pdf

Pensieve header: The Kerler Algebra and the Alexander polynomial.

pdf

In[∘]:=
```
Once[<< KnotTheory`];
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, ■, ■]];
```

pdf

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

pdf

In[∘]:=

MT =

□	a	b	c	d	ka	kb	kc	kd
a	a	b	0	0	ka	kb	0	0
b	0	0	a	b	0	0	-ka	-kb
c	c	d	0	0	-kc	-kd	0	0
d	0	0	c	d	0	0	kc	kd
ka	ka	kb	0	0	a	b	0	0
kb	0	0	ka	kb	0	0	-a	-b
kc	kc	kd	0	0	-c	-d	0	0
kd	0	0	kc	kd	0	0	c	d

;

```
ℰ_ // m_{i_,j_→k_} := Expand[ℰ] /. Flatten@Table[MT⟦α, 1⟧_i MT⟦1, β⟧_j →
    (MT⟦α, β⟧ /. v : (a | b | c | d | ka | kb | kc | kd) :→ v_k), {α, 2, 9}, {β, 2, 9}];
```

pdf

In[∘]:=
```
KBasis[{i_}] := {a_i, b_i, c_i, d_i, ka_i, kb_i, kc_i, kd_i};
KBasis[{i_, is__}] := Flatten@Outer[Times, KBasis[{i}], KBasis[{is}]]
```

pdf

In[∘]:=
```
η_{i_} := a_i + d_i;
γ_{i_} := ka_i + kd_i;
```

pdf

In[∘]:=
```
lhs = η_1 KBasis[{2}] // m_{1,2→1};
HL[lhs == KBasis[{1}]]
```

pdf

Out[∘]= True

pdf

In[∘]:=
```
lhs = η_1 KBasis[{2}] // Expand // m_{1,2→1};
HL[lhs == KBasis[{1}]]
```

pdf

Out[∘]= True

pdf

In[∘]:=
```
Short[lhs = KBasis[{1, 2, 3}] // m1,2→1 // m1,3→1]
rhs = KBasis[{1, 2, 3}] // m2,3→2 // m1,2→1;
lhs == rhs // HL
```

pdf

Out[∘]//Short= $\{a_1, b_1, 0, 0, ka_1, kb_1, 0, 0, 0, 0, a_1, b_1, 0, 0, -ka_1, \ll 482 \gg, d_1, 0, 0, -kc_1, -kd_1, 0, 0, 0, 0, c_1, d_1, 0, 0, kc_1, kd_1\}$

pdf

Out[∘]= True

pdf

In[∘]:= $R_{i\_,j\_} := a_i\, a_j + d_i\, a_j + T\, a_i\, d_j - (1 - T)\, kc_i\, kb_j - T\, d_i\, d_j$

pdf

In[∘]:=
```
Short[lhs = R1,2 R4,3 R5,6 // m1,4→1 // m2,5→2 // m3,6→3];
rhs = R2,3 R1,4 R5,6 // m1,5→1 // m2,6→2 // m3,4→3;
lhs == rhs // HL
```

pdf

Out[∘]= True

pdf

In[∘]:= $\overline{R}_{i\_,j\_} := a_i\, a_j + d_i\, a_j + T^{-1}\, a_i\, d_j - \left(1 - T^{-1}\right) kc_i\, kb_j - T^{-1}\, d_i\, d_j$

pdf

In[∘]:=
```
Short[lhs = R1,2 R̄3,4 // m1,3→1 // m2,4→2]
rhs = η1 η2 // Expand;
lhs == rhs // HL
```

pdf

Out[∘]//Short= $a_1\, a_2 + a_2\, d_1 + a_1\, d_2 + d_1\, d_2$

pdf

Out[∘]= True

pdf

In[∘]:=
```
Short[lhs = R1,2 R̄3,4 // m1,3→1 // m4,2→2]
rhs = η1 η2 // Expand;
Simplify[lhs - rhs]
```

pdf

Out[∘]//Short= $a_1\, a_2 + a_2\, d_1 + a_1\, d_2 + d_1\, d_2 - 2\, kb_2\, kc_1 + 2\, T\, kb_2\, kc_1$

pdf

Out[∘]= $2 \times (-1 + T)\, kb_2\, kc_1$

pdf

In[∘]:=
```
lhs = R1,4 R̄5,2 γ3 // m2,4→2 // m1,3→1 // m1,5→1
rhs = γ1 η2 // Expand;
lhs == rhs // HL
```

pdf

Out[∘]= $a_2\, ka_1 + d_2\, ka_1 + a_2\, kd_1 + d_2\, kd_1$

pdf

Out[∘]= True

pdf

RVK and Z

pdf

RVK, rot, Z modified from 2016-09/OneSmidgen.nb. See also in AP/Projects/SL2Invariant/.

pdf

Some details of the code below are at http://drorbn.net/bbs/show?shot=Dror-160920-151350.jpg.

pdf

In[∘]:=
```
RVK::usage =
  "RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings
    xs and a length 2n list of rotation numbers rots. Crossing
    sites are indexed 1 through 2n, and rots⟦k⟧ is the rotation
    between site k-1 and site k. RVK is also a casting operator
    converting to the RVK presentation from other knot presentations.";
```

pdf

In[∘]:=
```
RVK[pd_PD] := Module[{n, xs, x, rots, front = {0}, k},
   n = Length@pd; rots = Table[0, {2 n}];
   xs = Cases[pd, x_X :> { Xp[x⟦4⟧, x⟦1⟧]  PositiveQ@x
                           Xm[x⟦2⟧, x⟦1⟧]  True        ];
   For[k = 0, k < 2 n, ++k, If[k == 0 ∨ FreeQ[front, -k],
     front = Flatten@Replace[front, k → (xs /. {
              Xp[k + 1, l_] | Xm[l_, k + 1] :> {l, k + 1, 1 - l},
              Xp[l_, k + 1] | Xm[k + 1, l_] :> (++rots⟦l⟧; {1 - l, k + 1, l}),
              _Xp | _Xm :> {}
            }), {1}],
     Cases[front, k | -k] /. {k, -k} :> --rots⟦k + 1⟧;
    ]];
   RVK[xs, rots] ];
RVK[K_] := RVK[PD[K]];
```

pdf

In[∘]:=
```
rot_i_[n_] := { η_i  EvenQ[n]
                γ_i  OddQ[n]
```

pdf

In[∘]:=
```
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := Module[{ζ, done, st, c, χ, i, j, k},
  ζ = 1; done = {}; st = Range[2 Length[rvk[[1]]]];
  Do[
   {i, j} = List @@ c;
   χ = (c /. {_Xp :> R_{i,j}, _Xm :> R̄_{i,j}}) (ka_0 - kd_0) // m_{j,0→j};
   Do[χ = (rot_0[rvk[[2, k]]] χ) // m_{0,k→k}, {k, {i, j}}];
   ζ *= χ;
   Do[
    If[MemberQ[done, k + 1], ζ = ζ // m_{k,k+1→k}; st = st /. k + 1 → k];
    If[MemberQ[done, k - 1], ζ = ζ // m_{st[[k-1]],k→st[[k-1]]}; st = st /. k → st[[k - 1]]],
    {k, {i, j}}];
   done = done ⋃ {i, j},
   {c, rvk[[1]]}
   ];
  Factor@ζ
  ]
```

pdf

In[∘]:=
```
K = Knot[8, 17]; Factor@Alexander[K][T]
z = Z[K]
```

pdf

Out[∘]= $-\frac{1-4T+8T^2-11T^3+8T^4-4T^5+T^6}{T^3}$

pdf

Out[∘]= $-\frac{\left(1-4T+8T^2-11T^3+8T^4-4T^5+T^6\right)(a_1+d_1)}{T^4}$

pdf

In[∘]:=
```
Timing@Union@Table[Simplify[Alexander[K][T] (a_1 + d_1) / Z[K]], {K, AllKnots[{3, 10}]}]
```

pdf

Out[∘]= $\left\{44.8281, \left\{1, \frac{1}{T^5}, \frac{1}{T^4}, \frac{1}{T^3}, \frac{1}{T^2}, \frac{1}{T}, T, T^2, T^3, T^4, T^5, T^6\right\}\right\}$

pdf

In[∘]:=
```
ThinPosition[K_] := Module[{todo, done, pd, c},
  todo = List @@ PD@K; done = {}; pd = PD[];
  While[todo =!= {},
   AppendTo[pd, c = RandomChoice@MaximalBy[todo, Length[done ⋂ List @@ #] &]];
   todo = DeleteCases[todo, c];
   done = done ⋃ List @@ c];
  pd ]
```

In[∘]:=
```
ZF[K_] := Z@ThinPosition@K;
```

pdf

In[ ]:=
```
Timing@Union@Table[Simplify[Alexander[K][T] (a1 + d1) / ZF[K]], {K, AllKnots[{3, 10}]}]
```

pdf

Out[ ]= $\left\{11.3281, \left\{1, \frac{1}{T^5}, \frac{1}{T^4}, \frac{1}{T^3}, \frac{1}{T^2}, \frac{1}{T}, T, T^2, T^3, T^4, T^5, T^6\right\}\right\}$