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Pensieve header: The WG Algebra with care for conjugacy invariant sets and for tangles.

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```
In[ ]:=
Once[<< KnotTheory`];
HL[ $\mathcal{E}_-$ ] := Style[ $\mathcal{E}$ , Background  $\rightarrow$  If[TrueQ@ $\mathcal{E}$ , ■, ■]];
 $\chi$ [cond_] := If[TrueQ[cond], 1, 0];
```

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Loading **KnotTheory`** version of February 2, 2020, 10:53:45.2097.  
Read more at <http://katlas.org/wiki/KnotTheory>.

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```
In[ ]:=
DeclareGroup[S $_k$ ] := Module[{ $\alpha$ ,  $\beta$ , e,  $\gamma$ s},
  Clear[G, n, g,  $\iota$ , m,  $\eta$ , inv];
  G = PermutationCycles /@ (Permutations@Range@k);
  n = Length[G];
  Do[g[ $\alpha$ ] = e = G[ $\alpha$ ];  $\iota$ [e] =  $\alpha$ , { $\alpha$ , n}];
   $\eta$  =  $\iota$ [Cycles[{ }]];
  Do[m[ $\alpha$ ,  $\beta$ ] =  $\iota$ [g[ $\alpha$ ] ~ PermutationProduct ~ g[ $\beta$ ]], { $\alpha$ , n}, { $\beta$ , n}];
  m[ $\alpha$ ] :=  $\alpha$ ; m[ $\alpha$ ,  $\beta$ ,  $\gamma$ s_] := m[m[ $\alpha$ ,  $\beta$ ],  $\gamma$ s];
  Do[inv[ $\alpha$ ] =  $\iota$ [InversePermutation[g[ $\alpha$ ]]], { $\alpha$ , n}]
]
```

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```
In[ ]:=
ConjugacyClasses := Module[{sea = Range@n, ccs = { }, cc},
  While[Length@sea > 0,
    cc = Union[Table[m[inv[ $\alpha$ ], First@sea,  $\alpha$ ], { $\alpha$ , n}]];
    AppendTo[ccs, cc];
    sea = Complement[sea, cc];
  ]; ccs]
```

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```
In[ ]:=
m $_{i,j \rightarrow k}$ [ $\mathcal{E}_-$ ] := Expand[ $\mathcal{E}$ ] /. W $_i$ [ $\alpha$ , 1] W $_j$ [ $\beta$ ,  $\alpha$ ]  $\Rightarrow$   $\chi$ [m[ $\alpha$ ,  $\beta$ ] == m[ $\beta$ ,  $\gamma$ ]] W $_k$ [ $\alpha$ , m[ $\beta$ ,  $\delta$ ]]
```

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```
In[ ]:=
(* $CIS is a Conjugation Invariant Set *)
R $_{i,j}$  := Sum[W $_i$ [ $\alpha$ , 1] W $_j$ [ $\beta$ ,  $\alpha$ ], { $\alpha$ , $CIS}, { $\beta$ , $CIS}];
R $_{i,j}$  := Sum[W $_i$ [ $\alpha$ , 1] W $_j$ [ $\beta$ , inv@ $\alpha$ ], { $\alpha$ , $CIS}, { $\beta$ , $CIS}];
 $\eta$  $_i$  := Sum[W $_i$ [ $\alpha$ , 1], { $\alpha$ , $CIS}];
```

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```
In[ ]:= DeclareGroup[S3];
Table[m[i, j], {i, n}, {j, n}] // MatrixForm
```

Out[ ]:=**MatrixForm**=

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$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 3 & 6 & 5 \\ 3 & 5 & 1 & 6 & 2 & 4 \\ 4 & 6 & 2 & 5 & 1 & 3 \\ 5 & 3 & 6 & 1 & 4 & 2 \\ 6 & 4 & 5 & 2 & 3 & 1 \end{pmatrix}$$

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In[ ]:= **G**

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```
Out[ ]:= {Cycles[{}], Cycles[{{2, 3}}], Cycles[{{1, 2}}],
Cycles[{{1, 2, 3}}], Cycles[{{1, 3, 2}}], Cycles[{{1, 3}}]}
```

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In[ ]:= **ConjugacyClasses**[\$G]

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```
Out[ ]:= {{1}, {2, 3, 6}, {4, 5}}
```

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```
In[ ]:= Table[{
Short[lhs = R1,2 R4,3 R5,6 // m1,4→1 // m2,5→2 // m3,6→3,
rhs = R2,3 R1,4 R5,6 // m1,5→1 // m2,6→2 // m3,4→3; HL[lhs == rhs]],
{$CIS, ConjugacyClasses}]
```

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```
Out[ ]:= {{W1[1, 1] W2[1, 1] W3[1, 1], True},
{W1[2, 1] W2[2, 2] W3[2, 1] + <<25>> + W1[3, 1] W2[6, 3] W3[6, 5], True},
{W1[5, 1] W2[4, 5] W3[4, 1] + <<6>> + W1[4, 1] W2[4, 4] W3[5, 5], True}}
```

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```
In[ ]:= Table[
lhs = R1,2  $\bar{R}$ 3,4 // m1,3→1 // m2,4→2; rhs =  $\eta_1 \eta_2$  // Expand; HL[lhs == rhs],
{$CIS, ConjugacyClasses}]
```

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```
Out[ ]:= {True, True, True}
```

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```
In[ ]:= Table[
lhs = R1,2  $\bar{R}$ 3,4 // m1,3→1 // m4,2→2; rhs =  $\eta_1 \eta_2$  // Expand; HL[lhs == rhs],
{$CIS, ConjugacyClasses}]
```

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```
Out[ ]:= {True, True, True}
```

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```
In[ ]:= Z[K_] := Module[{z}, Sum[
  z = Expand[Times@@PD[K] /. x : X[i_, j_, k_, l_] => If[PositiveQ[x, Rl,i, R̄j,i]];
  Do[z = z // mc[[1],c[[j]]->c[[1]], {c, Skeleton[K]}, {j, 2, Length@c}];
  z,
  {$CIS, ConjugacyClasses}]]
```

```
In[ ]:= tab = Table[Echo@Timing[K -> Z[K]], {K, AllKnots[{3, 7}]}];
```

**KnotTheory:** Loading precomputed data in PD4Knots`.

- » {0.015625, Knot[3, 1] -> W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 2] + 3 W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + 3 W<sub>1</sub>[6, 6]}
- » {0.09375, Knot[4, 1] -> W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 1]}
- » {0.890625, Knot[5, 1] -> W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + W<sub>1</sub>[6, 6]}
- » {0.921875, Knot[5, 2] -> W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + W<sub>1</sub>[6, 6]}
- » {8.5625, Knot[6, 1] -> W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 1] + 3 W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + 3 W<sub>1</sub>[6, 1]}
- » {8.92188, Knot[6, 2] -> W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + W<sub>1</sub>[6, 1]}
- » {11., Knot[6, 3] -> W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 1]}
- » {90.3438, Knot[7, 1] -> W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 6]}
- » {85.8594, Knot[7, 2] -> W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 6]}
- » {84.4531, Knot[7, 3] -> W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + W<sub>1</sub>[6, 6]}
- » {84.7969, Knot[7, 4] -> W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 2] + 3 W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + 3 W<sub>1</sub>[6, 6]}
- » {84.9063, Knot[7, 5] -> W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 6]}
- » {85.5469, Knot[7, 6] -> W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 6]}
- » {86.9063, Knot[7, 7] -> W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 2] + 3 W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + 3 W<sub>1</sub>[6, 6]}

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Edge-vertex convention: an oriented edge carries the same label as the vertex ending it.

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In[ ]:=

```

ZF[K_] := ZF[PD@K];
ZF[pd_PD] := Module[{z, done, st, c, mn, k},
  Sum[
    z = 1; done = {}; st = Range[2 Length@pd];
    Do[
      z *= c /. X[i_, j_, _, L_] => If[PositiveQ@c, mn = {i, L}; R_{L,i}, mn = {i, j};
        R_{j,i}];
      Do[
        If[MemberQ[done, k + 1], z = z / m_{k,k+1->k}; st = st /. k + 1 -> k];
        If[MemberQ[done, k - 1], z = z / m_{st[[k-1]],k->st[[k-1]]}; st = st /. k -> st[[k-1]],
          {k, mn}];
        done = done ∪ mn,
        {c, List@@pd}];
    z,
    {$CIS, ConjugacyClasses} ] ]

```

```
In[ ]:= tab = Table[Echo@Timing[K → ZF[K]], {K, AllKnots[{3, 9}]}];
```

- » {0., Knot[3, 1] → W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 2] + 3 W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + 3 W<sub>1</sub>[6, 6]}
- » {0.015625, Knot[4, 1] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 1]}
- » {0.109375, Knot[5, 1] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + W<sub>1</sub>[6, 6]}
- » {0.03125, Knot[5, 2] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + W<sub>1</sub>[6, 6]}
- » {0.046875, Knot[6, 1] → W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 1] + 3 W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + 3 W<sub>1</sub>[6, 1]}
- » {0.03125, Knot[6, 2] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + W<sub>1</sub>[6, 1]}
- » {0.03125, Knot[6, 3] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 1]}
- » {0.859375, Knot[7, 1] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 6]}
- » {0.28125, Knot[7, 2] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 6]}
- » {0.1875, Knot[7, 3] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + W<sub>1</sub>[6, 6]}
- » {0.046875, Knot[7, 4] → W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 2] + 3 W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + 3 W<sub>1</sub>[6, 6]}
- » {0.234375, Knot[7, 5] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 6]}
- » {0.109375, Knot[7, 6] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 6]}
- » {0.046875, Knot[7, 7] → W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 2] + 3 W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + 3 W<sub>1</sub>[6, 6]}
- » {0.09375, Knot[8, 1] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 1]}
- » {0.109375, Knot[8, 2] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 1]}
- » {0.1875, Knot[8, 3] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 1]}
- » {0.140625, Knot[8, 4] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 1]}
- » {0.046875, Knot[8, 5] → W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 1] + 3 W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + 3 W<sub>1</sub>[6, 1]}
- » {0.125, Knot[8, 6] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 1]}
- » {0.234375, Knot[8, 7] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 1]}

- » {0.1875, Knot [8, 8] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.0625, Knot [8, 9] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.15625, Knot [8, 10] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]$  }
- » {0.046875, Knot [8, 11] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]$  }
- » {0.046875, Knot [8, 12] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.078125, Knot [8, 13] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.0625, Knot [8, 14] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.140625, Knot [8, 15] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]$  }
- » {0.296875, Knot [8, 16] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$  }
- » {0.015625, Knot [8, 17] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.4375, Knot [8, 18] →  $W_1[1, 1] + 9 W_1[2, 1] + 9 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 9 W_1[6, 1]$  }
- » {0.171875, Knot [8, 19] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]$  }
- » {0.15625, Knot [8, 20] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]$  }
- » {0.15625, Knot [8, 21] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]$  }
- » {9.07813, Knot [9, 1] →  $W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]$  }
- » {3., Knot [9, 2] →  $W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]$  }
- » {1.0625, Knot [9, 3] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]$  }
- » {1.42188, Knot [9, 4] →  $W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]$  }
- » {0.21875, Knot [9, 5] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]$  }
- » {2.54688, Knot [9, 6] →  $W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]$  }
- » {0.46875, Knot [9, 7] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]$  }
- » {0.953125, Knot [9, 8] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]$  }
- » {1.04688, Knot [9, 9] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]$  }
- » {0.203125, Knot [9, 10] →  $W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]$  }
- » {0.140625, Knot [9, 11] →  $W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 6]$  }
- » {0.453125, Knot [9, 12] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]$  }
- » {0.234375, Knot [9, 13] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]$  }
- » {0.09375, Knot [9, 14] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]$  }
- » {0.0625, Knot [9, 15] →  $W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 6]$  }
- » {0.328125, Knot [9, 16] →  $W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]$  }
- » {0.078125, Knot [9, 17] →  $W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 6]$  }
- » {0.71875, Knot [9, 18] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]$  }
- » {0.046875, Knot [9, 19] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]$  }
- » {0.984375, Knot [9, 20] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]$  }
- » {0.0625, Knot [9, 21] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]$  }
- » {0.046875, Knot [9, 22] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]$  }

- » {1.14063, Knot[9, 23] →  $W_1[1, 1] + 3W_1[2, 2] + 3W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3W_1[6, 6]$ }
- » {0.953125, Knot[9, 24] →  $W_1[1, 1] + 3W_1[2, 2] + 3W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 3W_1[6, 6]$ }
- » {0.40625, Knot[9, 25] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]$ }
- » {0.0625, Knot[9, 26] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]$ }
- » {0.375, Knot[9, 27] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]$ }
- » {0.25, Knot[9, 28] →  $W_1[1, 1] + 3W_1[2, 2] + 3W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3W_1[6, 6]$ }
- » {0.15625, Knot[9, 29] →  $W_1[1, 1] + 3W_1[2, 2] + 3W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 3W_1[6, 6]$ }
- » {0.078125, Knot[9, 30] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]$ }
- » {0.140625, Knot[9, 31] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]$ }
- » {0.171875, Knot[9, 32] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]$ }
- » {0.109375, Knot[9, 33] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]$ }
- » {0.03125, Knot[9, 34] →  $W_1[1, 1] + 3W_1[2, 2] + 3W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 3W_1[6, 6]$ }
- » {0.3125, Knot[9, 35] →  $W_1[1, 1] + 9W_1[2, 2] + 9W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 9W_1[6, 6]$ }
- » {0.0625, Knot[9, 36] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]$ }
- » {0.125, Knot[9, 37] →  $W_1[1, 1] + 9W_1[2, 2] + 9W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 9W_1[6, 6]$ }
- » {0.296875, Knot[9, 38] →  $W_1[1, 1] + 3W_1[2, 2] + 3W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3W_1[6, 6]$ }
- » {1.09375, Knot[9, 39] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]$ }
- » {0.0625, Knot[9, 40] →  $W_1[1, 1] + 3W_1[2, 2] + 3W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3W_1[6, 6]$ }
- » {0.1875, Knot[9, 41] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]$ }
- » {0.0625, Knot[9, 42] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]$ }
- » {0.09375, Knot[9, 43] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]$ }
- » {0.078125, Knot[9, 44] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]$ }
- » {0.0625, Knot[9, 45] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]$ }
- » {0.078125, Knot[9, 46] →  $W_1[1, 1] + 9W_1[2, 2] + 9W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 9W_1[6, 6]$ }
- » {0.03125, Knot[9, 47] →  $W_1[1, 1] + 9W_1[2, 2] + 9W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 9W_1[6, 6]$ }
- » {0.109375, Knot[9, 48] →  $W_1[1, 1] + 9W_1[2, 2] + 9W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 9W_1[6, 6]$ }
- » {0.25, Knot[9, 49] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]$ }

pdf

In[ ]:=

```

ThinPosition[K_] := Module[{todo, done, pd, c},
  todo = List@@PD@K; done = {}; pd = PD[];
  While[todo != {},
    AppendTo[pd, c = RandomChoice@MaximalBy[todo, Length[done ∩ List@@#] &]];
    todo = DeleteCases[todo, c];
    done = done ∪ List@@c];
  pd
]

```

```
In[ ]:= ThinPosition[Knot[3, 1]]
```

```
Out[ ]:= PD[X[3, 6, 4, 1], X[1, 4, 2, 5], X[5, 2, 6, 3]]
```

pdf

```
In[ ]:= ZEF[K_] := ZF@ThinPosition@K;
```

```
In[ ]:= tab = Table[Echo@Timing[K → ZEF[K]], {K, AllKnots[{3, 10}]}];
```

- » {0.015625, Knot[3, 1] → W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 2] + 3 W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + 3 W<sub>1</sub>[6, 6]}
- » {0., Knot[4, 1] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 1]}
- » {0., Knot[5, 1] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + W<sub>1</sub>[6, 6]}
- » {0.015625, Knot[5, 2] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + W<sub>1</sub>[6, 6]}
- » {0.015625, Knot[6, 1] → W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 1] + 3 W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + 3 W<sub>1</sub>[6, 1]}
- » {0., Knot[6, 2] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + W<sub>1</sub>[6, 1]}
- » {0.015625, Knot[6, 3] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 1]}
- » {0.015625, Knot[7, 1] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 6]}
- » {0.03125, Knot[7, 2] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 6]}
- » {0.03125, Knot[7, 3] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + W<sub>1</sub>[6, 6]}
- » {0.015625, Knot[7, 4] → W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 2] + 3 W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + 3 W<sub>1</sub>[6, 6]}
- » {0.015625, Knot[7, 5] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 6]}
- » {0.046875, Knot[7, 6] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 2] + W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 6]}
- » {0.015625, Knot[7, 7] → W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 2] + 3 W<sub>1</sub>[3, 3] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + 3 W<sub>1</sub>[6, 6]}
- » {0.03125, Knot[8, 1] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 1]}
- » {0.015625, Knot[8, 2] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 1]}
- » {0.015625, Knot[8, 3] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 1]}
- » {0.0625, Knot[8, 4] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 1]}
- » {0.015625, Knot[8, 5] → W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 1] + 3 W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + 3 W<sub>1</sub>[6, 1]}
- » {0.015625, Knot[8, 6] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 1]}
- » {0.03125, Knot[8, 7] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 1]}
- » {0.03125, Knot[8, 8] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 1]}
- » {0.015625, Knot[8, 9] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 1]}
- » {0.015625, Knot[8, 10] → W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 1] + 3 W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + 3 W<sub>1</sub>[6, 1]}
- » {0.015625, Knot[8, 11] → W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 1] + 3 W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + 3 W<sub>1</sub>[6, 1]}
- » {0.078125, Knot[8, 12] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 1] + W<sub>1</sub>[5, 1] + W<sub>1</sub>[6, 1]}
- » {0.015625, Knot[8, 13] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 1]}
- » {0.015625, Knot[8, 14] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 5] + W<sub>1</sub>[5, 4] + W<sub>1</sub>[6, 1]}
- » {0.046875, Knot[8, 15] → W<sub>1</sub>[1, 1] + 3 W<sub>1</sub>[2, 1] + 3 W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + 3 W<sub>1</sub>[6, 1]}
- » {0.046875, Knot[8, 16] → W<sub>1</sub>[1, 1] + W<sub>1</sub>[2, 1] + W<sub>1</sub>[3, 1] + W<sub>1</sub>[4, 4] + W<sub>1</sub>[5, 5] + W<sub>1</sub>[6, 1]}

- »  $\{0.09375, \text{Knot}[8, 17] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[8, 18] \rightarrow W_1[1, 1] + 9 W_1[2, 1] + 9 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 9 W_1[6, 1]\}$
- »  $\{0.015625, \text{Knot}[8, 19] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.015625, \text{Knot}[8, 20] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[8, 21] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[9, 1] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 2] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]\}$
- »  $\{0.015625, \text{Knot}[9, 3] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 4] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]\}$
- »  $\{0.015625, \text{Knot}[9, 5] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 6] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]\}$
- »  $\{0.015625, \text{Knot}[9, 7] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 8] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 9] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 10] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]\}$
- »  $\{0.046875, \text{Knot}[9, 11] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 6]\}$
- »  $\{0.046875, \text{Knot}[9, 12] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]\}$
- »  $\{0.015625, \text{Knot}[9, 13] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]\}$
- »  $\{0.0625, \text{Knot}[9, 14] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 15] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 6]\}$
- »  $\{0.046875, \text{Knot}[9, 16] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 17] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 18] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 19] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]\}$
- »  $\{0.015625, \text{Knot}[9, 20] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]\}$
- »  $\{0.015625, \text{Knot}[9, 21] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 22] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 23] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 24] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 25] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]\}$
- »  $\{0.015625, \text{Knot}[9, 26] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 27] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]\}$
- »  $\{0.0625, \text{Knot}[9, 28] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 29] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 30] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 31] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]\}$

- »  $\{0.03125, \text{Knot}[9, 32] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]\}$
- »  $\{0.0625, \text{Knot}[9, 33] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]\}$
- »  $\{0.078125, \text{Knot}[9, 34] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 6]\}$
- »  $\{0.046875, \text{Knot}[9, 35] \rightarrow W_1[1, 1] + 9 W_1[2, 2] + 9 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 9 W_1[6, 6]\}$
- »  $\{0.015625, \text{Knot}[9, 36] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]\}$
- »  $\{0.015625, \text{Knot}[9, 37] \rightarrow W_1[1, 1] + 9 W_1[2, 2] + 9 W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 9 W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 38] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]\}$
- »  $\{0.078125, \text{Knot}[9, 39] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]\}$
- »  $\{0.078125, \text{Knot}[9, 40] \rightarrow W_1[1, 1] + 3 W_1[2, 2] + 3 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 6]\}$
- »  $\{0.0625, \text{Knot}[9, 41] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]\}$
- »  $\{0.015625, \text{Knot}[9, 42] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]\}$
- »  $\{0.046875, \text{Knot}[9, 43] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]\}$
- »  $\{0.03125, \text{Knot}[9, 44] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6]\}$
- »  $\{0.234375, \text{Knot}[9, 45] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6]\}$
- »  $\{0.0625, \text{Knot}[9, 46] \rightarrow W_1[1, 1] + 9 W_1[2, 2] + 9 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 9 W_1[6, 6]\}$
- »  $\{0.0625, \text{Knot}[9, 47] \rightarrow W_1[1, 1] + 9 W_1[2, 2] + 9 W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + 9 W_1[6, 6]\}$
- »  $\{0.0625, \text{Knot}[9, 48] \rightarrow W_1[1, 1] + 9 W_1[2, 2] + 9 W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + 9 W_1[6, 6]\}$
- »  $\{0.09375, \text{Knot}[9, 49] \rightarrow W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6]\}$
- »  $\{0.015625, \text{Knot}[10, 1] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 2] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 3] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 4] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 5] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 6] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 7] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 8] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 9] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 10] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]\}$
- »  $\{0.015625, \text{Knot}[10, 11] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.015625, \text{Knot}[10, 12] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 13] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 14] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 15] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 16] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 17] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0., \text{Knot}[10, 18] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$

- » {0.015625, Knot [10, 19] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]$  }
- » {0.03125, Knot [10, 20] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 21] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]$  }
- » {0.03125, Knot [10, 22] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 23] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 24] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.0625, Knot [10, 25] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 26] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.015625, Knot [10, 27] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.015625, Knot [10, 28] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 29] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]$  }
- » {0.0625, Knot [10, 30] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 31] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]$  }
- » {0.078125, Knot [10, 32] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]$  }
- » {0.015625, Knot [10, 33] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.015625, Knot [10, 34] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$  }
- » {0.046875, Knot [10, 35] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.0625, Knot [10, 36] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]$  }
- » {0.03125, Knot [10, 37] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 38] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.015625, Knot [10, 39] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.046875, Knot [10, 40] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]$  }
- » {0.03125, Knot [10, 41] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 42] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]$  }
- » {0.03125, Knot [10, 43] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 44] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$  }
- » {0.046875, Knot [10, 45] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.015625, Knot [10, 46] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 47] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 48] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.078125, Knot [10, 49] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.0625, Knot [10, 50] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.046875, Knot [10, 51] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$  }
- » {0.078125, Knot [10, 52] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.046875, Knot [10, 53] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.078125, Knot [10, 54] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }

- »  $\{0.0625, \text{Knot}[10, 55] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 56] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 57] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.078125, \text{Knot}[10, 58] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 59] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 60] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 61] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 62] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 63] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 64] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 65] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 66] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 67] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 68] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 69] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 70] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]\}$
- »  $\{0.078125, \text{Knot}[10, 71] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 72] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 73] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 74] \rightarrow W_1[1, 1] + 9 W_1[2, 1] + 9 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 9 W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 75] \rightarrow W_1[1, 1] + 9 W_1[2, 1] + 9 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 9 W_1[6, 1]\}$
- »  $\{0.078125, \text{Knot}[10, 76] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 77] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 78] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 79] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 80] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 81] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 82] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 83] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 84] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]\}$
- »  $\{0.09375, \text{Knot}[10, 85] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 86] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 87] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 88] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 89] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 90] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]\}$

- »  $\{0.0625, \text{Knot}[10, 91] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 92] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.078125, \text{Knot}[10, 93] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 94] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 95] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 96] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 97] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 98] \rightarrow W_1[1, 1] + 9 W_1[2, 1] + 9 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 9 W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 99] \rightarrow W_1[1, 1] + 9 W_1[2, 1] + 9 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 9 W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 100] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 101] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.015625, \text{Knot}[10, 102] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]\}$
- »  $\{0.109375, \text{Knot}[10, 103] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 104] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 105] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]\}$
- »  $\{0.09375, \text{Knot}[10, 106] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 107] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]\}$
- »  $\{0.09375, \text{Knot}[10, 108] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 109] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 110] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 111] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.09375, \text{Knot}[10, 112] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 113] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]\}$
- »  $\{0.078125, \text{Knot}[10, 114] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]\}$
- »  $\{0.09375, \text{Knot}[10, 115] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 116] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.015625, \text{Knot}[10, 117] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.046875, \text{Knot}[10, 118] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 119] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 120] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.125, \text{Knot}[10, 121] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]\}$
- »  $\{0.09375, \text{Knot}[10, 122] \rightarrow W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]\}$
- »  $\{0.109375, \text{Knot}[10, 123] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.03125, \text{Knot}[10, 124] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]\}$
- »  $\{0.0625, \text{Knot}[10, 125] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]\}$
- »  $\{0.078125, \text{Knot}[10, 126] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]\}$

- » {0.046875, Knot [10, 127] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 128] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$  }
- » {0.078125, Knot [10, 129] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.046875, Knot [10, 130] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 131] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.046875, Knot [10, 132] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 133] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.015625, Knot [10, 134] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 135] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.046875, Knot [10, 136] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]$  }
- » {0.03125, Knot [10, 137] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$  }
- » {0.140625, Knot [10, 138] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.0625, Knot [10, 139] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]$  }
- » {0.03125, Knot [10, 140] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]$  }
- » {0.0625, Knot [10, 141] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]$  }
- » {0.0625, Knot [10, 142] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]$  }
- » {0.015625, Knot [10, 143] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]$  }
- » {0.125, Knot [10, 144] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]$  }
- » {0.09375, Knot [10, 145] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]$  }
- » {0.03125, Knot [10, 146] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]$  }
- » {0.078125, Knot [10, 147] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]$  }
- » {0.03125, Knot [10, 148] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 149] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.0625, Knot [10, 150] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 151] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$  }
- » {0.09375, Knot [10, 152] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 153] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.109375, Knot [10, 154] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$  }
- » {0.03125, Knot [10, 155] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.046875, Knot [10, 156] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.046875, Knot [10, 157] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1]$  }
- » {0.078125, Knot [10, 158] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]$  }
- » {0.078125, Knot [10, 159] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + 3 W_1[6, 1]$  }
- » {0.046875, Knot [10, 160] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]$  }
- » {0.078125, Knot [10, 161] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 5] + W_1[5, 4] + W_1[6, 1]$  }
- » {0.046875, Knot [10, 162] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1]$  }

- » {0.046875, Knot [10, 163] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 3 W_1[6, 1]$ }
- » {0.078125, Knot [10, 164] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]$ }
- » {0.078125, Knot [10, 165] →  $W_1[1, 1] + 3 W_1[2, 1] + 3 W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + 3 W_1[6, 1]$ }

In[*n*]:= **Max**[**First** /@ **tab**]

Out[*n*]= 0.234375

In[*n*]:= **DeclareGroup**[**S**<sub>4</sub>];

**tab** = **Table**[**Echo**@**Timing**[**K** → **ZEF**[**K**]], {**K**, **AllKnots** [{3, 7}]}];

- » {0.703125, Knot [3, 1] →  $W_1[1, 1] + 5 W_1[2, 2] + 5 W_1[3, 3] + 4 W_1[4, 1] + 4 W_1[5, 1] + 5 W_1[6, 6] + 5 W_1[7, 7] + W_1[8, 8] + 4 W_1[9, 1] + W_1[10, 10] + 4 W_1[10, 19] + W_1[11, 11] + 4 W_1[11, 14] + 4 W_1[12, 1] + 4 W_1[13, 1] + 4 W_1[14, 11] + W_1[14, 14] + 5 W_1[15, 15] + 4 W_1[16, 1] + W_1[17, 17] + W_1[18, 18] + 4 W_1[18, 23] + 4 W_1[19, 10] + W_1[19, 19] + 4 W_1[20, 1] + 4 W_1[21, 1] + 5 W_1[22, 22] + 4 W_1[23, 18] + W_1[23, 23] + W_1[24, 24]$ }
- » {1.14063, Knot [4, 1] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + 4 W_1[4, 1] + 4 W_1[5, 1] + W_1[6, 1] + W_1[7, 1] + W_1[8, 1] + 4 W_1[9, 1] + W_1[10, 1] + W_1[11, 1] + 4 W_1[12, 1] + 4 W_1[13, 1] + W_1[14, 1] + W_1[15, 1] + 4 W_1[16, 1] + W_1[17, 1] + W_1[18, 1] + W_1[19, 1] + 4 W_1[20, 1] + 4 W_1[21, 1] + W_1[22, 1] + W_1[23, 1] + W_1[24, 1]$ }
- » {1.35938, Knot [5, 1] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6] + W_1[7, 7] + W_1[8, 8] + W_1[9, 9] + W_1[10, 19] + W_1[11, 14] + W_1[12, 12] + W_1[13, 13] + W_1[14, 11] + W_1[15, 15] + W_1[16, 16] + W_1[17, 17] + W_1[18, 23] + W_1[19, 10] + W_1[20, 20] + W_1[21, 21] + W_1[22, 22] + W_1[23, 18] + W_1[24, 24]$ }
- » {0.28125, Knot [5, 2] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + W_1[6, 6] + W_1[7, 7] + W_1[8, 8] + W_1[9, 9] + W_1[10, 19] + W_1[11, 14] + W_1[12, 12] + W_1[13, 13] + W_1[14, 11] + W_1[15, 15] + W_1[16, 16] + W_1[17, 17] + W_1[18, 23] + W_1[19, 10] + W_1[20, 20] + W_1[21, 21] + W_1[22, 22] + W_1[23, 18] + W_1[24, 24]$ }
- » {0.328125, Knot [6, 1] →  $W_1[1, 1] + 5 W_1[2, 1] + 5 W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + 5 W_1[6, 1] + 5 W_1[7, 1] + W_1[8, 1] + W_1[9, 9] + 4 W_1[10, 1] + W_1[10, 17] + 4 W_1[11, 1] + W_1[11, 24] + W_1[12, 12] + W_1[13, 13] + 4 W_1[14, 1] + W_1[14, 24] + 5 W_1[15, 1] + W_1[16, 16] + W_1[17, 1] + 4 W_1[18, 1] + W_1[18, 8] + 4 W_1[19, 1] + W_1[19, 17] + W_1[20, 20] + W_1[21, 21] + 5 W_1[22, 1] + 4 W_1[23, 1] + W_1[23, 8] + W_1[24, 1]$ }
- » {1.375, Knot [6, 2] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 4] + W_1[5, 5] + W_1[6, 1] + W_1[7, 1] + W_1[8, 1] + W_1[9, 9] + W_1[10, 17] + W_1[11, 24] + W_1[12, 12] + W_1[13, 13] + W_1[14, 24] + W_1[15, 1] + W_1[16, 16] + W_1[17, 1] + W_1[18, 8] + W_1[19, 17] + W_1[20, 20] + W_1[21, 21] + W_1[22, 1] + W_1[23, 8] + W_1[24, 1]$ }
- » {1.3125, Knot [6, 3] →  $W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + W_1[4, 1] + W_1[5, 1] + W_1[6, 1] + W_1[7, 1] + W_1[8, 1] + W_1[9, 1] + W_1[10, 1] + W_1[11, 1] + W_1[12, 1] + W_1[13, 1] + W_1[14, 1] + W_1[15, 1] + W_1[16, 1] + W_1[17, 1] + W_1[18, 1] + W_1[19, 1] + W_1[20, 1] + W_1[21, 1] + W_1[22, 1] + W_1[23, 1] + W_1[24, 1]$ }
- » {3.03125, Knot [7, 1] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6] + W_1[7, 7] + W_1[8, 8] + W_1[9, 13] + W_1[10, 10] + W_1[11, 11] + W_1[12, 20] + W_1[13, 9] + W_1[14, 14] + W_1[15, 15] + W_1[16, 21] + W_1[17, 17] + W_1[18, 18] + W_1[19, 19] + W_1[20, 12] + W_1[21, 16] + W_1[22, 22] + W_1[23, 23] + W_1[24, 24]$ }
- » {4.1875, Knot [7, 2] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + 4 W_1[4, 5] + 4 W_1[5, 4] + W_1[6, 6] + W_1[7, 7] + W_1[8, 8] + 4 W_1[9, 13] + W_1[10, 10] + W_1[11, 11] + 4 W_1[12, 20] + 4 W_1[13, 9] + W_1[14, 14] + W_1[15, 15] + 4 W_1[16, 21] + W_1[17, 17] + W_1[18, 18] + W_1[19, 19] + 4 W_1[20, 12] + 4 W_1[21, 16] + W_1[22, 22] + W_1[23, 23] + W_1[24, 24]$ }

- » {0.5, Knot[7, 3] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + 4 W_1[4, 4] + 4 W_1[5, 5] + W_1[6, 6] + W_1[7, 7] + W_1[8, 8] + 4 W_1[9, 9] + W_1[10, 19] + W_1[11, 14] + 4 W_1[12, 12] + 4 W_1[13, 13] + W_1[14, 11] + W_1[15, 15] + 4 W_1[16, 16] + W_1[17, 17] + W_1[18, 23] + W_1[19, 10] + 4 W_1[20, 20] + 4 W_1[21, 21] + W_1[22, 22] + W_1[23, 18] + W_1[24, 24]$ }
- » {2.73438, Knot[7, 4] →  $W_1[1, 1] + 5 W_1[2, 2] + 5 W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + 5 W_1[6, 6] + 5 W_1[7, 7] + W_1[8, 8] + W_1[9, 9] + 4 W_1[10, 10] + W_1[10, 19] + 4 W_1[11, 11] + W_1[11, 14] + W_1[12, 12] + W_1[13, 13] + W_1[14, 11] + 4 W_1[14, 14] + 5 W_1[15, 15] + W_1[16, 16] + W_1[17, 17] + 4 W_1[18, 18] + W_1[18, 23] + W_1[19, 10] + 4 W_1[19, 19] + W_1[20, 20] + W_1[21, 21] + 5 W_1[22, 22] + W_1[23, 18] + 4 W_1[23, 23] + W_1[24, 24]$ }
- » {1.125, Knot[7, 5] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 5] + W_1[5, 4] + W_1[6, 6] + W_1[7, 7] + W_1[8, 8] + W_1[9, 13] + W_1[10, 10] + W_1[11, 11] + W_1[12, 20] + W_1[13, 9] + W_1[14, 14] + W_1[15, 15] + W_1[16, 21] + W_1[17, 17] + W_1[18, 18] + W_1[19, 19] + W_1[20, 12] + W_1[21, 16] + W_1[22, 22] + W_1[23, 23] + W_1[24, 24]$ }
- » {2.03125, Knot[7, 6] →  $W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 1] + W_1[5, 1] + W_1[6, 6] + W_1[7, 7] + W_1[8, 8] + W_1[9, 1] + W_1[10, 10] + W_1[11, 11] + W_1[12, 1] + W_1[13, 1] + W_1[14, 14] + W_1[15, 15] + W_1[16, 1] + W_1[17, 17] + W_1[18, 18] + W_1[19, 19] + W_1[20, 1] + W_1[21, 1] + W_1[22, 22] + W_1[23, 23] + W_1[24, 24]$ }
- » {1.4375, Knot[7, 7] →  $W_1[1, 1] + 5 W_1[2, 2] + 5 W_1[3, 3] + W_1[4, 4] + W_1[5, 5] + 5 W_1[6, 6] + 5 W_1[7, 7] + W_1[8, 8] + W_1[9, 9] + W_1[10, 10] + 4 W_1[10, 19] + W_1[11, 11] + 4 W_1[11, 14] + W_1[12, 12] + W_1[13, 13] + 4 W_1[14, 11] + W_1[14, 14] + 5 W_1[15, 15] + W_1[16, 16] + W_1[17, 17] + W_1[18, 18] + 4 W_1[18, 23] + 4 W_1[19, 10] + W_1[19, 19] + W_1[20, 20] + W_1[21, 21] + 5 W_1[22, 22] + 4 W_1[23, 18] + W_1[23, 23] + W_1[24, 24]$ }

In[ ]:= `DeclareGroup[S5];`

`tab = Table[Echo@Timing[K → ZEF[K]], {K, AllKnots[{3, 7]}}];`

» {26.4688, Knot[3, 1] →

$$\begin{aligned}
 & W_1[1, 1] + 7 W_1[2, 2] + 7 W_1[3, 3] + 7 W_1[4, 1] + 7 W_1[5, 1] + 7 W_1[6, 6] + 7 W_1[7, 7] + 5 W_1[8, 8] + \\
 & 7 W_1[9, 1] + W_1[10, 10] + 4 W_1[10, 19] + W_1[11, 11] + 4 W_1[11, 14] + 7 W_1[12, 1] + 7 W_1[13, 1] + \\
 & 4 W_1[14, 11] + W_1[14, 14] + 7 W_1[15, 15] + 7 W_1[16, 1] + 5 W_1[17, 17] + W_1[18, 18] + 4 W_1[18, 23] + \\
 & 4 W_1[19, 10] + W_1[19, 19] + 7 W_1[20, 1] + 7 W_1[21, 1] + 7 W_1[22, 22] + 4 W_1[23, 18] + W_1[23, 23] + \\
 & 5 W_1[24, 24] + 7 W_1[25, 25] + 5 W_1[26, 26] + 5 W_1[27, 27] + W_1[28, 25] + W_1[29, 25] + 5 W_1[30, 30] + \\
 & 7 W_1[31, 1] + W_1[32, 2] + W_1[33, 33] + 4 W_1[33, 73] + W_1[34, 65] + 5 W_1[34, 91] + W_1[35, 72] + \\
 & 5 W_1[35, 116] + W_1[36, 36] + 4 W_1[36, 98] + W_1[37, 37] + 4 W_1[37, 51] + W_1[38, 94] + 5 W_1[38, 113] + \\
 & 7 W_1[39, 1] + W_1[40, 40] + 4 W_1[40, 99] + W_1[41, 6] + 5 W_1[42, 69] + W_1[42, 86] + 5 W_1[43, 90] + \\
 & W_1[43, 117] + W_1[44, 44] + 4 W_1[44, 54] + W_1[45, 45] + 4 W_1[45, 76] + 7 W_1[46, 1] + 5 W_1[47, 64] + \\
 & W_1[47, 109] + W_1[48, 3] + 7 W_1[49, 1] + W_1[50, 2] + 4 W_1[51, 37] + W_1[51, 51] + W_1[52, 90] + \\
 & 5 W_1[52, 117] + 5 W_1[53, 94] + W_1[53, 113] + 4 W_1[54, 44] + W_1[54, 54] + 7 W_1[55, 55] + 5 W_1[56, 56] + \\
 & 7 W_1[57, 1] + W_1[58, 58] + 4 W_1[58, 103] + W_1[59, 59] + 4 W_1[59, 80] + 7 W_1[60, 1] + 5 W_1[61, 61] + \\
 & W_1[62, 55] + W_1[63, 63] + 4 W_1[63, 85] + W_1[64, 47] + 5 W_1[64, 78] + 5 W_1[65, 34] + W_1[65, 97] + \\
 & W_1[66, 15] + W_1[67, 55] + 5 W_1[68, 68] + W_1[69, 42] + 5 W_1[69, 101] + W_1[70, 70] + 4 W_1[70, 110] + \\
 & W_1[71, 22] + 5 W_1[72, 35] + W_1[72, 74] + 4 W_1[73, 33] + W_1[73, 73] + 5 W_1[74, 72] + W_1[74, 116] + \\
 & 7 W_1[75, 1] + 4 W_1[76, 45] + W_1[76, 76] + W_1[77, 6] + W_1[78, 64] + 5 W_1[78, 109] + 7 W_1[79, 1] + \\
 & 4 W_1[80, 59] + W_1[80, 80] + 7 W_1[81, 81] + 7 W_1[82, 1] + 5 W_1[83, 83] + W_1[84, 84] + 4 W_1[84, 107] + \\
 & 4 W_1[85, 63] + W_1[85, 85] + 5 W_1[86, 42] + W_1[86, 101] + 5 W_1[87, 87] + W_1[88, 7] + W_1[89, 81] + \\
 & W_1[90, 43] + 5 W_1[90, 52] + W_1[91, 34] + 5 W_1[91, 97] + W_1[92, 22] + W_1[93, 81] + 5 W_1[94, 38] + \\
 & W_1[94, 53] + 5 W_1[95, 95] + W_1[96, 96] + 4 W_1[96, 119] + 5 W_1[97, 65] + W_1[97, 91] + 4 W_1[98, 36] + \\
 & W_1[98, 98] + 4 W_1[99, 40] + W_1[99, 99] + 7 W_1[100, 1] + W_1[101, 69] + 5 W_1[101, 86] + W_1[102, 3] + \\
 & 4 W_1[103, 58] + W_1[103, 103] + 7 W_1[104, 1] + 7 W_1[105, 1] + 7 W_1[106, 106] + 4 W_1[107, 84] + \\
 & W_1[107, 107] + 5 W_1[108, 108] + 5 W_1[109, 47] + W_1[109, 78] + 4 W_1[110, 70] + W_1[110, 110] + W_1[111, 7] + \\
 & 5 W_1[112, 112] + W_1[113, 38] + 5 W_1[113, 53] + W_1[114, 106] + W_1[115, 15] + W_1[116, 35] + 5 W_1[116, 74] + \\
 & 5 W_1[117, 43] + W_1[117, 52] + W_1[118, 106] + 4 W_1[119, 96] + W_1[119, 119] + 5 W_1[120, 120] \}
 \end{aligned}$$

» {45.3281,

$$\begin{aligned}
 & \text{Knot}[4, 1] \rightarrow W_1[1, 1] + W_1[2, 1] + W_1[3, 1] + 7 W_1[4, 1] + 7 W_1[5, 1] + W_1[6, 1] + W_1[7, 1] + 9 W_1[8, 1] + \\
 & 7 W_1[9, 1] + W_1[10, 1] + W_1[11, 1] + 7 W_1[12, 1] + 7 W_1[13, 1] + W_1[14, 1] + W_1[15, 1] + 7 W_1[16, 1] + \\
 & 9 W_1[17, 1] + W_1[18, 1] + W_1[19, 1] + 7 W_1[20, 1] + 7 W_1[21, 1] + W_1[22, 1] + W_1[23, 1] + 9 W_1[24, 1] + \\
 & W_1[25, 1] + 9 W_1[26, 1] + 9 W_1[27, 1] + W_1[28, 1] + 6 W_1[28, 4] + 6 W_1[28, 5] + W_1[29, 1] + 6 W_1[29, 4] + \\
 & 6 W_1[29, 5] + 9 W_1[30, 1] + 7 W_1[31, 1] + W_1[32, 1] + 6 W_1[32, 31] + 6 W_1[32, 49] + W_1[33, 1] + W_1[34, 1] + \\
 & W_1[35, 1] + W_1[36, 1] + W_1[37, 1] + W_1[38, 1] + 7 W_1[39, 1] + W_1[40, 1] + W_1[41, 1] + 6 W_1[41, 39] + \\
 & 6 W_1[41, 75] + W_1[42, 1] + W_1[43, 1] + W_1[44, 1] + W_1[45, 1] + 7 W_1[46, 1] + W_1[47, 1] + W_1[48, 1] + \\
 & 6 W_1[48, 46] + 6 W_1[48, 100] + 7 W_1[49, 1] + W_1[50, 1] + 6 W_1[50, 31] + 6 W_1[50, 49] + W_1[51, 1] + \\
 & W_1[52, 1] + W_1[53, 1] + W_1[54, 1] + W_1[55, 1] + 9 W_1[56, 1] + 7 W_1[57, 1] + W_1[58, 1] + W_1[59, 1] + \\
 & 7 W_1[60, 1] + 9 W_1[61, 1] + W_1[62, 1] + 6 W_1[62, 16] + 6 W_1[62, 21] + W_1[63, 1] + W_1[64, 1] + W_1[65, 1] + \\
 & W_1[66, 1] + 6 W_1[66, 60] + 6 W_1[66, 104] + W_1[67, 1] + 6 W_1[67, 16] + 6 W_1[67, 21] + 9 W_1[68, 1] + \\
 & W_1[69, 1] + W_1[70, 1] + W_1[71, 1] + 6 W_1[71, 57] + 6 W_1[71, 79] + W_1[72, 1] + W_1[73, 1] + W_1[74, 1] + \\
 & 7 W_1[75, 1] + W_1[76, 1] + W_1[77, 1] + 6 W_1[77, 39] + 6 W_1[77, 75] + W_1[78, 1] + 7 W_1[79, 1] + W_1[80, 1] + \\
 & W_1[81, 1] + 7 W_1[82, 1] + 9 W_1[83, 1] + W_1[84, 1] + W_1[85, 1] + W_1[86, 1] + 9 W_1[87, 1] + W_1[88, 1] + \\
 & 6 W_1[88, 82] + 6 W_1[88, 105] + W_1[89, 1] + 6 W_1[89, 12] + 6 W_1[89, 20] + W_1[90, 1] + W_1[91, 1] + \\
 & W_1[92, 1] + 6 W_1[92, 57] + 6 W_1[92, 79] + W_1[93, 1] + 6 W_1[93, 12] + 6 W_1[93, 20] + W_1[94, 1] + \\
 & 9 W_1[95, 1] + W_1[96, 1] + W_1[97, 1] + W_1[98, 1] + W_1[99, 1] + 7 W_1[100, 1] + W_1[101, 1] + W_1[102, 1] + \\
 & 6 W_1[102, 46] + 6 W_1[102, 100] + W_1[103, 1] + 7 W_1[104, 1] + 7 W_1[105, 1] + W_1[106, 1] + W_1[107, 1] + \\
 & 9 W_1[108, 1] + W_1[109, 1] + W_1[110, 1] + W_1[111, 1] + 6 W_1[111, 82] + 6 W_1[111, 105] + 9 W_1[112, 1] + \\
 & W_1[113, 1] + W_1[114, 1] + 6 W_1[114, 9] + 6 W_1[114, 13] + W_1[115, 1] + 6 W_1[115, 60] + 6 W_1[115, 104] + \\
 & W_1[116, 1] + W_1[117, 1] + W_1[118, 1] + 6 W_1[118, 9] + 6 W_1[118, 13] + W_1[119, 1] + 9 W_1[120, 1] \}
 \end{aligned}$$

» {58.4531,

$$\begin{aligned} \text{Knot}[5, 1] \rightarrow & W_1[1, 1] + W_1[2, 2] + W_1[3, 3] + W_1[4, 4] + 6 W_1[4, 5] + 6 W_1[5, 4] + W_1[5, 5] + W_1[6, 6] + \\ & W_1[7, 7] + 9 W_1[8, 8] + W_1[9, 9] + 6 W_1[9, 13] + 4 W_1[10, 10] + W_1[10, 19] + 4 W_1[11, 11] + W_1[11, 14] + \\ & W_1[12, 12] + 6 W_1[12, 20] + 6 W_1[13, 9] + W_1[13, 13] + W_1[14, 11] + 4 W_1[14, 14] + W_1[15, 15] + \\ & W_1[16, 16] + 6 W_1[16, 21] + 9 W_1[17, 17] + 4 W_1[18, 18] + W_1[18, 23] + W_1[19, 10] + 4 W_1[19, 19] + \\ & 6 W_1[20, 12] + W_1[20, 20] + 6 W_1[21, 16] + W_1[21, 21] + W_1[22, 22] + W_1[23, 18] + 4 W_1[23, 23] + \\ & 9 W_1[24, 24] + W_1[25, 25] + 9 W_1[26, 26] + 9 W_1[27, 27] + W_1[28, 28] + 6 W_1[28, 29] + 6 W_1[29, 28] + \\ & W_1[29, 29] + 9 W_1[30, 30] + W_1[31, 31] + 6 W_1[31, 49] + W_1[32, 32] + 6 W_1[32, 50] + 4 W_1[33, 33] + \\ & W_1[33, 73] + 6 W_1[34, 1] + 6 W_1[35, 1] + 4 W_1[36, 36] + W_1[36, 98] + 4 W_1[37, 37] + W_1[37, 51] + \\ & 6 W_1[38, 1] + W_1[39, 39] + 6 W_1[39, 75] + 4 W_1[40, 40] + W_1[40, 99] + W_1[41, 41] + 6 W_1[41, 77] + \\ & 6 W_1[42, 1] + 6 W_1[43, 1] + 4 W_1[44, 44] + W_1[44, 54] + 4 W_1[45, 45] + W_1[45, 76] + W_1[46, 46] + \\ & 6 W_1[46, 100] + 6 W_1[47, 1] + W_1[48, 48] + 6 W_1[48, 102] + 6 W_1[49, 31] + W_1[49, 49] + \\ & 6 W_1[50, 32] + W_1[50, 50] + W_1[51, 37] + 4 W_1[51, 51] + 6 W_1[52, 1] + 6 W_1[53, 1] + W_1[54, 44] + \\ & 4 W_1[54, 54] + W_1[55, 55] + 9 W_1[56, 56] + W_1[57, 57] + 6 W_1[57, 79] + 4 W_1[58, 58] + W_1[58, 103] + \\ & 4 W_1[59, 59] + W_1[59, 80] + W_1[60, 60] + 6 W_1[60, 104] + 9 W_1[61, 61] + W_1[62, 62] + 6 W_1[62, 67] + \\ & 4 W_1[63, 63] + W_1[63, 85] + 6 W_1[64, 1] + 6 W_1[65, 1] + W_1[66, 66] + 6 W_1[66, 115] + 6 W_1[67, 62] + \\ & W_1[67, 67] + 9 W_1[68, 68] + 6 W_1[69, 1] + 4 W_1[70, 70] + W_1[70, 110] + W_1[71, 71] + 6 W_1[71, 92] + \\ & 6 W_1[72, 1] + W_1[73, 33] + 4 W_1[73, 73] + 6 W_1[74, 1] + 6 W_1[75, 39] + W_1[75, 75] + W_1[76, 45] + \\ & 4 W_1[76, 76] + 6 W_1[77, 41] + W_1[77, 77] + 6 W_1[78, 1] + 6 W_1[79, 57] + W_1[79, 79] + W_1[80, 59] + \\ & 4 W_1[80, 80] + W_1[81, 81] + W_1[82, 82] + 6 W_1[82, 105] + 9 W_1[83, 83] + 4 W_1[84, 84] + \\ & W_1[84, 107] + W_1[85, 63] + 4 W_1[85, 85] + 6 W_1[86, 1] + 9 W_1[87, 87] + W_1[88, 88] + 6 W_1[88, 111] + \\ & W_1[89, 89] + 6 W_1[89, 93] + 6 W_1[90, 1] + 6 W_1[91, 1] + 6 W_1[92, 71] + W_1[92, 92] + 6 W_1[93, 89] + \\ & W_1[93, 93] + 6 W_1[94, 1] + 9 W_1[95, 95] + 4 W_1[96, 96] + W_1[96, 119] + 6 W_1[97, 1] + W_1[98, 36] + \\ & 4 W_1[98, 98] + W_1[99, 40] + 4 W_1[99, 99] + 6 W_1[100, 46] + W_1[100, 100] + 6 W_1[101, 1] + \\ & 6 W_1[102, 48] + W_1[102, 102] + W_1[103, 58] + 4 W_1[103, 103] + 6 W_1[104, 60] + W_1[104, 104] + \\ & 6 W_1[105, 82] + W_1[105, 105] + W_1[106, 106] + W_1[107, 84] + 4 W_1[107, 107] + 9 W_1[108, 108] + \\ & 6 W_1[109, 1] + W_1[110, 70] + 4 W_1[110, 110] + 6 W_1[111, 88] + W_1[111, 111] + 9 W_1[112, 112] + \\ & 6 W_1[113, 1] + W_1[114, 114] + 6 W_1[114, 118] + 6 W_1[115, 66] + W_1[115, 115] + 6 W_1[116, 1] + \\ & 6 W_1[117, 1] + 6 W_1[118, 114] + W_1[118, 118] + W_1[119, 96] + 4 W_1[119, 119] + 9 W_1[120, 120] \} \end{aligned}$$

Out[\*]= \$Aborted

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To do: Implement and verify  $\Delta$  etc.