 2 c_{1,13} + \frac{2 c_{1,13}}{T^2} - \frac{4 c_{1,13}}{T} - \frac{2 c_{1,14}}{T^2} + \frac{2 c_{1,14}}{T} + T^2 d_{1,11},$$
$$0 == \frac{c_{1,12}}{T^2} + d_{1,12},\ 0 == c_{1,9} + T d_{1,9} + 2 T d_{1,12} - 2 T^2 d_{1,12},\ 0 == c_{1,12} + T^2 d_{1,12},$$
$$0 == c_{1,6} + d_{1,6} + d_{1,9} - T d_{1,9} + d_{1,12} - 2 T d_{1,12} + T^2 d_{1,12},\ 0 == -\frac{2 c_{1,12}}{T^2} + \frac{2 c_{1,12}}{T} + \frac{c_{1,13}}{T^2} + T d_{1,13},$$
$$0 == 2 c_{1,9} - \frac{2 c_{1,9}}{T} + \frac{c_{1,10}}{T} + T d_{1,10} + 2 T d_{1,13} - 2 T^2 d_{1,13},\ 0 == 2 c_{1,12} - \frac{2 c_{1,12}}{T} + \frac{c_{1,13}}{T} + T^2 d_{1,13},$$
$$0 == 2 c_{1,6} - \frac{2 c_{1,6}}{T} + \frac{c_{1,7}}{T} + d_{1,7} + d_{1,10} - T d_{1,10} + d_{1,13} - 2 T d_{1,13} + T^2 d_{1,13},$$
$$0 == c_{1,9} + \frac{c_{1,9}}{T^2} - \frac{2 c_{1,9}}{T} - \frac{c_{1,10}}{T^2} + \frac{c_{1,10}}{T} + \frac{c_{1,11}}{T^2} + T d_{1,11} + 2 T d_{1,14} - 2 T^2 d_{1,14},$$
$$0 == c_{1,12} + \frac{c_{1,12}}{T^2} - \frac{2 c_{1,12}}{T} - \frac{c_{1,13}}{T^2} + \frac{c_{1,13}}{T} + \frac{c_{1,14}}{T^2} + T^2 d_{1,14},$$
$$0 == c_{1,6} + \frac{c_{1,6}}{T^2} - \frac{2 c_{1,6}}{T} - \frac{c_{1,7}}{T^2} + \frac{c_{1,7}}{T} + \frac{c_{1,8}}{T^2} + d_{1,8} + d_{1,11} - T d_{1,11} + d_{1,14} - 2 T d_{1,14} + T^2 d_{1,14},$$

$$0 == d_{1,1} - d_{1,4} + 2\,d_{1,12} + e_{1,1},\ 0 == \frac{d_{1,2}}{T} + d_{1,3} + \frac{d_{1,4}}{T} + d_{1,5} - \frac{2\,d_{1,9}}{T} - d_{1,10} - \frac{4\,d_{1,12}}{T} - 2\,d_{1,13} + \frac{e_{1,2}}{T},$$

$$0 == c_{1,6} + \frac{c_{1,7}}{T} + \frac{c_{1,8}}{T^2} + c_{1,9} + \frac{c_{1,10}}{T} + \frac{c_{1,11}}{T^2} + c_{1,12} + \frac{c_{1,13}}{T} + \frac{c_{1,14}}{T^2} + \frac{e_{1,3}}{T^2},$$

$$0 == \frac{d_{1,6}}{T^2} + \frac{d_{1,7}}{T} + d_{1,8} + \frac{d_{1,9}}{T^2} + \frac{d_{1,10}}{T} + d_{1,11} + \frac{d_{1,12}}{T^2} + \frac{d_{1,13}}{T} + d_{1,14} + \frac{e_{1,3}}{T^2},$$

$$0 == c_{1,1} - \frac{c_{1,3}}{T} + \frac{2\,c_{1,8}}{T^2} + e_{1,1} + e_{1,2} - \frac{e_{1,2}}{T} + 2\,e_{1,3} + \frac{2\,e_{1,3}}{T^2} - \frac{4\,e_{1,3}}{T},$$

$$0 == c_{1,2} + \frac{c_{1,3}}{T} + c_{1,4} + \frac{c_{1,5}}{T} - \frac{2\,c_{1,7}}{T} - \frac{4\,c_{1,8}}{T^2} - \frac{c_{1,10}}{T} - \frac{2\,c_{1,11}}{T^2} + \frac{e_{1,2}}{T} - \frac{4\,e_{1,3}}{T^2} + \frac{4\,e_{1,3}}{T},$$

$$0 == -\frac{c_{1,3}}{T} + c_{1,4} + \frac{2\,c_{1,8}}{T^2} - 2\,c_{1,12} + e_{1,1} + e_{1,2} - \frac{e_{1,2}}{T} + 2\,e_{1,3} + \frac{2\,e_{1,3}}{T^2} - \frac{4\,e_{1,3}}{T} - f_{1,1},$$

$$0 == c_{1,1} - c_{1,4} + 2\,c_{1,12} + f_{1,1},\ 0 == e_{1,1} + f_{1,1},\ 0 == e_{1,2} + f_{1,2},$$

$$0 == c_{1,2} - T\,c_{1,2} - c_{1,3} + \frac{c_{1,3}}{T} + c_{1,4} - T\,c_{1,4} - c_{1,5} + \frac{c_{1,5}}{T} - \frac{2\,c_{1,7}}{T} - \frac{4\,c_{1,8}}{T^2} +$$
$$2\,T\,c_{1,9} + c_{1,10} - \frac{c_{1,10}}{T} - \frac{2\,c_{1,11}}{T^2} + 4\,T\,c_{1,12} + 2\,c_{1,13} + \frac{e_{1,2}}{T} - \frac{4\,e_{1,3}}{T^2} + \frac{4\,e_{1,3}}{T} - T\,f_{1,2},$$

$$0 == T\,c_{1,2} + c_{1,3} + T\,c_{1,4} + c_{1,5} - 2\,T\,c_{1,9} - c_{1,10} - 4\,T\,c_{1,12} - 2\,c_{1,13} + T\,f_{1,2},$$

$$0 == -c_{1,2} + T\,c_{1,2} + c_{1,3} - 2\,c_{1,4} + \frac{c_{1,4}}{T} + T\,c_{1,4} + c_{1,5} - \frac{c_{1,5}}{T} + 4\,c_{1,9} - \frac{2\,c_{1,9}}{T} - 2\,T\,c_{1,9} - c_{1,10} + \frac{c_{1,10}}{T} +$$
$$12\,c_{1,12} + \frac{4\,c_{1,12}}{T^2} - \frac{12\,c_{1,12}}{T} - 4\,T\,c_{1,12} - 2\,c_{1,13} - \frac{2\,c_{1,13}}{T^2} + \frac{4\,c_{1,13}}{T} + T\,d_{1,3} - f_{1,2} + T\,f_{1,2},$$

$$0 == e_{1,3} + f_{1,3},\ 0 == -2\,c_{1,6} + 2\,T\,c_{1,6} + c_{1,7} - 4\,c_{1,9} + \frac{2\,c_{1,9}}{T} + 2\,T\,c_{1,9} + c_{1,10} - \frac{c_{1,10}}{T} -$$
$$6\,c_{1,12} - \frac{2\,c_{1,12}}{T^2} + \frac{6\,c_{1,12}}{T} + 2\,T\,c_{1,12} + c_{1,13} + \frac{c_{1,13}}{T^2} - \frac{2\,c_{1,13}}{T} + T\,d_{1,7} - 2\,f_{1,3} + 2\,T\,f_{1,3},$$

$$0 == d_{1,2} + T\,d_{1,3} + d_{1,4} + T\,d_{1,5} - 2\,T\,d_{1,7} - 4\,T^2\,d_{1,8} - T\,d_{1,10} - 2\,T^2\,d_{1,11} + T\,f_{1,2} + 4\,T\,f_{1,3} - 4\,T^2\,f_{1,3},$$

$$0 == c_{1,6} - T^2\,c_{1,6} + \frac{c_{1,7}}{T} - T\,c_{1,7} - c_{1,8} + \frac{c_{1,8}}{T^2} + c_{1,9} - T^2\,c_{1,9} + \frac{c_{1,10}}{T} - T\,c_{1,10} -$$
$$c_{1,11} + \frac{c_{1,11}}{T^2} + c_{1,12} - T^2\,c_{1,12} + \frac{c_{1,13}}{T} - T\,c_{1,13} - c_{1,14} + \frac{c_{1,14}}{T^2} + \frac{e_{1,3}}{T^2} - T^2\,f_{1,3},$$

$$0 == T^2\,c_{1,6} + T\,c_{1,7} + c_{1,8} + T^2\,c_{1,9} + T\,c_{1,10} + c_{1,11} + T^2\,c_{1,12} + T\,c_{1,13} + c_{1,14} + T^2\,f_{1,3},$$

$$0 == d_{1,6} + T\,d_{1,7} + T^2\,d_{1,8} + d_{1,9} + T\,d_{1,10} + T^2\,d_{1,11} + d_{1,12} + T\,d_{1,13} + T^2\,d_{1,14} + T^2\,f_{1,3},$$

$$0 == c_{1,6} - 2\,T\,c_{1,6} + T^2\,c_{1,6} - c_{1,7} + T\,c_{1,7} + c_{1,8} + 3\,c_{1,9} - \frac{c_{1,9}}{T} - 3\,T\,c_{1,9} + T^2\,c_{1,9} - 2\,c_{1,10} +$$
$$\frac{c_{1,10}}{T} + T\,c_{1,10} + c_{1,11} - \frac{c_{1,11}}{T} + 6\,c_{1,12} + \frac{c_{1,12}}{T^2} - \frac{4\,c_{1,12}}{T} - 4\,T\,c_{1,12} + T^2\,c_{1,12} - 3\,c_{1,13} -$$
$$\frac{c_{1,13}}{T^2} + \frac{3\,c_{1,13}}{T} + T\,c_{1,13} + c_{1,14} + \frac{c_{1,14}}{T^2} - \frac{2\,c_{1,14}}{T} + T^2\,d_{1,8} + f_{1,3} - 2\,T\,f_{1,3} + T^2\,f_{1,3},$$

$$0 == d_{1,1} - T\,d_{1,3} + 2\,T^2\,d_{1,8} + f_{1,1} + f_{1,2} - T\,f_{1,2} + 2\,f_{1,3} - 4\,T\,f_{1,3} + 2\,T^2\,f_{1,3}\}$$

(Alt) In[ ]:=
```
{sol} = Solve[eqns, unknowns]
```

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[∘]= $\left\{\left\{c_{1,1}\to -\frac{c_{1,2}}{2}-\frac{c_{1,5}}{2T},\ c_{1,3}\to -T\,c_{1,2}-c_{1,5},\ c_{1,4}\to 0,\ c_{1,6}\to 0,\ c_{1,8}\to -\frac{1}{2}\times(1-T)\,c_{1,10},\ c_{1,9}\to 0,\ c_{1,11}\to -T\,c_{1,7}-\frac{1}{2}\times(-1+3T)\,c_{1,10},\ c_{1,12}\to 0,\ c_{1,13}\to 0,\ c_{1,14}\to 0,\ d_{1,1}\to \frac{c_{1,2}}{2}+\frac{c_{1,5}}{2T},\ d_{1,2}\to -c_{1,2},\ d_{1,3}\to \frac{c_{1,2}}{T}+\frac{c_{1,5}}{T^2},\ d_{1,4}\to 0,\ d_{1,5}\to -\frac{c_{1,5}}{T^2},\ d_{1,6}\to 0,\ d_{1,7}\to -\frac{c_{1,7}}{T}-\frac{(-1+T)\,c_{1,10}}{T^2},\ d_{1,8}\to -\frac{(1-T)\,c_{1,10}}{2T^3},\ d_{1,9}\to 0,\ d_{1,10}\to -\frac{c_{1,10}}{T^2},\ d_{1,11}\to \frac{c_{1,7}}{T^2}-\frac{(-1-T)\,c_{1,10}}{2T^3},\ d_{1,12}\to 0,\ d_{1,13}\to 0,\ d_{1,14}\to 0,\ e_{1,1}\to -\frac{c_{1,2}}{2}-\frac{c_{1,5}}{2T},\ e_{1,2}\to -\frac{c_{1,10}}{T},\ e_{1,3}\to 0,\ f_{1,1}\to \frac{c_{1,2}}{2}+\frac{c_{1,5}}{2T},\ f_{1,2}\to \frac{c_{1,10}}{T},\ f_{1,3}\to 0\right\}\right\}$

(Alt) In[∘]:= **sol /. (*a*_ → *b*_) ⧴ (*a* = *b*)**

(Alt) Out[∘]= $\left\{-\frac{c_{1,2}}{2}-\frac{c_{1,5}}{2T},\ -T\,c_{1,2}-c_{1,5},\ 0,\ 0,\ -\frac{1}{2}\times(1-T)\,c_{1,10},\ 0,\ -T\,c_{1,7}-\frac{1}{2}\times(-1+3T)\,c_{1,10},\ 0,\ 0,\ 0,\ \frac{c_{1,2}}{2}+\frac{c_{1,5}}{2T},\ -c_{1,2},\ \frac{c_{1,2}}{T}+\frac{c_{1,5}}{T^2},\ 0,\ -\frac{c_{1,5}}{T^2},\ 0,\ -\frac{c_{1,7}}{T}-\frac{(-1+T)\,c_{1,10}}{T^2},\ -\frac{(1-T)\,c_{1,10}}{2T^3},\ 0,\ -\frac{c_{1,10}}{T^2},\ \frac{c_{1,7}}{T^2}-\frac{(-1-T)\,c_{1,10}}{2T^3},\ 0,\ 0,\ 0,\ -\frac{c_{1,2}}{2}-\frac{c_{1,5}}{2T},\ -\frac{c_{1,10}}{T},\ 0,\ \frac{c_{1,2}}{2}+\frac{c_{1,5}}{2T},\ \frac{c_{1,10}}{T},\ 0\right\}$

(Alt) In[∘]:= $\{R_{1,2},\ \overline{R}_{1,2},\ C_1,\ \overline{C}_1\}$

(Alt) Out[∘]= $\Big\{\mathbb{E}_{\{\}\to\{1,2\}}\Big[\sqrt{T},\ (-1+T)\,(p_1-p_2)\,x_2,\ \epsilon\text{Series}\Big[0,\ -\frac{c_{1,2}}{2}+p_1x_1c_{1,2}+p_1x_2\,(-T\,c_{1,2}-c_{1,5})-\frac{c_{1,5}}{2T}+p_2x_2c_{1,5}+p_1^2x_1x_2c_{1,7}+p_1p_2x_1x_2c_{1,10}-\frac{1}{2}\times(1-T)\,p_1^2x_2^2c_{1,10}+p_1p_2x_2^2\left(-T\,c_{1,7}-\frac{1}{2}\times(-1+3T)\,c_{1,10}\right)\Big]\Big],$

$\mathbb{E}_{\{\}\to\{1,2\}}\Big[\frac{1}{\sqrt{T}},\ \left(-1+\frac{1}{T}\right)(p_1-p_2)\,x_2,\ \epsilon\text{Series}\Big[0,\ \frac{c_{1,2}}{2}-p_1x_1c_{1,2}+\frac{c_{1,5}}{2T}-\frac{p_2x_2c_{1,5}}{T^2}+p_1x_2\left(\frac{c_{1,2}}{T}+\frac{c_{1,5}}{T^2}\right)-\frac{p_1p_2x_1x_2c_{1,10}}{T^2}-\frac{(1-T)\,p_1^2x_2^2c_{1,10}}{2T^3}+p_1p_2x_2^2\left(\frac{c_{1,7}}{T^2}-\frac{(-1-T)\,c_{1,10}}{2T^3}\right)+p_1^2x_1x_2\left(-\frac{c_{1,7}}{T}-\frac{(-1+T)\,c_{1,10}}{T^2}\right)\Big]\Big],$

$\mathbb{E}_{\{\}\to\{1\}}\Big[\sqrt{T},\ 0,\ \epsilon\text{Series}\Big[0,\ -\frac{c_{1,2}}{2}-\frac{c_{1,5}}{2T}-\frac{p_1x_1c_{1,10}}{T}\Big]\Big],$

$\mathbb{E}_{\{\}\to\{1\}}\Big[\frac{1}{\sqrt{T}},\ 0,\ \epsilon\text{Series}\Big[0,\ \frac{c_{1,2}}{2}+\frac{c_{1,5}}{2T}+\frac{p_1x_1c_{1,10}}{T}\Big]\Big]\Big\}$

(Alt) In[∘]:= **Cases**$\left[\{R_{1,2},\ \overline{R}_{1,2},\ C_1,\ \overline{C}_1\},\ (c\mid d\mid e\mid f)_{\$k,\_},\ \infty\right]$ **// Union**

(Alt) Out[∘]= $\{c_{1,2},\ c_{1,5},\ c_{1,7},\ c_{1,10}\}$

(Alt) In[∘]:= $\{c_{1,2} = 0, c_{1,5} = 0, c_{1,7} = 0, c_{1,10} = 1\};$
$\{R_{1,2}, \overline{R}_{1,2}, C_1, \overline{C}_1\}$

(Alt) Out[∘]= $\Big\{\mathbb{E}_{\{\}\to\{1,2\}}\Big[\sqrt{T}, (-1+T)(p_1-p_2)x_2, \epsilon\text{Series}\Big[0, p_1 p_2 x_1 x_2 + \frac{1}{2}\times(-1+T) p_1^2 x_2^2 + \frac{1}{2}\times(1-3T) p_1 p_2 x_2^2\Big]\Big],$
$\mathbb{E}_{\{\}\to\{1,2\}}\Big[\frac{1}{\sqrt{T}}, \left(-1+\frac{1}{T}\right)(p_1-p_2)x_2, \epsilon\text{Series}\Big[0, -\frac{(-1+T)p_1^2 x_1 x_2}{T^2} - \frac{p_1 p_2 x_1 x_2}{T^2} - \frac{(1-T)p_1^2 x_2^2}{2T^3} - \frac{(-1-T)p_1 p_2 x_2^2}{2T^3}\Big]\Big],$
$\mathbb{E}_{\{\}\to\{1\}}\Big[\sqrt{T}, 0, \epsilon\text{Series}\Big[0, -\frac{p_1 x_1}{T}\Big]\Big], \mathbb{E}_{\{\}\to\{1\}}\Big[\frac{1}{\sqrt{T}}, 0, \epsilon\text{Series}\Big[0, \frac{p_1 x_1}{T}\Big]\Big]\Big\}$

(Alt) In[∘]:= **RMoves**

(Alt) Out[∘]= {True, True, True, True, True, True, True, True, True}

Some Knot Theory at $k=1

(Alt) In[∘]:= **ZF[Knot[3, 1]]**

(Alt) Out[∘]= $\mathbb{E}_{\{\}\to\{1\}}\Big[\frac{T}{1-T+T^2}, 0, \epsilon\text{Series}\Big[0, \frac{-2+3T-2T^2+T^3}{T-2T^2+3T^3-2T^4+T^5}\Big]\Big]$

(Alt) In[∘]:= **ZF /@ {Knot[6, 1], Knot[9, 46]}**

(Alt) Out[∘]= $\Big\{\mathbb{E}_{\{\}\to\{1\}}\Big[\frac{T}{2-5T+2T^2}, 0, \epsilon\text{Series}\Big[0, \frac{-5+16T-10T^2-4T^3+3T^4}{4T-20T^2+33T^3-20T^4+4T^5}\Big]\Big],$
$\mathbb{E}_{\{\}\to\{1\}}\Big[\frac{T}{2-5T+2T^2}, 0, \epsilon\text{Series}\Big[0, \frac{-7+28T-30T^2+8T^3+T^4}{4T-20T^2+33T^3-20T^4+4T^5}\Big]\Big]\Big\}$

(Alt) In[∘]:=
```
equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};
Length@Union@Echo[ZF /@ equiv]
```

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

» $\left\{\mathbb{E}_{\{\}\to\{1\}}\left[\frac{T^4}{1-4T+9T^2-15T^3+17T^4-15T^5+9T^6-4T^7+T^8}, 0, \epsilon\text{Series}\left[0, \left(-3+20T-69T^2+161T^3-272T^4+328T^5-225T^6-92T^7+548T^8-952T^9+1113T^{10}-980T^{11}+668T^{12}-349T^{13}+135T^{14}-36T^{15}+5T^{16}\right)\Big/\left(T-8T^2+34T^3-102T^4+235T^5-436T^6+669T^7-860T^8+935T^9-860T^{10}+669T^{11}-436T^{12}+235T^{13}-102T^{14}+34T^{15}-8T^{16}+T^{17}\right)\right]\right],\right.$
$\left.\mathbb{E}_{\{\}\to\{1\}}\left[\frac{T^4}{1-4T+9T^2-15T^3+17T^4-15T^5+9T^6-4T^7+T^8}, 0, \epsilon\text{Series}\left[0, \left(-3+20T-69T^2+161T^3-272T^4+328T^5-225T^6-92T^7+548T^8-952T^9+1113T^{10}-980T^{11}+668T^{12}-349T^{13}+135T^{14}-36T^{15}+5T^{16}\right)\Big/\left(T-8T^2+34T^3-102T^4+235T^5-436T^6+669T^7-860T^8+935T^9-860T^{10}+669T^{11}-436T^{12}+235T^{13}-102T^{14}+34T^{15}-8T^{16}+T^{17}\right)\right]\right]\right\}$

(Alt) Out[∘]= 1

(Alt) In[∘]:=
```
equiv =
  {Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};
Length@Union[ZF /@ equiv]
```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

(Alt) Out[∘]= 1

Solving for *R*, *C*, $k = 2

In[∘]:=
```
$k = 2;
{R1,2, C1}
Short[RMoves, 20]
```

Out[∘]=
$$\left\{\mathbb{E}_{\{\}\to\{1,2\}}\left[\sqrt{T},\ (-1+T)\ (p_1-p_2)\ x_2,\ \in\text{Series}\left[0,\ p_1\,p_2\,x_1\,x_2+\frac{1}{2}\times(-1+T)\ p_1^2\,x_2^2+\frac{1}{2}\times(1-3\,T)\ p_1\,p_2\,x_2^2,\right.\right.\right.$$
$$c_{2,1}+p_1\,x_1\,c_{2,2}+p_1\,x_2\,c_{2,3}+p_2\,x_1\,c_{2,4}+p_2\,x_2\,c_{2,5}+p_1^2\,x_1^2\,c_{2,6}+p_1^2\,x_1\,x_2\,c_{2,7}+p_1^2\,x_2^2\,c_{2,8}+p_1\,p_2\,x_1^2\,c_{2,9}+p_1\,p_2\,x_1\,x_2\,c_{2,10}+p_1\,p_2\,x_2^2\,c_{2,11}+p_2^2\,x_1^2\,c_{2,12}+p_2^2\,x_1\,x_2\,c_{2,13}+p_2^2\,x_2^2\,c_{2,14}+p_1^3\,x_1^3\,c_{2,15}+p_1^3\,x_1^2\,x_2\,c_{2,16}+p_1^3\,x_1\,x_2^2\,c_{2,17}+p_1^3\,x_2^3\,c_{2,18}+p_1^2\,p_2\,x_1^3\,c_{2,19}+p_1^2\,p_2\,x_1^2\,x_2\,c_{2,20}+p_1^2\,p_2\,x_1\,x_2^2\,c_{2,21}+p_1^2\,p_2\,x_2^3\,c_{2,22}+p_1\,p_2^2\,x_1^3\,c_{2,23}+p_1\,p_2^2\,x_1^2\,x_2\,c_{2,24}+p_1\,p_2^2\,x_1\,x_2^2\,c_{2,25}+p_1\,p_2^2\,x_2^3\,c_{2,26}+p_2^3\,x_1^3\,c_{2,27}+p_2^3\,x_1^2\,x_2\,c_{2,28}+p_2^3\,x_1\,x_2^2\,c_{2,29}+p_2^3\,x_2^3\,c_{2,30}\Big]\Big],$$
$$\left.\mathbb{E}_{\{\}\to\{1\}}\left[\sqrt{T},\ 0,\ \in\text{Series}\left[0,\ -\frac{p_1\,x_1}{T},\ e_{2,1}+p_1\,x_1\,e_{2,2}+p_1^2\,x_1^2\,e_{2,3}+p_1^3\,x_1^3\,e_{2,4}\right]\right]\right\}$$

Out[∘]//Short=
$$\Big\{(-1+T)\ p_1^2\,p_3\,x_1\,x_2\,x_3-3\,T\,p_1\,p_2\,p_3\,x_1\,x_2\,x_3+\left(-2\,T+4\,T^2\right)\ p_1\,p_2\,p_3\,x_1\,x_3^2+3\,c_{2,1}+2\,p_1\,x_1\,c_{2,2}+p_2\,x_1\,c_{2,4}+p_3\,x_1\,c_{2,4}+T\,p_3\,x_2\,c_{2,4}+p_1\,x_2\,(c_{2,2}-T\,c_{2,2}+c_{2,3}+c_{2,4}-T\,c_{2,4})+2\,T\,p_3\,x_3\,c_{2,5}+\ll 97\gg+$$
$$\frac{1}{2}\,p_1^3\,x_2\,x_3^2\,\left(1-3\,T+3\,T^2-T^3+2\,T^2\,c_{2,16}-4\,T^3\,c_{2,16}+2\,T^4\,c_{2,16}+2\,c_{2,17}-2\,T\,c_{2,17}-2\,T^2\,c_{2,17}+2\,T^3\,c_{2,17}+6\,c_{2,18}-12\,T\,c_{2,18}+6\,T^2\,c_{2,18}+2\,c_{2,21}-6\,T\,c_{2,21}+6\,T^2\,c_{2,21}-2\,T^3\,c_{2,21}+2\,c_{2,25}-6\,T\,c_{2,25}+6\,T^2\,c_{2,25}-2\,T^3\,c_{2,25}+2\,c_{2,29}-6\,T\,c_{2,29}+6\,T^2\,c_{2,29}-2\,T^3\,c_{2,29}\right)+$$
$$\frac{1}{2}\,p_1^2\,p_3\,x_2\,x_3^2\,\left(T-T^3+2\,T\,c_{2,21}-4\,T^2\,c_{2,21}+2\,T^3\,c_{2,21}+4\,T\,c_{2,25}-8\,T^2\,c_{2,25}+4\,T^3\,c_{2,25}+6\,T\,c_{2,29}-12\,T^2\,c_{2,29}+6\,T^3\,c_{2,29}\right)+2\,T^3\,p_3^3\,x_3^3\,c_{2,30}+$$
$$p_2^3\,x_2^3\,\left(T^3\,c_{2,15}+c_{2,30}\right)+p_2^3\,x_2^2\,x_3\,\left(T^3\,c_{2,16}+T\,c_{2,29}-T^2\,c_{2,29}+3\,c_{2,30}-3\,T\,c_{2,30}\right)+$$
$$p_2^3\,x_2\,x_3^2\,\left(T^3\,c_{2,17}+T^2\,c_{2,28}-2\,T^3\,c_{2,28}+T^4\,c_{2,28}+2\,T\,c_{2,29}-4\,T^2\,c_{2,29}+2\,T^3\,c_{2,29}+3\,c_{2,30}-6\,T\,c_{2,30}+3\,T^2\,c_{2,30}\right)+p_1\,p_3^2\,x_3^3\,\left(T^2\,c_{2,26}+3\,T^2\,c_{2,30}-3\,T^3\,c_{2,30}\right)+$$
$$\frac{1}{2}\,p_1^3\,x_3^3\,\left(-1+3\,T-5\,T^2+5\,T^3-2\,T^4+2\,T^3\,c_{2,15}-6\,T^4\,c_{2,15}+6\,T^5\,c_{2,15}-2\,T^6\,c_{2,15}+2\,T^2\,c_{2,16}-6\,T^3\,c_{2,16}+6\,T^4\,c_{2,16}-2\,T^5\,c_{2,16}+2\,T\,c_{2,17}-6\,T^2\,c_{2,17}+6\,T^3\,c_{2,17}-2\,T^4\,c_{2,17}+4\,c_{2,18}-12\,T\,c_{2,18}+12\,T^2\,c_{2,18}-2\,T^3\,c_{2,18}+2\,c_{2,22}-6\,T\,c_{2,22}+6\,T^2\,c_{2,22}-2\,T^3\,c_{2,22}+2\,c_{2,26}-6\,T\,c_{2,26}+6\,T^2\,c_{2,26}-2\,T^3\,c_{2,26}+2\,c_{2,30}-6\,T\,c_{2,30}+6\,T^2\,c_{2,30}-2\,T^3\,c_{2,30}\right)+$$
$$p_2^3\,x_3^3\,\left(T^3\,c_{2,18}+T^3\,c_{2,27}-3\,T^4\,c_{2,27}+3\,T^5\,c_{2,27}-T^6\,c_{2,27}+T^2\,c_{2,28}-3\,T^3\,c_{2,28}+3\,T^4\,c_{2,28}-T^5\,c_{2,28}+T\,c_{2,29}-3\,T^2\,c_{2,29}+3\,T^3\,c_{2,29}-T^4\,c_{2,29}+c_{2,30}-3\,T\,c_{2,30}+3\,T^2\,c_{2,30}-T^3\,c_{2,30}\right)+$$
$$p_1^2\,p_3\,x_3^3\,\left(T^2-4\,T^3+3\,T^4+T\,c_{2,22}-2\,T^2\,c_{2,22}+2\,T^3\,c_{2,22}+2\,T\,c_{2,26}-4\,T^2\,c_{2,26}+2\,T^3\,c_{2,26}+3\,T\,c_{2,30}-6\,T^2\,c_{2,30}+3\,T^3\,c_{2,30}\right)==$$
$$3\,c_{2,1}+\ll 144\gg+p_2^2\,p_3\ll 1\gg\ \left(T^3\,c_{2,22}+3\,T\,c_{2,30}-6\,T^2\,c_{2,\ll 2\gg}+3\,T^3\,c_{2,30}\right),\ \ll 8\gg\Big\}$$

In[∘]:=
```
unknowns = Cases[{R_{1,2}, R̄_{1,2}, C_1, C̄_1}, (c | d | e | f)_{$k,_}, ∞] // Union
```

Out[∘]=
$$\{c_{2,1}, c_{2,2}, c_{2,3}, c_{2,4}, c_{2,5}, c_{2,6}, c_{2,7}, c_{2,8}, c_{2,9}, c_{2,10}, c_{2,11}, c_{2,12}, c_{2,13}, c_{2,14}, c_{2,15}, c_{2,16}, c_{2,17}, c_{2,18}, c_{2,19}, c_{2,20}, c_{2,21}, c_{2,22}, c_{2,23}, c_{2,24}, c_{2,25}, c_{2,26}, c_{2,27}, c_{2,28}, c_{2,29}, c_{2,30}, d_{2,1}, d_{2,2}, d_{2,3}, d_{2,4}, d_{2,5}, d_{2,6}, d_{2,7}, d_{2,8}, d_{2,9}, d_{2,10}, d_{2,11}, d_{2,12}, d_{2,13}, d_{2,14}, d_{2,15}, d_{2,16}, d_{2,17}, d_{2,18}, d_{2,19}, d_{2,20}, d_{2,21}, d_{2,22}, d_{2,23}, d_{2,24}, d_{2,25}, d_{2,26}, d_{2,27}, d_{2,28}, d_{2,29}, d_{2,30}, e_{2,1}, e_{2,2}, e_{2,3}, e_{2,4}, f_{2,1}, f_{2,2}, f_{2,3}, f_{2,4}\}$$

In[∘]:=
```
Short[errors = CCF /@ Cases[RMoves, a_ == b_ :> a - b], 25]
```

Out[∘]//Short=
$$\Big\{\frac{1}{2}\times$$
$$\big(2 p_1 p_2 x_2 x_3 - 2 T p_1 p_2 x_2 x_3 - 2 p_1^2 p_2 x_1 x_2 x_3 + 2 T p_1^2 p_2 x_1 x_2 x_3 + 2 p_1 p_2^2 x_1 x_2 x_3 - 2 T p_1 p_2^2 x_1 x_2 x_3 -$$
$$2 p_1^2 p_3 x_1 x_2 x_3 + 2 T p_1^2 p_3 x_1 x_2 x_3 + 2 p_1^3 x_2^2 x_3 - 4 T p_1^3 x_2^2 x_3 + 2 T^2 p_1^3 x_2^2 x_3 - 6 p_1^2 p_2 x_2^2 x_3 +$$
$$16 T p_1^2 p_2 x_2^2 x_3 - 10 T^2 p_1^2 p_2 x_2^2 x_3 + p_1 p_2^2 x_2^2 x_3 - 6 T p_1 p_2^2 x_2^2 x_3 + 5 T^2 p_1 p_2^2 x_2^2 x_3 + 2 T p_1 p_2 p_3 x_2^2 x_3 -$$
$$2 T^2 p_1 p_2 p_3 x_2^2 x_3 + p_1^2 x_3^2 - 2 T p_1^2 x_3^2 + T^2 p_1^2 x_3^2 + p_1^2 p_2 x_1 x_3^2 - T^2 p_1^2 p_2 x_1 x_3^2 - p_1^2 p_3 x_1 x_3^2 +$$
$$4 T p_1^2 p_3 x_1 x_3^2 - 3 T^2 p_1^2 p_3 x_1 x_3^2 - 2 T p_1 p_2 p_3 x_1 x_3^2 + 2 T^2 p_1 p_2 p_3 x_1 x_3^2 + p_1^3 x_2 x_3^2 -$$
$$3 T p_1^3 x_2 x_3^2 + 3 T^2 p_1^3 x_2 x_3^2 - T^3 p_1^3 x_2 x_3^2 - 4 p_1^2 p_2 x_2 x_3^2 + 14 T p_1^2 p_2 x_2 x_3^2 - 16 T^2 p_1^2 p_2 x_2 x_3^2 +$$
$$6 T^3 p_1^2 p_2 x_2 x_3^2 - 3 T p_1 p_2^2 x_2 x_3^2 + 4 T^2 p_1 p_2^2 x_2 x_3^2 - T^3 p_1 p_2^2 x_2 x_3^2 + T p_1^2 p_3 x_2 x_3^2 - T^3 p_1^2 p_3 x_2 x_3^2 -$$
$$2 T^2 p_1 p_2 p_3 x_2 x_3^2 + 2 T^3 p_1 p_2 p_3 x_2 x_3^2 - p_1^3 x_3^3 + 3 T p_1^3 x_3^3 - 5 T^2 p_1^3 x_3^3 + 5 T^3 p_1^3 x_3^3 - 2 T^4 p_1^3 x_3^3 +$$
$$\ll 974 \gg + 2 T^2 p_3^3 x_1 x_3^2 c_{2,29} - 2 T^3 p_3^3 x_1 x_3^2 c_{2,29} + 2 p_1^3 x_2 x_3^2 c_{2,29} - 6 T p_1^3 x_2 x_3^2 c_{2,29} +$$
$$6 T^2 p_1^3 x_2 x_3^2 c_{2,29} - 2 T^3 p_1^3 x_2 x_3^2 c_{2,29} - 2 p_2^3 x_2 x_3^2 c_{2,29} + 12 T p_2^3 x_2 x_3^2 c_{2,29} - 20 T^2 p_2^3 x_2 x_3^2 c_{2,29} +$$
$$12 T^3 p_2^3 x_2 x_3^2 c_{2,29} - 2 T^4 p_2^3 x_2 x_3^2 c_{2,29} + 6 T p_1^2 p_3 x_2 x_3^2 c_{2,29} - 12 T^2 p_1^2 p_3 x_2 x_3^2 c_{2,29} +$$
$$6 T^3 p_1^2 p_3 x_2 x_3^2 c_{2,29} - 6 T p_2^2 p_3 x_2 x_3^2 c_{2,29} + 18 T^2 p_2^2 p_3 x_2 x_3^2 c_{2,29} - 18 T^3 p_2^2 p_3 x_2 x_3^2 c_{2,29} +$$
$$6 T^4 p_2^2 p_3 x_2 x_3^2 c_{2,29} + 6 T^2 p_1 p_3^2 x_2 x_3^2 c_{2,29} - 6 T^3 p_1 p_3^2 x_2 x_3^2 c_{2,29} - 6 T^2 p_2 p_3^2 x_2 x_3^2 c_{2,29} +$$
$$12 T^3 p_2 p_3^2 x_2 x_3^2 c_{2,29} - 6 T^4 p_2 p_3^2 x_2 x_3^2 c_{2,29} - 2 T^3 p_3^3 x_2 x_3^2 c_{2,29} + 2 T^4 p_3^3 x_2 x_3^2 c_{2,29} +$$
$$2 T p_2^3 x_3^3 c_{2,29} - 6 T^2 p_2^3 x_3^3 c_{2,29} + 6 T^3 p_2^3 x_3^3 c_{2,29} - 2 T^4 p_2^3 x_3^3 c_{2,29} + 6 p_2^3 x_2^2 x_3 c_{2,30} -$$
$$6 T p_2^3 x_2^2 x_3 c_{2,30} + 6 p_2^3 x_2 x_3^2 c_{2,30} - 12 T p_2^3 x_2 x_3^2 c_{2,30} + 6 T^2 p_2^3 x_2 x_3^2 c_{2,30} + 2 p_1^3 x_3^3 c_{2,30} -$$
$$6 T p_1^3 x_3^3 c_{2,30} + 6 T^2 p_1^3 x_3^3 c_{2,30} - 2 T^3 p_1^3 x_3^3 c_{2,30} + 6 T p_1^2 p_3 x_3^3 c_{2,30} - 12 T^2 p_1^2 p_3 x_3^3 c_{2,30} +$$
$$6 T^3 p_1^2 p_3 x_3^3 c_{2,30} - 6 T p_2^2 p_3 x_3^3 c_{2,30} + 12 T^2 p_2^2 p_3 x_3^3 c_{2,30} - 6 T^3 p_2^2 p_3 x_3^3 c_{2,30} + 6 T^2 p_1 p_3^2 x_3^3 c_{2,30} -$$
$$6 T^3 p_1 p_3^2 x_3^3 c_{2,30} - 6 T^2 p_2 p_3^2 x_3^3 c_{2,30} + 6 T^3 p_2 p_3^2 x_3^3 c_{2,30}\big), \ll 7 \gg, \frac{\ll 1 \gg}{2 \ll 1 \gg}\Big\}$$

In[∘]:=
```
Short[#, 10] &[eqns =
   Thread[0 == Union @@ (CoefficientRules[#, {x_1, x_2, x_3, p_1, p_2, p_3}][[;; , 2]] & /@ errors)]]
```

Out[∘]//Short=
$$\Big\{0 == c_{2,4} - T c_{2,4},\ 0 == -c_{2,4} + T c_{2,4},\ \ll 215 \gg,$$
$$0 == 1 + \frac{2}{T} + d_{2,2} + T d_{2,3} + d_{2,4} + T d_{2,5} - 2 T d_{2,7} - 4 T^2 d_{2,8} - T d_{2,10} - 2 T^2 d_{2,11} + 6 T^2 d_{2,17} +$$
$$18 T^3 d_{2,18} + 2 T^2 d_{2,21} + 6 T^3 d_{2,22} + T f_{2,2} + 4 T f_{2,3} - 4 T^2 f_{2,3} + 18 T f_{2,4} - 36 T^2 f_{2,4} + 18 T^3 f_{2,4}\Big\}$$

In[∘]:= {sol} = Solve[eqns, unknowns]

Solve: Equations may not give solutions for all "solve" variables.

Out[∘]=
$$\Big\{\Big\{c_{2,1}\to-\frac{c_{2,2}}{2}-\frac{c_{2,5}}{2\,T},\ c_{2,3}\to-T\,c_{2,2}-c_{2,5},\ c_{2,4}\to0,\ c_{2,6}\to0,\ c_{2,8}\to-\frac{1}{2}\times(1-T)\,c_{2,10},$$
$$c_{2,9}\to0,\ c_{2,11}\to-\frac{1}{2}-T\,c_{2,7}-\frac{1}{2}\times(-1+3\,T)\,c_{2,10},\ c_{2,12}\to0,\ c_{2,13}\to0,\ c_{2,14}\to0,$$
$$c_{2,15}\to0,\ c_{2,17}\to-((-1+T)\,c_{2,16}),\ c_{2,18}\to-\frac{-1+4\,T-3\,T^2}{6\,T},\ c_{2,19}\to0,\ c_{2,20}\to-\frac{1}{2\,T},$$
$$c_{2,21}\to-\frac{1-3\,T}{2\,T},\ c_{2,22}\to-\frac{1-11\,T+16\,T^2}{6\,T}-\left(T-T^2\right)c_{2,16},\ c_{2,23}\to0,\ c_{2,24}\to0,$$
$$c_{2,25}\to-\frac{1}{2},\ c_{2,26}\to\frac{1}{6}\times(-1+7\,T)-T^2\,c_{2,16},\ c_{2,27}\to0,\ c_{2,28}\to0,\ c_{2,29}\to0,\ c_{2,30}\to0,$$
$$d_{2,1}\to\frac{c_{2,2}}{2}+\frac{c_{2,5}}{2\,T},\ d_{2,2}\to-c_{2,2},\ d_{2,3}\to\frac{c_{2,2}}{T}+\frac{c_{2,5}}{T^2},\ d_{2,4}\to0,\ d_{2,5}\to-\frac{c_{2,5}}{T^2},\ d_{2,6}\to0,$$
$$d_{2,7}\to-\frac{1-T}{T^3}-\frac{c_{2,7}}{T}-\frac{(-1+T)\,c_{2,10}}{T^2},\ d_{2,8}\to-\frac{-1+T}{2\,T^4}-\frac{(1-T)\,c_{2,10}}{2\,T^3},\ d_{2,9}\to0,\ d_{2,10}\to\frac{1}{T^3}-\frac{c_{2,10}}{T^2},$$
$$d_{2,11}\to-\frac{1}{2\,T^4}+\frac{c_{2,7}}{T^2}-\frac{(-1-T)\,c_{2,10}}{2\,T^3},\ d_{2,12}\to0,\ d_{2,13}\to0,\ d_{2,14}\to0,\ d_{2,15}\to0,$$
$$d_{2,16}\to-\frac{-1+T}{2\,T^3}-\frac{c_{2,16}}{T},\ d_{2,17}\to-\frac{3-4\,T+T^2}{2\,T^4}-\frac{(-1+T)\,c_{2,16}}{T^2},\ d_{2,18}\to-\frac{-3+4\,T-T^2}{6\,T^5},\ d_{2,19}\to0,$$
$$d_{2,20}\to-\frac{1}{2\,T^3},\ d_{2,21}\to\frac{2}{T^4},\ d_{2,22}\to-\frac{4+T+T^2}{6\,T^5}-\frac{(1-T)\,c_{2,16}}{T^3},\ d_{2,23}\to0,\ d_{2,24}\to0,\ d_{2,25}\to-\frac{1}{2\,T^4},$$
$$d_{2,26}\to-\frac{-1+T}{6\,T^5}+\frac{c_{2,16}}{T^3},\ d_{2,27}\to0,\ d_{2,28}\to0,\ d_{2,29}\to0,\ d_{2,30}\to0,\ e_{2,1}\to-\frac{c_{2,2}}{2}-\frac{c_{2,5}}{2\,T},$$
$$e_{2,2}\to-\frac{c_{2,10}}{T},\ e_{2,3}\to0,\ e_{2,4}\to0,\ f_{2,1}\to\frac{c_{2,2}}{2}+\frac{c_{2,5}}{2\,T},\ f_{2,2}\to-\frac{1}{T^2}+\frac{c_{2,10}}{T},\ f_{2,3}\to0,\ f_{2,4}\to0\Big\}\Big\}$$

In[∘]:= **sol /. (a_ → b_) :→ (a = b)**

Out[∘]=

$$\left\{-\frac{c_{2,2}}{2}-\frac{c_{2,5}}{2\,T},\ -T\,c_{2,2}-c_{2,5},\ 0,\ 0,\ -\frac{1}{2}\times(1-T)\,c_{2,10},\ 0,\ -\frac{1}{2}-T\,c_{2,7}-\frac{1}{2}\times(-1+3\,T)\,c_{2,10},\ 0,\ 0,\ 0,\right.$$
$$0,\ -((-1+T)\,c_{2,16}),\ -\frac{-1+4\,T-3\,T^2}{6\,T},\ 0,\ -\frac{1}{2\,T},\ -\frac{1-3\,T}{2\,T},\ -\frac{1-11\,T+16\,T^2}{6\,T}-\left(T-T^2\right)c_{2,16},$$
$$0,\ 0,\ -\frac{1}{2},\ \frac{1}{6}\times(-1+7\,T)-T^2\,c_{2,16},\ 0,\ 0,\ 0,\ 0,\ \frac{c_{2,2}}{2}+\frac{c_{2,5}}{2\,T},\ -c_{2,2},\ \frac{c_{2,2}}{T}+\frac{c_{2,5}}{T^2},\ 0,$$
$$-\frac{c_{2,5}}{T^2},\ 0,\ -\frac{1-T}{T^3}-\frac{c_{2,7}}{T}-\frac{(-1+T)\,c_{2,10}}{T^2},\ -\frac{-1+T}{2\,T^4}-\frac{(1-T)\,c_{2,10}}{2\,T^3},\ 0,\ \frac{1}{T^3}-\frac{c_{2,10}}{T^2},$$
$$-\frac{1}{2\,T^4}+\frac{c_{2,7}}{T^2}-\frac{(-1-T)\,c_{2,10}}{2\,T^3},\ 0,\ 0,\ 0,\ 0,\ -\frac{-1+T}{2\,T^3}-\frac{c_{2,16}}{T},\ -\frac{3-4\,T+T^2}{2\,T^4}-\frac{(-1+T)\,c_{2,16}}{T^2},$$
$$-\frac{-3+4\,T-T^2}{6\,T^5},\ 0,\ -\frac{1}{2\,T^3},\ \frac{2}{T^4},\ -\frac{4+T+T^2}{6\,T^5}-\frac{(1-T)\,c_{2,16}}{T^3},\ 0,\ 0,\ -\frac{1}{2\,T^4},\ -\frac{-1+T}{6\,T^5}+\frac{c_{2,16}}{T^3},$$
$$\left.0,\ 0,\ 0,\ 0,\ -\frac{c_{2,2}}{2}-\frac{c_{2,5}}{2\,T},\ -\frac{c_{2,10}}{T},\ 0,\ 0,\ \frac{c_{2,2}}{2}+\frac{c_{2,5}}{2\,T},\ -\frac{1}{T^2}+\frac{c_{2,10}}{T},\ 0,\ 0\right\}$$

In[∘]:= **Cases**$\left[\left\{R_{1,2},\ \overline{R}_{1,2},\ C_1,\ \overline{C}_1\right\},\ (c\,|\,d\,|\,e\,|\,f)_{\$k,\_},\ \infty\right]$ **// Union**

Out[∘]= $\{c_{2,2},\ c_{2,5},\ c_{2,7},\ c_{2,10},\ c_{2,16}\}$

In[∘]:= $\{c_{2,2}=0,\ c_{2,5}=0,\ c_{2,7}=0,\ c_{2,10}=0,\ c_{2,16}=0\};$
$\left\{R_{1,2},\ \overline{R}_{1,2},\ C_1,\ \overline{C}_1\right\}$

Out[∘]=

$$\left\{\mathbb{E}_{\{\}\to\{1,2\}}\left[\sqrt{T},\ (-1+T)\,(p_1-p_2)\,x_2,\right.\right.$$
$$\epsilon\text{Series}\left[0,\ p_1\,p_2\,x_1\,x_2+\frac{1}{2}\times(-1+T)\,p_1^2\,x_2^2+\frac{1}{2}\times(1-3\,T)\,p_1\,p_2\,x_2^2,\right.$$
$$-\frac{p_1^2\,p_2\,x_1^2\,x_2}{2\,T}-\frac{1}{2}\,p_1\,p_2\,x_2^2-\frac{(1-3\,T)\,p_1^2\,p_2\,x_1\,x_2^2}{2\,T}-\frac{1}{2}\,p_1\,p_2^2\,x_1\,x_2^2-\frac{\left(-1+4\,T-3\,T^2\right)p_1^3\,x_2^3}{6\,T}-$$
$$\left.\left.\frac{\left(1-11\,T+16\,T^2\right)p_1^2\,p_2\,x_2^3}{6\,T}+\frac{1}{6}\times(-1+7\,T)\,p_1\,p_2^2\,x_2^3\right]\right],\ \mathbb{E}_{\{\}\to\{1,2\}}\left[\frac{1}{\sqrt{T}},\ \left(-1+\frac{1}{T}\right)(p_1-p_2)\,x_2,\right.$$
$$\epsilon\text{Series}\left[0,\ -\frac{(-1+T)\,p_1^2\,x_1\,x_2}{T^2}-\frac{p_1\,p_2\,x_1\,x_2}{T^2}-\frac{(1-T)\,p_1^2\,x_2^2}{2\,T^3}-\frac{(-1-T)\,p_1\,p_2\,x_2^2}{2\,T^3},\ -\frac{(1-T)\,p_1^2\,x_1\,x_2}{T^3}+\right.$$
$$\frac{p_1\,p_2\,x_1\,x_2}{T^3}-\frac{(-1+T)\,p_1^3\,x_1^2\,x_2}{2\,T^3}-\frac{p_1^2\,p_2\,x_1^2\,x_2}{2\,T^3}-\frac{(-1+T)\,p_1^2\,x_2^2}{2\,T^4}-\frac{p_1\,p_2\,x_2^2}{2\,T^4}-\frac{\left(3-4\,T+T^2\right)p_1^3\,x_1\,x_2^2}{2\,T^4}+$$
$$\left.\left.\frac{2\,p_1^2\,p_2\,x_1\,x_2^2}{T^4}-\frac{p_1\,p_2^2\,x_1\,x_2^2}{2\,T^4}-\frac{\left(-3+4\,T-T^2\right)p_1^3\,x_2^3}{6\,T^5}-\frac{\left(4+T+T^2\right)p_1^2\,p_2\,x_2^3}{6\,T^5}-\frac{(-1+T)\,p_1\,p_2^2\,x_2^3}{6\,T^5}\right]\right],$$
$$\left.\mathbb{E}_{\{\}\to\{1\}}\left[\sqrt{T},\ 0,\ \epsilon\text{Series}\left[0,\ -\frac{p_1\,x_1}{T},\ 0\right]\right],\ \mathbb{E}_{\{\}\to\{1\}}\left[\frac{1}{\sqrt{T}},\ 0,\ \epsilon\text{Series}\left[0,\ \frac{p_1\,x_1}{T},\ -\frac{p_1\,x_1}{T^2}\right]\right]\right\}$$

In[∘]:= **RMoves**

Out[∘]= {True, True, True, True, True, True, True, True, True}

Some Knot Theory at $k=2

According to 12XingStats.nb at pensieve://Projects/SL2Invariant/k=2/ the following pair have equal ρ_1 but different ρ_2:

In[∘]:=
```
equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};
res21 = ZF /@ equiv
Simplify[res21[[1]] ≡ res21[[2]]]
```

Out[∘]=

$$\Big\{\mathbb{E}_{\{\}\to\{1\}}\Big[\frac{T^4}{1-4T+9T^2-15T^3+17T^4-15T^5+9T^6-4T^7+T^8},\ 0,$$
$$\epsilon\text{Series}\Big[0,\ \left(-3+20T-69T^2+161T^3-272T^4+328T^5-225T^6-92T^7+548T^8-952T^9+1113T^{10}-980T^{11}+668T^{12}-349T^{13}+135T^{14}-36T^{15}+5T^{16}\right)\Big/\left(T-8T^2+34T^3-102T^4+235T^5-436T^6+669T^7-860T^8+935T^9-860T^{10}+669T^{11}-436T^{12}+235T^{13}-102T^{14}+34T^{15}-8T^{16}+T^{17}\right),$$
$$\left(3-40T+264T^2-1128T^3+3437T^4-7552T^5+10\,297T^6+2304T^7-67\,324T^8+259\,472T^9-699\,066T^{10}+1\,539\,252T^{11}-2\,919\,131T^{12}+4\,882\,760T^{13}-7\,290\,870T^{14}+9\,779\,044T^{15}-11\,816\,854T^{16}+12\,877\,354T^{17}-12\,651\,386T^{18}+11\,191\,592T^{19}-8\,896\,165T^{20}+6\,336\,738T^{21}-4\,030\,390T^{22}+2\,278\,962T^{23}-1\,139\,320T^{24}+500\,046T^{25}-190\,857T^{26}+62\,504T^{27}-17\,215T^{28}+3862T^{29}-668T^{30}+80T^{31}-5T^{32}\right)\Big/$$
$$\left(2T^2-32T^3+264T^4-1496T^5+6516T^6-23\,136T^7+69\,396T^8-180\,024T^9+410\,582T^{10}-833\,112T^{11}+1\,517\,288T^{12}-2\,496\,728T^{13}+3\,730\,618T^{14}-5\,080\,576T^{15}+6\,323\,208T^{16}-7\,205\,208T^{17}+7\,524\,878T^{18}-7\,205\,208T^{19}+6\,323\,208T^{20}-5\,080\,576T^{21}+3\,730\,618T^{22}-2\,496\,728T^{23}+1\,517\,288T^{24}-833\,112T^{25}+410\,582T^{26}-180\,024T^{27}+69\,396T^{28}-23\,136T^{29}+6516T^{30}-1496T^{31}+264T^{32}-32T^{33}+2T^{34}\right)\Big]\Big],$$

$$\mathbb{E}_{\{\}\to\{1\}}\Big[\frac{T^4}{1-4T+9T^2-15T^3+17T^4-15T^5+9T^6-4T^7+T^8},$$
$$0,$$
$$\epsilon\text{Series}\Big[0,\ \left(-3+20T-69T^2+161T^3-272T^4+328T^5-225T^6-92T^7+548T^8-952T^9+1113T^{10}-980T^{11}+668T^{12}-349T^{13}+135T^{14}-36T^{15}+5T^{16}\right)\Big/$$
$$\left(T-8T^2+34T^3-102T^4+235T^5-436T^6+669T^7-860T^8+935T^9-860T^{10}+669T^{11}-436T^{12}+235T^{13}-102T^{14}+34T^{15}-8T^{16}+T^{17}\right),$$
$$\left(3-40T+264T^2-1120T^3+3333T^4-6896T^5+7641T^6+9944T^7-83\,404T^8+283\,088T^9-716\,082T^{10}+1\,514\,140T^{11}-2\,796\,883T^{12}+4\,607\,952T^{13}-6\,839\,214T^{14}+9\,183\,044T^{15}-11\,164\,950T^{16}+12\,281\,354T^{17}-12\,199\,730T^{18}+10\,916\,784T^{19}-8\,773\,917T^{20}+6\,311\,626T^{21}-4\,047\,406T^{22}+2\,302\,578T^{23}-1\,155\,400T^{24}+507\,686T^{25}-193\,513T^{26}+63\,160T^{27}-17\,319T^{28}+3870T^{29}-668T^{30}+80T^{31}-5T^{32}\right)\Big/$$
$$\left(2T^2-32T^3+264T^4-1496T^5+6516T^6-23\,136T^7+69\,396T^8-180\,024T^9+410\,582T^{10}-833\,112T^{11}+1\,517\,288T^{12}-2\,496\,728T^{13}+3\,730\,618T^{14}-5\,080\,576T^{15}+6\,323\,208T^{16}-7\,205\,208T^{17}+7\,524\,878T^{18}-7\,205\,208T^{19}+6\,323\,208T^{20}-5\,080\,576T^{21}+3\,730\,618T^{22}-2\,496\,728T^{23}+1\,517\,288T^{24}-833\,112T^{25}+410\,582T^{26}-180\,024T^{27}+69\,396T^{28}-23\,136T^{29}+6516T^{30}-1496T^{31}+264T^{32}-32T^{33}+2T^{34}\right)\Big]\Big]\Big\}$$

Out[∘]=

$$\frac{(-1+T)\,T\,\left(1-3T+2T^2+5T^3-12T^4+18T^5-12T^6+5T^7+2T^8-3T^9+T^{10}\right)}{\left(1-T+T^2\right)\times\left(1-3T+5T^2-7T^3+5T^4-3T^5+T^6\right)}==0$$

According to 12XingStats.nb at pensieve://Projects/SL2Invariant/k=2/ the following triple have equal ρ_1 but different ρ_2:

In[∘]:=
```
equiv =
  {Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};
Length@Union[res22 = ZF /@ equiv]
```

Out[∘]= 3

Solving for *R*, *C*, $k = 3

In[∘]:=
```
$k = 3;
Short[RMoves, 20]
```

Out[∘]//Short= { ≪1≫ }

In[∘]:= unknowns = Cases$\left[\left\{R_{1,2}, \overline{R}_{1,2}, C_1, \overline{C}_1\right\}, (c \mid d \mid e \mid f)_{\$k,_}, \infty\right]$ // Union

Out[∘]= $\{c_{3,1}, c_{3,2}, c_{3,3}, c_{3,4}, c_{3,5}, c_{3,6}, c_{3,7}, c_{3,8}, c_{3,9}, c_{3,10}, c_{3,11}, c_{3,12}, c_{3,13}, c_{3,14}, c_{3,15}, c_{3,16}, c_{3,17}, c_{3,18}, c_{3,19}, c_{3,20}, c_{3,21}, c_{3,22}, c_{3,23}, c_{3,24}, c_{3,25}, c_{3,26}, c_{3,27}, c_{3,28}, c_{3,29}, c_{3,30}, c_{3,31}, c_{3,32}, c_{3,33}, c_{3,34}, c_{3,35}, c_{3,36}, c_{3,37}, c_{3,38}, c_{3,39}, c_{3,40}, c_{3,41}, c_{3,42}, c_{3,43}, c_{3,44}, c_{3,45}, c_{3,46}, c_{3,47}, c_{3,48}, c_{3,49}, c_{3,50}, c_{3,51}, c_{3,52}, c_{3,53}, c_{3,54}, c_{3,55}, d_{3,1}, d_{3,2}, d_{3,3}, d_{3,4}, d_{3,5}, d_{3,6}, d_{3,7}, d_{3,8}, d_{3,9}, d_{3,10}, d_{3,11}, d_{3,12}, d_{3,13}, d_{3,14}, d_{3,15}, d_{3,16}, d_{3,17}, d_{3,18}, d_{3,19}, d_{3,20}, d_{3,21}, d_{3,22}, d_{3,23}, d_{3,24}, d_{3,25}, d_{3,26}, d_{3,27}, d_{3,28}, d_{3,29}, d_{3,30}, d_{3,31}, d_{3,32}, d_{3,33}, d_{3,34}, d_{3,35}, d_{3,36}, d_{3,37}, d_{3,38}, d_{3,39}, d_{3,40}, d_{3,41}, d_{3,42}, d_{3,43}, d_{3,44}, d_{3,45}, d_{3,46}, d_{3,47}, d_{3,48}, d_{3,49}, d_{3,50}, d_{3,51}, d_{3,52}, d_{3,53}, d_{3,54}, d_{3,55}, e_{3,1}, e_{3,2}, e_{3,3}, e_{3,4}, e_{3,5}, f_{3,1}, f_{3,2}, f_{3,3}, f_{3,4}, f_{3,5}\}$

In[∘]:=
```
Short[errors = CCF /@ Cases[RMoves, a_ == b_ :> a - b], 25]
```

Out[∘]//Short=

$$\{\frac{1}{24\,T}(-24 p_1^2 p_2 x_1 x_2 x_3+24 T p_1^2 p_2 x_1 x_2 x_3+12 p_1^3 p_2 x_1^2 x_2 x_3-12 T p_1^3 p_2 x_1^2 x_2 x_3-48 p_1^2 p_2^2 x_1^2 x_2 x_3+$$

$$48 T p_1^2 p_2^2 x_1^2 x_2 x_3+12 p_1^3 p_3 x_1^2 x_2 x_3-12 T p_1^3 p_3 x_1^2 x_2 x_3+12 p_1^3 x_2^2 x_3-24 T p_1^3 x_2^2 x_3+12 T^2 p_1^3 x_2^2 x_3-$$

$$72 p_1^2 p_2 x_2^2 x_3+240 T p_1^2 p_2 x_2^2 x_3-168 T^2 p_1^2 p_2 x_2^2 x_3-60 T p_1 p_2^2 x_2^2 x_3+60 T^2 p_1 p_2^2 x_2^2 x_3+$$

$$96 p_1^3 p_2 x_1 x_2^2 x_3-240 T p_1^3 p_2 x_1 x_2^2 x_3+144 T^2 p_1^3 p_2 x_1 x_2^2 x_3-60 p_1^2 p_2^2 x_1 x_2^2 x_3+\ll 3662 \gg+$$

$$144 T^3 p_1^4 x_3^4 c_{3,55}-96 T^4 p_1^4 x_3^4 c_{3,55}+24 T^5 p_1^4 x_3^4 c_{3,55}+96 T^2 p_1^3 p_3 x_3^4 c_{3,55}-288 T^3 p_1^3 p_3 x_3^4 c_{3,55}+$$

$$288 T^4 p_1^3 p_3 x_3^4 c_{3,55}-96 T^5 p_1^3 p_3 x_3^4 c_{3,55}-96 T^2 p_2^3 p_3 x_3^4 c_{3,55}+288 T^3 p_2^3 p_3 x_3^4 c_{3,55}-$$

$$288 T^4 p_2^3 p_3 x_3^4 c_{3,55}+96 T^5 p_2^3 p_3 x_3^4 c_{3,55}+144 T^3 p_1^2 p_3^2 x_3^4 c_{3,55}-288 T^4 p_1^2 p_3^2 x_3^4 c_{3,55}+$$

$$144 T^5 p_1^2 p_3^2 x_3^4 c_{3,55}-144 T^3 p_2^2 p_3^2 x_3^4 c_{3,55}+288 T^4 p_2^2 p_3^2 x_3^4 c_{3,55}-144 T^5 p_2^2 p_3^2 x_3^4 c_{3,55}+$$

$$96 T^4 p_1 p_3^3 x_3^4 c_{3,55}-96 T^5 p_1 p_3^3 x_3^4 c_{3,55}-96 T^4 p_2 p_3^3 x_3^4 c_{3,55}+96 T^5 p_2 p_3^3 x_3^4 c_{3,55}),\ \frac{1}{24\,T^6}$$

$$(24 T^3 p_1 p_2 x_1 x_2-24 T^3 p_1^2 p_2 x_1^2 x_2-12 T^2 p_1 p_2 x_2^2+36 T^2 p_1^3 x_1 x_2^2-36 T^3 p_1^3 x_1 x_2^2+60 T^2 p_1^2 p_2 x_1 x_2^2+$$

$$\ll 842 \gg+144 T^{10} p_1^2 p_2^2 x_2^4 d_{3,55}+96 T^9 p_1 p_2^3 x_2^4 d_{3,55}-96 T^{10} p_1 p_2^3 x_2^4 d_{3,55}+24 T^{10} p_2^4 x_2^4 d_{3,55}),$$

$$\frac{-p_1 x_1+T^3 e_{3,1}+T^3 p_1 x_1 e_{3,2}+\ll 6 \gg+T^3 p_1^3 x_1^3 f_{3,4}+T^3 p_1^4 x_1^4 f_{3,5}}{T^3},$$

$$\frac{\ll 3127 \gg+24 \ll 3 \gg f_{\ll 1 \gg}}{24\,T^4},$$

$$\frac{\ll 1 \gg}{12\,T^6},$$

$$\frac{\ll 1 \gg}{12\,T^2},$$

$$\frac{108-216 T+\ll 209 \gg+12 T^7 p_1^4 x_1^4 f_{3,5}}{12\,T^3},$$

$$\frac{1}{12\,T^6}$$

$$(-12 T^2 p_1 x_1+18 T p_1^2 x_1^2+12 T^2 p_1^2 x_1^2+6 p_1^3 x_1^3+\ll 148 \gg+$$

$$12 T^5 p_1 x_1 e_{3,2}+12 T^4 p_1^2 x_1^2 e_{3,3}+12 T^3 p_1^3 x_1^3 e_{3,4}+12 T^2 p_1^4 x_1^4 e_{3,5}),$$

$$\frac{1}{12\,T^6}(12-144 T+312 T^2-180 T^3-36 p_1 x_1+216 T p_1 x_1+108 T^2 p_1 x_1-408 T^3 p_1 x_1+18 p_1^2 x_1^2+$$

$$60 T p_1^2 x_1^2-600 T^2 p_1^2 x_1^2+444 T^3 p_1^2 x_1^2-2 p_1^3 x_1^3-52 T p_1^3 x_1^3+166 T^2 p_1^3 x_1^3-54 T^3 p_1^3 x_1^3+$$

$$\ll 171 \gg+12 T^3 p_1^3 x_1^3 e_{3,4}+288 T^2 e_{3,5}-1152 T^3 e_{3,5}+1728 T^4 e_{3,5}-1152 T^5 e_{3,5}+288 T^6 e_{3,5}-$$

$$1152 T^2 p_1 x_1 e_{3,5}+3456 T^3 p_1 x_1 e_{3,5}-3456 T^4 p_1 x_1 e_{3,5}+1152 T^5 p_1 x_1 e_{3,5}+864 T^2 p_1^2 x_1^2 e_{3,5}-$$

$$1728 T^3 p_1^2 x_1^2 e_{3,5}+864 T^4 p_1^2 x_1^2 e_{3,5}-192 T^2 p_1^3 x_1^3 e_{3,5}+192 T^3 p_1^3 x_1^3 e_{3,5}+12 T^2 p_1^4 x_1^4 e_{3,5})\}$$

In[∘]:=
```
Short[#, 10] &[eqns =
  Thread[0 == Union @@ (CoefficientRules[#, {x1, x2, x3, p1, p2, p3}][[;; , 2]] & /@ errors)]]
```

Out[∘]//Short=

$$\Big\{0 == c_{3,4} - T\,c_{3,4},\ 0 == -c_{3,4} + T\,c_{3,4},\ 0 == T\,c_{3,4} - T^2\,c_{3,4},$$
$$\ll 489 \gg,\ 0 == -1 + \frac{9}{T^3} - \frac{18}{T^2} + \frac{10}{T} + d_{3,1} - T\,d_{3,3} + 2T^2 d_{3,8} - 6T^3 d_{3,18} +$$
$$24T^4 d_{3,35} + f_{3,1} + f_{3,2} - T f_{3,2} + 2 f_{3,3} - 4T f_{3,3} + 2T^2 f_{3,3} + 6 f_{3,4} - 18 T f_{3,4} +$$
$$18T^2 f_{3,4} - 6T^3 f_{3,4} + 24 f_{3,5} - 96 T f_{3,5} + 144 T^2 f_{3,5} - 96 T^3 f_{3,5} + 24 T^4 f_{3,5},$$
$$0 == \frac{3}{2} - \frac{4}{T^3} + \frac{23}{T^2} - \frac{25}{T} + d_{3,6} + T d_{3,7} + T^2 d_{3,8} + d_{3,9} + T d_{3,10} + T^2 d_{3,11} + d_{3,12} + T d_{3,13} + T^2 d_{3,14} -$$
$$3T d_{3,16} - 6T^2 d_{3,17} - 9T^3 d_{3,18} - 2T d_{3,20} - 4T^2 d_{3,21} - 6T^3 d_{3,22} - T d_{3,24} - 2T^2 d_{3,25} -$$
$$3T^3 d_{3,26} + 12T^2 d_{3,33} + 36T^3 d_{3,34} + 72T^4 d_{3,35} + 6T^2 d_{3,38} + 18T^3 d_{3,39} + 36T^4 d_{3,40} + 2T^2 d_{3,43} +$$
$$6T^3 d_{3,44} + 12T^4 d_{3,45} + T^2 f_{3,3} + 9T^2 f_{3,4} - 9T^3 f_{3,4} + 72T^2 f_{3,5} - 144T^3 f_{3,5} + 72T^4 f_{3,5}\Big\}$$

In[∘]:=
```
{sol} = Solve[eqns, unknowns]
```

Solve: Equations may not give solutions for all "solve" variables.

Out[∘]=

$$\Big\{\Big\{c_{3,1} \to -\frac{c_{3,2}}{2} - \frac{c_{3,5}}{2T},\ c_{3,3} \to -T c_{3,2} - c_{3,5},\ c_{3,4} \to 0,\ c_{3,6} \to 0,$$
$$c_{3,8} \to -\frac{1}{2} \times (1-T)\,c_{3,10},\ c_{3,9} \to 0,\ c_{3,11} \to -T c_{3,7} - \frac{1}{2} \times (-1+3T)\,c_{3,10},\ c_{3,12} \to 0,$$
$$c_{3,13} \to 0,\ c_{3,14} \to 0,\ c_{3,15} \to 0,\ c_{3,17} \to -((-1+T)\,c_{3,16}),\ c_{3,18} \to -\frac{1-T}{6T},\ c_{3,19} \to 0,$$
$$c_{3,20} \to 0,\ c_{3,21} \to \frac{1}{2T},\ c_{3,22} \to -\frac{-2+5T}{2T} - (T-T^2)\,c_{3,16},\ c_{3,23} \to 0,\ c_{3,24} \to 0,$$
$$c_{3,25} \to 0,\ c_{3,26} \to \frac{5}{6} - T^2 c_{3,16},\ c_{3,27} \to 0,\ c_{3,28} \to 0,\ c_{3,29} \to 0,\ c_{3,30} \to 0,\ c_{3,31} \to 0,$$
$$c_{3,33} \to -\frac{3}{2} \times (-1+T)\,c_{3,32},\ c_{3,34} \to -((-1+2T-T^2)\,c_{3,32}),\ c_{3,35} \to -\frac{1-12T+27T^2-16T^3}{24T^2},$$
$$c_{3,36} \to 0,\ c_{3,37} \to \frac{1}{6T^2},\ c_{3,38} \to -\frac{-1+3T}{4T^2},\ c_{3,39} \to -\frac{-1+11T-16T^2}{6T^2},$$
$$c_{3,40} \to -\frac{-1+31T-131T^2+125T^3}{24T^2} - (T-2T^2+T^3)\,c_{3,32},\ c_{3,41} \to 0,\ c_{3,42} \to 0,\ c_{3,43} \to \frac{1}{T},$$
$$c_{3,44} \to -\frac{-5+23T}{6T},\ c_{3,45} \to -\frac{-5+69T-142T^2}{24T} + \frac{3}{2} \times (-1+T)\,T^2 c_{3,32},\ c_{3,46} \to 0,\ c_{3,47} \to 0,$$
$$c_{3,48} \to 0,\ c_{3,49} \to \frac{1}{6},\ c_{3,50} \to \frac{1}{24} \times (1-15T) - T^3 c_{3,32},\ c_{3,51} \to 0,\ c_{3,52} \to 0,\ c_{3,53} \to 0,\ c_{3,54} \to 0,$$
$$c_{3,55} \to 0,\ d_{3,1} \to \frac{c_{3,2}}{2} + \frac{c_{3,5}}{2T},\ d_{3,2} \to -c_{3,2},\ d_{3,3} \to \frac{c_{3,2}}{T} + \frac{c_{3,5}}{T^2},\ d_{3,4} \to 0,\ d_{3,5} \to -\frac{c_{3,5}}{T^2},$$
$$d_{3,6} \to 0,\ d_{3,7} \to -\frac{-1+T}{T^4} - \frac{c_{3,7}}{T} - \frac{(-1+T)\,c_{3,10}}{T^2},\ d_{3,8} \to -\frac{1-T}{2T^5} - \frac{(1-T)\,c_{3,10}}{2T^3},\ d_{3,9} \to 0,$$
$$d_{3,10} \to -\frac{1}{T^4} - \frac{c_{3,10}}{T^2},\ d_{3,11} \to \frac{1}{2T^5} + \frac{c_{3,7}}{T^2} - \frac{(-1-T)\,c_{3,10}}{2T^3},\ d_{3,12} \to 0,\ d_{3,13} \to 0,\ d_{3,14} \to 0,\ d_{3,15} \to 0,$$

$d_{3,16} \to -\frac{1-T}{T^4} - \frac{c_{3,16}}{T}$, $d_{3,17} \to -\frac{-7+9T-2T^2}{2T^5} - \frac{(-1+T)\,c_{3,16}}{T^2}$, $d_{3,18} \to -\frac{7-9T+2T^2}{6T^6}$, $d_{3,19} \to 0$,
$d_{3,20} \to \frac{1}{T^4}$, $d_{3,21} \to -\frac{9-T}{2T^5}$, $d_{3,22} \to \frac{3}{2T^6} - \frac{(1-T)\,c_{3,16}}{T^3}$, $d_{3,23} \to 0$, $d_{3,24} \to 0$, $d_{3,25} \to \frac{1}{T^5}$,
$d_{3,26} \to -\frac{1}{3T^6} + \frac{c_{3,16}}{T^3}$, $d_{3,27} \to 0$, $d_{3,28} \to 0$, $d_{3,29} \to 0$, $d_{3,30} \to 0$, $d_{3,31} \to 0$, $d_{3,32} \to -\frac{-1+T}{6T^4} - \frac{c_{3,32}}{T}$,
$d_{3,33} \to -\frac{2-3T+T^2}{T^5} - \frac{3\times(-1+T)\,c_{3,32}}{2T^2}$, $d_{3,34} \to -\frac{-16+27T-12T^2+T^3}{6T^6} - \frac{(1-2T+T^2)\,c_{3,32}}{T^3}$,
$d_{3,35} \to -\frac{16-27T+12T^2-T^3}{24T^7}$, $d_{3,36} \to 0$, $d_{3,37} \to -\frac{1}{6T^4}$, $d_{3,38} \to -\frac{-3+T}{T^5}$,
$d_{3,39} \to \frac{3\times(-3+T)}{2T^6}$, $d_{3,40} \to -\frac{-27+5T-T^2-T^3}{24T^7} - \frac{(-1+2T-T^2)\,c_{3,32}}{T^4}$, $d_{3,41} \to 0$,
$d_{3,42} \to 0$, $d_{3,43} \to -\frac{1}{T^5}$, $d_{3,44} \to \frac{2}{T^6}$, $d_{3,45} \to -\frac{12-T-5T^2}{24T^7} + \frac{3\times(-1+T)\,c_{3,32}}{2T^4}$,
$d_{3,46} \to 0$, $d_{3,47} \to 0$, $d_{3,48} \to 0$, $d_{3,49} \to -\frac{1}{6T^6}$, $d_{3,50} \to -\frac{-1-T}{24T^7} + \frac{c_{3,32}}{T^4}$, $d_{3,51} \to 0$,
$d_{3,52} \to 0$, $d_{3,53} \to 0$, $d_{3,54} \to 0$, $d_{3,55} \to 0$, $e_{3,1} \to -\frac{c_{3,2}}{2} - \frac{c_{3,5}}{2T}$, $e_{3,2} \to -\frac{c_{3,10}}{T}$, $e_{3,3} \to 0$,
$e_{3,4} \to 0$, $e_{3,5} \to 0$, $f_{3,1} \to \frac{c_{3,2}}{2} + \frac{c_{3,5}}{2T}$, $f_{3,2} \to \frac{1}{T^3} + \frac{c_{3,10}}{T}$, $f_{3,3} \to 0$, $f_{3,4} \to 0$, $f_{3,5} \to 0\}\}$

In[ ]:=
```
sol /. (a_ → b_) :→ (a = b)
```

Out[•]=
$$\Big\{-\frac{c_{3,2}}{2}-\frac{c_{3,5}}{2T},\ -T c_{3,2}-c_{3,5},\ 0,\ 0,\ -\frac{1}{2}\times(1-T)\,c_{3,10},\ 0,\ -T c_{3,7}-\frac{1}{2}\times(-1+3T)\,c_{3,10},\ 0,\ 0,\ 0,$$
$$0,\ -((-1+T)\,c_{3,16}),\ -\frac{1-T}{6T},\ 0,\ 0,\ \frac{1}{2T},\ -\frac{-2+5T}{2T}-\left(T-T^2\right)c_{3,16},\ 0,\ 0,\ 0,\ \frac{5}{6}-T^2 c_{3,16},\ 0,$$
$$0,\ 0,\ 0,\ 0,\ -\frac{3}{2}\times(-1+T)\,c_{3,32},\ -\left(\left(-1+2T-T^2\right)c_{3,32}\right),\ -\frac{1-12T+27T^2-16T^3}{24T^2},\ 0,\ \frac{1}{6T^2},$$
$$-\frac{-1+3T}{4T^2},\ -\frac{-1+11T-16T^2}{6T^2},\ -\frac{-1+31T-131T^2+125T^3}{24T^2}-\left(T-2T^2+T^3\right)c_{3,32},\ 0,\ 0,\ \frac{1}{T},$$
$$-\frac{-5+23T}{6T},\ -\frac{-5+69T-142T^2}{24T}+\frac{3}{2}\times(-1+T)\,T^2 c_{3,32},\ 0,\ 0,\ 0,\ \frac{1}{6},\ \frac{1}{24}\times(1-15T)-T^3 c_{3,32},$$
$$0,\ 0,\ 0,\ 0,\ 0,\ \frac{c_{3,2}}{2}+\frac{c_{3,5}}{2T},\ -c_{3,2},\ \frac{c_{3,2}}{T}+\frac{c_{3,5}}{T^2},\ 0,\ -\frac{c_{3,5}}{T^2},\ 0,\ -\frac{-1+T}{T^4}-\frac{c_{3,7}}{T}-\frac{(-1+T)\,c_{3,10}}{T^2},$$
$$-\frac{1-T}{2T^5}-\frac{(1-T)\,c_{3,10}}{2T^3},\ 0,\ -\frac{1}{T^4}-\frac{c_{3,10}}{T^2},\ \frac{1}{2T^5}+\frac{c_{3,7}}{T^2}-\frac{(-1-T)\,c_{3,10}}{2T^3},\ 0,\ 0,\ 0,\ 0,\ -\frac{1-T}{T^4}-\frac{c_{3,16}}{T},$$
$$-\frac{-7+9T-2T^2}{2T^5}-\frac{(-1+T)\,c_{3,16}}{T^2},\ -\frac{7-9T+2T^2}{6T^6},\ 0,\ \frac{1}{T^4},\ -\frac{9-T}{2T^5},\ \frac{3}{2T^6}-\frac{(1-T)\,c_{3,16}}{T^3},\ 0,$$
$$0,\ \frac{1}{T^5},\ -\frac{1}{3T^6}+\frac{c_{3,16}}{T^3},\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{-1+T}{6T^4}-\frac{c_{3,32}}{T},\ -\frac{2-3T+T^2}{T^5}-\frac{3\times(-1+T)\,c_{3,32}}{2T^2},$$
$$-\frac{-16+27T-12T^2+T^3}{6T^6}-\frac{\left(1-2T+T^2\right)c_{3,32}}{T^3},\ -\frac{16-27T+12T^2-T^3}{24T^7},\ 0,\ -\frac{1}{6T^4},$$
$$-\frac{-3+T}{T^5},\ \frac{3\times(-3+T)}{2T^6},\ -\frac{-27+5T-T^2-T^3}{24T^7}-\frac{\left(-1+2T-T^2\right)c_{3,32}}{T^4},\ 0,\ 0,\ -\frac{1}{T^5},$$
$$\frac{2}{T^6},\ -\frac{12-T-5T^2}{24T^7}+\frac{3\times(-1+T)\,c_{3,32}}{2T^4},\ 0,\ 0,\ 0,\ -\frac{1}{6T^6},\ -\frac{-1-T}{24T^7}+\frac{c_{3,32}}{T^4},\ 0,$$
$$0,\ 0,\ 0,\ 0,\ -\frac{c_{3,2}}{2}-\frac{c_{3,5}}{2T},\ -\frac{c_{3,10}}{T},\ 0,\ 0,\ 0,\ \frac{c_{3,2}}{2}+\frac{c_{3,5}}{2T},\ \frac{1}{T^3}+\frac{c_{3,10}}{T},\ 0,\ 0,\ 0\Big\}$$

In[•]:=
```
Cases[{R_{1,2}, R̄_{1,2}, C_1, C̄_1}, (c | d | e | f)_{$k,_}, ∞] // Union
```

Out[•]= $\{c_{3,2}, c_{3,5}, c_{3,7}, c_{3,10}, c_{3,16}, c_{3,32}\}$

In[•]:=
```
{c_{3,2} = 0, c_{3,5} = 0, c_{3,7} = 0, c_{3,10} = 0, c_{3,16} = 0, c_{3,32} = 1};
{R_{1,2}, R̄_{1,2}, C_1, C̄_1}
```

Out[•]=
$$\Big\{\mathbb{E}_{\{\}\to\{1,2\}}\Big[\sqrt{T},\ (-1+T)\,(p_1-p_2)\,x_2,$$
$$\epsilon\text{Series}\Big[0,\ p_1 p_2 x_1 x_2+\frac{1}{2}\times(-1+T)\,p_1^2 x_2^2+\frac{1}{2}\times(1-3T)\,p_1 p_2 x_2^2,$$
$$-\frac{p_1^2 p_2 x_1^2 x_2}{2T}-\frac{1}{2}p_1 p_2 x_2^2-\frac{(1-3T)\,p_1^2 p_2 x_1 x_2^2}{2T}-\frac{1}{2}p_1 p_2^2 x_1 x_2^2-$$
$$\frac{\left(-1+4T-3T^2\right)p_1^3 x_2^3}{6T}-\frac{\left(1-11T+16T^2\right)p_1^2 p_2 x_2^3}{6T}+\frac{1}{6}\times(-1+7T)\,p_1 p_2^2 x_2^3,$$

$$p_1^4 x_1^3 x_2 + \frac{p_1^3 p_2 x_1^3 x_2}{6\,T^2} + \frac{p_1^2 p_2 x_1 x_2^2}{2\,T} - \frac{3}{2} \times (-1+T)\, p_1^4 x_1^2 x_2^2 - \frac{(-1+3\,T)\, p_1^3 p_2 x_1^2 x_2^2}{4\,T^2} +$$

$$\frac{p_1^2 p_2^2 x_1^2 x_2^2}{T} - \frac{(1-T)\, p_1^3 x_2^3}{6\,T} - \frac{(-2+5\,T)\, p_1^2 p_2 x_2^3}{2\,T} + \frac{5}{6} p_1 p_2^2 x_2^3 + \left(1-2\,T+T^2\right) p_1^4 x_1 x_2^3 -$$

$$\frac{\left(-1+11\,T-16\,T^2\right) p_1^3 p_2 x_1 x_2^3}{6\,T^2} - \frac{(-5+23\,T)\, p_1^2 p_2^2 x_1 x_2^3}{6\,T} + \frac{1}{6} p_1 p_2^3 x_1 x_2^3 -$$

$$\frac{\left(1-12\,T+27\,T^2-16\,T^3\right) p_1^4 x_2^4}{24\,T^2} + \left(-T+2\,T^2-T^3-\frac{-1+31\,T-131\,T^2+125\,T^3}{24\,T^2}\right) p_1^3 p_2 x_2^4 +$$

$$\left(\frac{3}{2} \times (-1+T)\, T^2 - \frac{-5+69\,T-142\,T^2}{24\,T}\right) p_1^2 p_2^2 x_2^4 + \left(\frac{1}{24} \times (1-15\,T) - T^3\right) p_1 p_2^3 x_2^4 \Big]\Big],$$

$$\mathbb{E}_{\{\}\to\{1,2\}}\Big[\frac{1}{\sqrt{T}},\ \left(-1+\frac{1}{T}\right)(p_1-p_2)\, x_2,\ \epsilon\text{Series}\Big[0,\ -\frac{(-1+T)\, p_1^2 x_1 x_2}{T^2} - \frac{p_1 p_2 x_1 x_2}{T^2} -$$

$$\frac{(1-T)\, p_1^2 x_2^2}{2\,T^3} - \frac{(-1-T)\, p_1 p_2 x_2^2}{2\,T^3},\ -\frac{(1-T)\, p_1^2 x_1 x_2}{T^3} + \frac{p_1 p_2 x_1 x_2}{T^3} - \frac{(-1+T)\, p_1^3 x_1^2 x_2}{2\,T^3} -$$

$$\frac{p_1^2 p_2 x_1^2 x_2}{2\,T^3} - \frac{(-1+T)\, p_1^2 x_2^2}{2\,T^4} - \frac{p_1 p_2 x_2^2}{2\,T^4} - \frac{\left(3-4\,T+T^2\right) p_1^3 x_1 x_2^2}{2\,T^4} + \frac{2\, p_1^2 p_2 x_1 x_2^2}{T^4} -$$

$$\frac{p_1 p_2^2 x_1 x_2^2}{2\,T^4} - \frac{\left(-3+4\,T-T^2\right) p_1^3 x_2^3}{6\,T^5} - \frac{\left(4+T+T^2\right) p_1^2 p_2 x_2^3}{6\,T^5} - \frac{(-1+T)\, p_1 p_2^2 x_2^3}{6\,T^5},$$

$$-\frac{(-1+T)\, p_1^2 x_1 x_2}{T^4} - \frac{p_1 p_2 x_1 x_2}{T^4} - \frac{(1-T)\, p_1^3 x_1^2 x_2}{T^4} + \frac{p_1^2 p_2 x_1^2 x_2}{T^4} + \left(-\frac{-1+T}{6\,T^4} - \frac{1}{T}\right) p_1^4 x_1^3 x_2 -$$

$$\frac{p_1^3 p_2 x_1^3 x_2}{6\,T^4} - \frac{(1-T)\, p_1^2 x_2^2}{2\,T^5} + \frac{p_1 p_2 x_2^2}{2\,T^5} - \frac{\left(-7+9\,T-2\,T^2\right) p_1^3 x_1 x_2^2}{2\,T^5} -$$

$$\frac{(9-T)\, p_1^2 p_2 x_1 x_2^2}{2\,T^5} + \frac{p_1 p_2^2 x_1 x_2^2}{T^5} + \left(-\frac{3 \times (-1+T)}{2\,T^2} - \frac{2-3\,T+T^2}{T^5}\right) p_1^4 x_1^2 x_2^2 -$$

$$\frac{(-3+T)\, p_1^3 p_2 x_1^2 x_2^2}{T^5} - \frac{p_1^2 p_2^2 x_1^2 x_2^2}{T^5} - \frac{\left(7-9\,T+2\,T^2\right) p_1^3 x_2^3}{6\,T^6} + \frac{3\, p_1^2 p_2 x_2^3}{2\,T^6} - \frac{p_1 p_2^2 x_2^3}{3\,T^6} +$$

$$\left(-\frac{1-2\,T+T^2}{T^3} - \frac{-16+27\,T-12\,T^2+T^3}{6\,T^6}\right) p_1^4 x_1 x_2^3 + \frac{3 \times (-3+T)\, p_1^3 p_2 x_1 x_2^3}{2\,T^6} + \frac{2\, p_1^2 p_2^2 x_1 x_2^3}{T^6} -$$

$$\frac{p_1 p_2^3 x_1 x_2^3}{6\,T^6} - \frac{\left(16-27\,T+12\,T^2-T^3\right) p_1^4 x_2^4}{24\,T^7} + \left(-\frac{-1+2\,T-T^2}{T^4} - \frac{-27+5\,T-T^2-T^3}{24\,T^7}\right) p_1^3 p_2 x_2^4 +$$

$$\left(\frac{3 \times (-1+T)}{2\,T^4} - \frac{12-T-5\,T^2}{24\,T^7}\right) p_1^2 p_2^2 x_2^4 + \left(-\frac{-1-T}{24\,T^7} + \frac{1}{T^4}\right) p_1 p_2^3 x_2^4 \Big]\Big],$$

$$\mathbb{E}_{\{\}\to\{1\}}\Big[\sqrt{T},\ 0,\ \epsilon\text{Series}\Big[0,\ -\frac{p_1 x_1}{T},\ 0,\ 0\Big]\Big],$$

$$\mathbb{E}_{\{\}\to\{1\}}\Big[\frac{1}{\sqrt{T}},\ 0,\ \epsilon\text{Series}\Big[0,\ \frac{p_1 x_1}{T},\ -\frac{p_1 x_1}{T^2},\ \frac{p_1 x_1}{T^3}\Big]\Big]\Big\}$$

In[∘]:= **RMoves**

Out[ ]= {True, True, True, True, True, True, True, True, True}