Pensieve header: Implementing the Jones polynomial.

Based on http://drorbn.net/syd3

```
Setting Knots to contain all from Knot[3,1] to Knot[10,165] as well as GST48...
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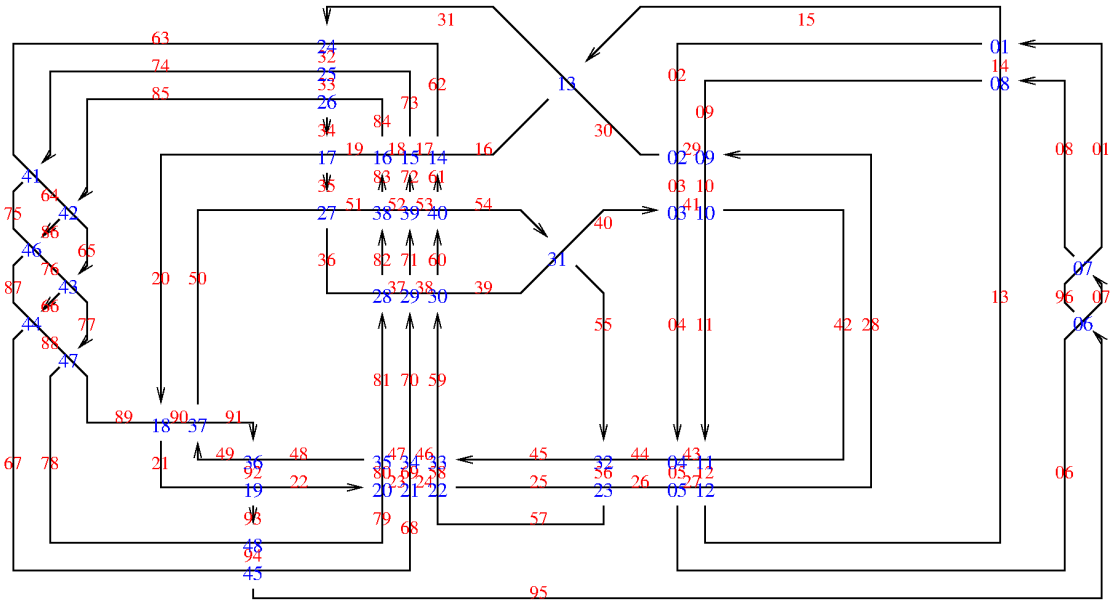
Out[•]=



In[•]:=
```
1 + 1
```
Out[•]= 2

In[•]:=
```
2 + 2
```
Out[•]= 4

In[•]:=
```
7!
```
Out[•]= 5040

In[•]:=
```
Integrate[1 / (1 + x^2), x]
```
Out[•]= ArcTan[x]

In[•]:=
```
K = PD[X[1, 5, 2, 4], X[5, 3, 6, 2], X[3, 1, 4, 6]]
```
Out[•]= PD[X[1, 5, 2, 4], X[5, 3, 6, 2], X[3, 1, 4, 6]]

In[•]:=
```
prod = 1;
Do[prod = prod * k, {k, 1, 7}];
prod
```
Out[•]= 5040

In[•]:=
```
prod = 1;
Do[prod = prod * k, {k, 1, 28}];
prod
```
Out[•]= 304 888 344 611 713 860 501 504 000 000

```
In[•]:= {10, 9, 180, 26} /. 9 → 23
Out[•]= {10, 23, 180, 26}

In[•]:= {10, 9, 180, 26} /. x_ → x^2
Out[•]= {100, 81, 32400, 676}

In[•]:= {10, 9, 180, 26, Tembel} /. x_ → x^2
Out[•]= {100, 81, 32400, 676, Tembel^2}

In[•]:= {10, 9, 180, 26, Tembel + Silly} /. x_ → x^2
Out[•]= {100, 81, 32400, 676, (Silly + Tembel)^2}

In[•]:= Expand[{10, 9, 180, 26, Tembel + Silly} /. x_ → x^2]
Out[•]= {100, 81, 32400, 676, Silly^2 + 2 Silly Tembel + Tembel^2}

In[•]:= K
Out[•]= PD[X[1, 5, 2, 4], X[5, 3, 6, 2], X[3, 1, 4, 6]]

In[•]:= t1 = K /. X[i_, j_, k_, l_] → A * p[i, j] * p[k, l] + B * p[i, l] p[j, k]
Out[•]= PD[A p[1, 5] p[2, 4] + B p[1, 4] p[5, 2],
  B p[3, 6] p[5, 2] + A p[5, 3] p[6, 2], B p[1, 4] p[3, 6] + A p[3, 1] p[4, 6]]

In[•]:= t1

In[•]:= Expand[(a + b) (a - b)]
Out[•]= a^2 - b^2

In[•]:= Factor[a^2 - b^2]
Out[•]= (a - b) (a + b)

In[•]:= Expand[t1]

In[•]:= Factor[a^2 - b^2] // FullForm

In[•]:= t1 /. PD → Times
Out[•]= (B p[1, 4] p[3, 6] + A p[3, 1] p[4, 6])
   (A p[1, 5] p[2, 4] + B p[1, 4] p[5, 2]) (B p[3, 6] p[5, 2] + A p[5, 3] p[6, 2])

In[•]:= t2 = Expand[t1 /. PD → Times]
Out[•]= A B^2 p[1, 4] p[1, 5] p[2, 4] p[3, 6]^2 p[5, 2] + A^2 B p[1, 5] p[2, 4] p[3, 1] p[3, 6] p[4, 6] p[5, 2] +
  B^3 p[1, 4]^2 p[3, 6]^2 p[5, 2]^2 + A B^2 p[1, 4] p[3, 1] p[3, 6] p[4, 6] p[5, 2]^2 +
  A^2 B p[1, 4] p[1, 5] p[2, 4] p[3, 6] p[5, 3] p[6, 2] +
  A^3 p[1, 5] p[2, 4] p[3, 1] p[4, 6] p[5, 3] p[6, 2] +
  A B^2 p[1, 4]^2 p[3, 6] p[5, 2] p[5, 3] p[6, 2] + A^2 B p[1, 4] p[3, 1] p[4, 6] p[5, 2] p[5, 3] p[6, 2]

In[•]:= {7, 1} /. {{0, n_} → n, {k_, n_} → {k - 1, n * k}}
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In[•]:= {7, 1} //. {{0, n_} → n, {k_, n_} → {k - 1, n * k}}

In[•]:= t3 = t2 //. p[i_, j_] p[j_, k_] → p[i, k]

In[•]:= t4 = t3 /. {p[i_, i_] → d, p[i_, j_]^2 → d}

In[•]:= t5 = Expand[t4 /. {B → 1 / A, d → -A^2 - 1 / A^2}]

In[•]:= Knots

In[•]:= Knot[3, 1] /. Knots

In[•]:= Knot[10, 165] /. Knots

In[•]:= K = Knot[10, 73] /. Knots;
        t1 = K /. X[i_, j_, k_, l_] → A * p[i, j] * p[k, l] + B * p[i, l] p[j, k];
        t2 = Expand[t1 /. PD → Times];
        t3 = t2 //. {p[i_, j_] p[j_, k_] → p[i, k], p[i_, j_] p[k_, j_] → p[i, k]};
        t4 = t3 /. {p[i_, i_] → d, p[i_, j_]^2 → d};
        Expand[t4 /. {B → 1 / A, d → -A^2 - 1 / A^2}]

In[•]:= K = GST48 /. Knots

In[•]:= Length[K]

In[•]:= K = GST48 /. Knots;
        t1 = K /. X[i_, j_, k_, l_] → A * p[i, j] * p[k, l] + B * p[i, l] p[j, k];
        t2 = Expand[t1 /. PD → Times];
        t3 = t2 //. {p[i_, j_] p[j_, k_] → p[i, k],
            p[i_, j_] p[k_, j_] → p[i, k], p[j_, i_] p[j_, k_] → p[i, k]};
        t4 = t3 /. {p[i_, i_] → d, p[i_, j_]^2 → d};
        Expand[t4 /. {B → 1 / A, d → -A^2 - 1 / A^2}]

In[•]:= K = Knot[8, 17] /. Knots;
        SetAttributes[p, Orderless];
        t1 = K /. X[i_, j_, k_, l_] → A * p[i, j] * p[k, l] + B * p[i, l] p[j, k];
        t2 = Expand[t1 /. PD → Times];
        t3 = t2 //. {p[i_, j_] p[j_, k_] → p[i, k]};
        t4 = t3 /. {p[i_, i_] → d, p[i_, j_]^2 → d};
        Expand[t4 /. {B → 1 / A, d → -A^2 - 1 / A^2}]
```