

pdf

Pensieve header: The WG Algebra with testing.

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```
In[ ]:= Once[<< KnotTheory`];
HL[ $\mathcal{E}$ _] := Style[ $\mathcal{E}$ , Background  $\rightarrow$  If[TrueQ@ $\mathcal{E}$ , ■, ■]];
 $\chi$ [cond_] := If[TrueQ[cond], 1, 0];
```

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Loading **KnotTheory`** version of February 2, 2020, 10:53:45.2097.
Read more at <http://katlas.org/wiki/KnotTheory>.

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```
In[ ]:= DeclareGroup[ $S_k$ ] := Module[{ $\alpha$ ,  $\beta$ , e,  $\gamma S$ },
Clear[G, n, g,  $\iota$ , m, inv];
G = PermutationCycles /@ (Permutations@Range@k);
n = Length[G];
Do[g[ $\alpha$ ] = e = G[ $\alpha$ ];  $\iota$ [e] =  $\alpha$ , { $\alpha$ , n }];
m[] =  $\iota$ [Cycles[{ }]];
Do[m[ $\alpha$ ,  $\beta$ ] =  $\iota$ [g[ $\alpha$ ] ~ PermutationProduct ~ g[ $\beta$ ]], { $\alpha$ , n}, { $\beta$ , n }];
m[ $\alpha$ _] :=  $\alpha$ ; m[ $\alpha$ _,  $\beta$ _,  $\gamma S$ _] := m[m[ $\alpha$ ,  $\beta$ ],  $\gamma S$ ];
Do[inv[ $\alpha$ ] =  $\iota$ [InversePermutation[g[ $\alpha$ ]]], { $\alpha$ , n}]
]
```

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```
In[ ]:= DeclareGroup[ $S_3$ ];
Table[m[i, j], {i, n}, {j, n}] // MatrixForm
```

Out[]//MatrixForm=
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$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 3 & 6 & 5 \\ 3 & 5 & 1 & 6 & 2 & 4 \\ 4 & 6 & 2 & 5 & 1 & 3 \\ 5 & 3 & 6 & 1 & 4 & 2 \\ 6 & 4 & 5 & 2 & 3 & 1 \end{pmatrix}$$

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```
In[ ]:= Basis[] = {1};
Basis[i_, is_] := Flatten@Table[Wi[ $\alpha$ ,  $\beta$ ] Basis[is], { $\alpha$ , n}, { $\beta$ , n}]
```

```
In[ ]:= Basis[1, 3]
```

Out[]:=

$$\{W_1[1, 1] W_3[1, 1], W_1[1, 1] W_3[1, 2], W_1[1, 1] W_3[1, 3], W_1[1, 1] W_3[1, 4], W_1[1, 1] W_3[1, 5], W_1[1, 1] W_3[1, 6], W_1[1, 1] W_3[2, 1], W_1[1, 1] W_3[2, 2], W_1[1, 1] W_3[2, 3], \dots 1278 \dots, W_1[6, 6] W_3[5, 4], W_1[6, 6] W_3[5, 5], W_1[6, 6] W_3[5, 6], W_1[6, 6] W_3[6, 1], W_1[6, 6] W_3[6, 2], W_1[6, 6] W_3[6, 3], W_1[6, 6] W_3[6, 4], W_1[6, 6] W_3[6, 5], W_1[6, 6] W_3[6, 6]\}$$

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```

mi,j→k[ε-] :=
  Expand[ε / . Wi[α-, β-] Wj[γ-, δ-] ⇒ χ[m[α, β] = m[β, γ]] Wk[α, m[β, δ]];
ηi[ε-] := Expand[ε Sum[Wi[α, m[]], {α, n}]];
Δi→j, k[ε-] := Expand[ε / . Wi[α-, β-] ⇒ Sum[Wj[γ, β] Wk[m[α, inv[γ]], β], {γ, n}]];
εi[ε-] := Expand[ε / . Wi[α-, β-] ⇒ χ[α = m[]]];
Si[ε-] := Expand[ε / . Wi[α-, β-] ⇒ Wi[m[inv[β]], inv[α], β, inv[β]]];
    
```

In[]:= **b = Basis[1, 2, 3]; HL[(b // m_{1,2→1} // m_{1,3→1}) == (b // m_{2,3→2} // m_{1,2→1)]}**

Out[]:= **True**

In[]:= **b = Basis[2]; HL[(b // η₁ // m_{1,2→2}) == b]**

Out[]:= **True**

In[]:= **b = Basis[1]; HL[(b // η₂ // m_{1,2→1}) == b]**

Out[]:= **True**

In[]:= **b = Basis[1]; HL[(b // Δ_{1→1,2} // Δ_{2→2,3}) == (b // Δ_{1→1,3} // Δ_{1→1,2})]**

Out[]:= **True**

In[]:= **b = Basis[1, 3]; HL[(b // Δ_{1→1,2} // Δ_{3→3,4} // m_{1,3→1} // m_{2,4→2}) == (b // m_{1,3→1} // Δ_{1→1,2})]**

Out[]:= **True**

In[]:= **b = Basis[1, 3]; HL[(b // Δ_{1→1,2} // S₂ // m_{1,2→1}) == (b // ε₁ // η₁)]**

Out[]:= **True**

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```

Ri,j := Sum[Wi[α, 1] Wj[β, α], {α, n}, {β, n}];
R̄i,j := Sum[Wi[α, 1] Wj[β, inv@α], {α, n}, {β, n}];
    
```

In[]:= **HL[(R_{1,2} R_{4,3} R_{5,6} // m_{1,4→1} // m_{2,5→2} // m_{3,6→3}) == (R_{2,3} R_{1,4} R_{5,6} // m_{1,5→1} // m_{2,6→2} // m_{3,4→3})]**

Out[]:= **True**

In[]:= **HL[(R_{1,2} R̄_{3,4} // m_{1,3→1} // m_{2,4→2}) == (1 // η₁ // η₂)]**

Out[]:= **True**

In[]:= **HL[(R_{1,2} R̄_{3,4} // m_{1,3→1} // m_{4,2→2}) == (1 // η₁ // η₂)]**

Out[]:= **True**

In[]:= **HL[(R_{1,3} // Δ_{1→1,2}) == (R_{2,3} R_{1,4} // m_{3,4→3})]**

Out[]:= **True**

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```
In[ ]:= Z[K_] := Module[{z},
  z = Expand[Times @@ PD[K] /. x : X[i_, j_, k_, L_] := If[PositiveQ[x, RL,i, R̄j,i]];
  Do[z = z // mc[[1],c[[j]]→c[[1]], {c, Skeleton[K]}, {j, 2, Length@c}];
  z]
```

```
In[ ]:= tab = Table[Echo@Timing[K → Z[K]], {K, AllKnots[{3, 7}]}];
```

KnotTheory: Loading precomputed data in PD4Knots`.

- » {0.625, Knot[3, 1] → W₁[1, 1] + 3 W₁[2, 2] + 3 W₁[3, 3] + W₁[4, 1] + W₁[5, 1] + 3 W₁[6, 6]}
- » {24.0313, Knot[4, 1] → W₁[1, 1] + W₁[2, 1] + W₁[3, 1] + W₁[4, 1] + W₁[5, 1] + W₁[6, 1]}

Out[]:= \$Aborted

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Edge-vertex convention: an oriented edge carries the same label as the vertex ending it.

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```
In[ ]:= ZF[K_] := ZF[PD@K];
ZF[pd_PD] := Module[{z, done, st, c, mn, k},
  z = 1; done = {}; st = Range[2 Length@pd];
  Do[
    z *= c /. X[i_, j_, _, L_] := If[PositiveQ@c, mn = {i, L}; RL,i, mn = {i, j};
    R̄j,i];
  Do[
    If[MemberQ[done, k + 1], z = z // mk,k+1→k; st = st /. k + 1 → k];
    If[MemberQ[done, k - 1], z = z // mst[[k-1],k→st[[k-1]]; st = st /. k → st[[k - 1]],
    {k, mn}];
  done = done ∪ mn,
  {c, List @@ pd}];
  z]
```

```
In[ ]:= tab = Table[Echo@Timing[K → ZF[K]], {K, AllKnots[{3, 7}]}];
```

- » {0.1875, Knot[3, 1] → W₁[1, 1] + 3 W₁[2, 2] + 3 W₁[3, 3] + W₁[4, 1] + W₁[5, 1] + 3 W₁[6, 6]}
- » {0.234375, Knot[4, 1] → W₁[1, 1] + W₁[2, 1] + W₁[3, 1] + W₁[4, 1] + W₁[5, 1] + W₁[6, 1]}
- » {6.26563, Knot[5, 1] → W₁[1, 1] + W₁[2, 2] + W₁[3, 3] + W₁[4, 4] + W₁[5, 5] + W₁[6, 6]}
- » {1.125, Knot[5, 2] → W₁[1, 1] + W₁[2, 2] + W₁[3, 3] + W₁[4, 4] + W₁[5, 5] + W₁[6, 6]}
- » {1.54688, Knot[6, 1] → W₁[1, 1] + 3 W₁[2, 1] + 3 W₁[3, 1] + W₁[4, 4] + W₁[5, 5] + 3 W₁[6, 1]}
- » {1.07813, Knot[6, 2] → W₁[1, 1] + W₁[2, 1] + W₁[3, 1] + W₁[4, 4] + W₁[5, 5] + W₁[6, 1]}
- » {1.67188, Knot[6, 3] → W₁[1, 1] + W₁[2, 1] + W₁[3, 1] + W₁[4, 1] + W₁[5, 1] + W₁[6, 1]}

Out[]:= \$Aborted

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```

In[ ]:= ThinPosition[K_] := Module[{todo, done, pd, c},
  todo = List@@PD@K; done = {}; pd = PD[];
  While[todo != {},
    AppendTo[pd, c = RandomChoice@MaximalBy[todo, Length[done ∩ List@@#] &]];
    todo = DeleteCases[todo, c];
    done = done ∪ List@@c];
  pd
]

```

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```

In[ ]:= ZEF[K_] := ZF@ThinPosition@K;

```

```

In[ ]:= tab = Table[Echo@Timing[K → ZEF[K]], {K, AllKnots[{3, 7}]}];

```

- » {0.125, Knot[3, 1] → $W_1[1, 1] + 3W_1[2, 2] + 3W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3W_1[6, 6]$ }
- » {0.203125, Knot[4, 1] → $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$ }
- » {0.328125, Knot[5, 1] → $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]$ }
- » {0.0625, Knot[5, 2] → $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]$ }
- » {0.4375, Knot[6, 1] → $W_1[1, 1] + 3W_1[2, 1] + 3W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3W_1[6, 1]$ }
- » {0.234375, Knot[6, 2] → $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$ }
- » {0.328125, Knot[6, 3] → $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$ }
- » {0.484375, Knot[7, 1] → $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]$ }
- » {0.5, Knot[7, 2] → $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]$ }
- » {0.109375, Knot[7, 3] → $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]$ }
- » {0.484375, Knot[7, 4] → $W_1[1, 1] + 3W_1[2, 2] + 3W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + 3W_1[6, 6]$ }
- » {0.359375, Knot[7, 5] → $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]$ }
- » {0.234375, Knot[7, 6] → $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]$ }
- » {0.265625, Knot[7, 7] → $W_1[1, 1] + 3W_1[2, 2] + 3W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + 3W_1[6, 6]$ }

```

In[ ]:= Max[First/@tab]

```

```

Out[ ]:= 0.5

```

```

In[ ]:= DeclareGroup[S4];

```

```

tab = Table[Echo@Timing[K → ZEF[K]], {K, AllKnots[{3, 7}]}];

```

- » {4.82813, Knot[3, 1] → $W_1[1, 1] + 5W_1[2, 2] + 5W_1[3, 3] + 4W_1[4, 1] + 4W_1[5, 1] + 5W_1[6, 6] + 5W_1[7, 7] + W_1[8, 8] + 4W_1[9, 1] + W_1[10, 10] + 4W_1[10, 19] + W_1[11, 11] + 4W_1[11, 14] + 4W_1[12, 1] + 4W_1[13, 1] + 4W_1[14, 11] + W_1[14, 14] + 5W_1[15, 15] + 4W_1[16, 1] + W_1[17, 17] + W_1[18, 18] + 4W_1[18, 23] + 4W_1[19, 10] + W_1[19, 19] + 4W_1[20, 1] + 4W_1[21, 1] + 5W_1[22, 22] + 4W_1[23, 18] + W_1[23, 23] + W_1[24, 24]$ }

```

Out[ ]:= $Aborted

```

```
In[ ]:= DeclareGroup[S5];  
      tab = Table[Echo@Timing[K → ZEF[K]], {K, AllKnots [{3, 7}]};  
Out[ ]:= $Aborted
```