Pensieve header: Solution of HW5 Q1.

Question 1.

m is the number of vertices, *l* the number of edges.

(Alt) In[∘]:=
$$\text{Module}\left[\{l, m\}, \text{Log}\left[\text{Sum}\left[l = 3m/2;\ \frac{(3m)!\,\epsilon^m}{2^l\, l!\, 6^m\, m!},\ \{m, 0, 30, 2\}\right] + O[\epsilon]^{31}\right]\right]$$

(Alt) Out[∘]=
$$\frac{5\epsilon^2}{24} + \frac{5\epsilon^4}{16} + \frac{1105\epsilon^6}{1152} + \frac{565\epsilon^8}{128} + \frac{82825\epsilon^{10}}{3072} + \frac{19675\epsilon^{12}}{96} + \frac{1282031525\epsilon^{14}}{688128} + \frac{80727925\epsilon^{16}}{4096} + \frac{1683480621875\epsilon^{18}}{7077888} + \frac{13209845125\epsilon^{20}}{4096} + \frac{2239646759308375\epsilon^{22}}{46137344} + \frac{19739117098375\epsilon^{24}}{24576} + \frac{6320791709083309375\epsilon^{26}}{436207616} + \frac{32468078556378125\epsilon^{28}}{114688} + \frac{38362676768845045751875\epsilon^{30}}{6442450944} + O[\epsilon]^{31}$$

In[∘]:=
$$\text{Sum}\left[l = 2m;\ \frac{(4m)!\,\epsilon^m}{2^l\, l!\, 24^m\, m!},\ \{m, 0, 15\}\right] + O[\epsilon]^{16}$$

Out[∘]=
$$1 + \frac{\epsilon}{8} + \frac{35\epsilon^2}{384} + \frac{385\epsilon^3}{3072} + \frac{25025\epsilon^4}{98304} + \frac{1616615\epsilon^5}{2359296} + \frac{260275015\epsilon^6}{113246208} + \frac{929553625\epsilon^7}{100663296} + \frac{835668708875\epsilon^8}{19327352832} + \frac{321732452916875\epsilon^9}{1391569403904} + \frac{30950661970603375\epsilon^{10}}{22265110462464} + \frac{4960547004924886375\epsilon^{11}}{534362651099136} + \frac{1165728546157348298125\epsilon^{12}}{17099604835172352} + \frac{74696298380697779410625\epsilon^{13}}{136796838681378816} + \frac{31105672825676289568853125\epsilon^{14}}{6566248256706183168} + \frac{6973891847516624121336870625\epsilon^{15}}{157589958160948396032} + O[\epsilon]^{16}$$

(Alt) In[∘]:=
$$\text{Module}\left[\{l, m\}, \text{Log}\left[\text{Sum}\left[l = 2m;\ \frac{(4m)!\,\epsilon^m}{2^l\, l!\, 24^m\, m!},\ \{m, 0, 15\}\right] + O[\epsilon]^{16}\right]\right]$$

(Alt) Out[∘]=
$$\frac{\epsilon}{8} + \frac{\epsilon^2}{12} + \frac{11\epsilon^3}{96} + \frac{17\epsilon^4}{72} + \frac{619\epsilon^5}{960} + \frac{709\epsilon^6}{324} + \frac{858437\epsilon^7}{96768} + \frac{54193\epsilon^8}{1296} + \frac{18639247\epsilon^9}{82944} + \frac{2197187\epsilon^{10}}{1620} + \frac{33152545703\epsilon^{11}}{3649536} + \frac{1169890097\epsilon^{12}}{17496} + \frac{41657327595361\epsilon^{13}}{77635584} + \frac{31722037141\epsilon^{14}}{6804} + \frac{6944870083473751\epsilon^{15}}{159252480} + O[\epsilon]^{16}$$

(Alt) In[∘]:=
$$\frac{1}{2^3}$$

(Alt) Out[∘]=
$$\frac{1}{8}$$

(Alt) In[ ]:= $\frac{1}{2\times 4\,!}+\frac{1}{2^4}$

(Alt) Out[ ]= $\frac{1}{12}$

See also https://oeis.org/A226259

Question 2.

$g\left(m_k^{ij}\right)=\text{Exp}\left[\left(\alpha_i+\alpha_j\right)a_k+\left(e^{-\alpha_j}\,\xi_i+\xi_j\right)x_k\right].$