pdf

In[∘]:=
```
Once[<< KnotTheory`]
```

pdf

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

pdf

In[∘]:=
```
χ[cond_] := If[TrueQ[cond], 1, 0];
```

pdf

In[∘]:=
```
ℰ_ // m[i_, j_ → k_] :=
 Expand[Expand[ℰ] /. e_{r_,s_,t_}[i] e_{u_,v_,w_}[j] :→ χ[t == v] (-1)^(u (s+t)) e_{(r+u)~Mod~2,s,w}[k]]
```

pdf

In[∘]:=
```
KBasis[{i_}] :=
 {e_{0,0,0}[i], e_{0,0,1}[i], e_{0,1,0}[i], e_{0,1,1}[i], e_{1,0,0}[i], e_{1,0,1}[i], e_{1,1,0}[i], e_{1,1,1}[i]}
KBasis[{i_, is__}] := Flatten@Outer[Times, KBasis[{i}], KBasis[{is}]]
```

pdf

In[∘]:=
```
η[i_] := e_{0,0,0}[i] + e_{0,1,1}[i];
γ[i_] := e_{1,0,0}[i] + e_{1,1,1}[i];
```

pdf

In[∘]:=
```
KBasis[{3}]
```

pdf

Out[∘]= $\{e_{0,0,0}[3], e_{0,0,1}[3], e_{0,1,0}[3], e_{0,1,1}[3], e_{1,0,0}[3], e_{1,0,1}[3], e_{1,1,0}[3], e_{1,1,1}[3]\}$

In[∘]:=
```
MatrixForm[Table[p q // m[1, 2 → 3],
   {p, {η[1]}~Join~KBasis[{1}]}, {q, {η[2]}~Join~KBasis[{2}]}
  ] /. {e_{0,0,0}[3] → a, e_{0,0,1}[3] → b, e_{0,1,0}[3] → c,
   e_{0,1,1}[3] → d, e_{1,0,0}[3] → ka, e_{1,0,1}[3] → kb, e_{1,1,0}[3] → kc, e_{1,1,1}[3] → kd}]
```

Out[∘]//MatrixForm=

a + d	a	b	c	d	ka	kb	kc	kd
a	a	b	0	0	ka	kb	0	0
b	0	0	a	b	0	0	-ka	-kb
c	c	d	0	0	-kc	-kd	0	0
d	0	0	c	d	0	0	kc	kd
ka	ka	kb	0	0	a	b	0	0
kb	0	0	ka	kb	0	0	-a	-b
kc	kc	kd	0	0	-c	-d	0	0
kd	0	0	kc	kd	0	0	c	d

pdf

In[∘]:=
```
lhs = η[1] × KBasis[{2}] // Expand // m[1, 2 → 1]
lhs == KBasis[{1}]
```

pdf

Out[∘]= $\{e_{0,0,0}[1], e_{0,0,1}[1], e_{0,1,0}[1], e_{0,1,1}[1], e_{1,0,0}[1], e_{1,0,1}[1], e_{1,1,0}[1], e_{1,1,1}[1]\}$

pdf

Out[∘]= True

pdf

In[∘]:=
```
lhs = η[2] × KBasis[{1}] // Expand // m[1, 2 → 1]
lhs == KBasis[{1}]
```

pdf

Out[∘]= $\{e_{0,0,0}[1], e_{0,0,1}[1], e_{0,1,0}[1], e_{0,1,1}[1], e_{1,0,0}[1], e_{1,0,1}[1], e_{1,1,0}[1], e_{1,1,1}[1]\}$

pdf

Out[∘]= True

pdf

In[∘]:=
```
Short[lhs = KBasis[{1, 2, 3}] // m[1, 2 → 1] // m[1, 3 → 1]]
rhs = KBasis[{1, 2, 3}] // m[2, 3 → 2] // m[1, 2 → 1];
lhs == rhs
```

pdf

Out[∘]//Short= $\{e_{0,0,0}[1], e_{0,0,1}[1], 0, 0, e_{1,0,0}[1], e_{1,0,1}[1], \ll 501 \gg, e_{0,1,1}[1], 0, 0, e_{1,1,0}[1], e_{1,1,1}[1]\}$

pdf

Out[∘]= True

pdf

In[∘]:=
```
R[i_, j_] := e_{0,0,0}[i] e_{0,0,0}[j] + e_{0,1,1}[i] e_{0,0,0}[j] +
   T e_{0,0,0}[i] e_{0,1,1}[j] - (1 - T) e_{1,1,0}[i] e_{1,0,1}[j] - T e_{0,1,1}[i] e_{0,1,1}[j]
```

pdf

In[∘]:=
```
Short[lhs = R[1, 2] × R[4, 3] × R[5, 6] // Expand // m[1, 4 → 1] // m[2, 5 → 2] // m[3, 6 → 3]]
rhs = R[2, 3] × R[1, 4] × R[5, 6] // Expand // m[1, 5 → 1] // m[2, 6 → 2] // m[3, 4 → 3];
lhs == rhs
```

pdf

Out[∘]//Short= $e_{0,0,0}[1]\, e_{0,0,0}[2]\, e_{0,0,0}[3] + e_{0,0,0}[2] \ll 1 \gg e_{0,\ll 1 \gg \ll 1 \gg 1}[1] + \ll 28 \gg$

pdf

Out[∘]= True

pdf

In[∘]:=
```
R̄[i_, j_] := e_{0,0,0}[i] e_{0,0,0}[j] + e_{0,1,1}[i] e_{0,0,0}[j] +
   T^-1 e_{0,0,0}[i] e_{0,1,1}[j] - (1 - T^-1) e_{1,1,0}[i] e_{1,0,1}[j] - T^-1 e_{0,1,1}[i] e_{0,1,1}[j]
```

pdf

In[∘]:=
```
lhs = R[1, 2] R̄[3, 4] // m[1, 3 → 1] // m[2, 4 → 2]
rhs = η[1] × η[2] // Expand;
lhs == rhs
```

pdf

Out[∘]= $e_{0,0,0}[1]\, e_{0,0,0}[2] + e_{0,0,0}[2]\, e_{0,1,1}[1] + e_{0,0,0}[1]\, e_{0,1,1}[2] + e_{0,1,1}[1]\, e_{0,1,1}[2]$

pdf

Out[∘]= True

pdf

In[∘]:=
```
lhs = R[1, 2] R̄[3, 4] // Expand // m[1, 3 → 1] // m[4, 2 → 2];
rhs = η[1] × η[2] // Expand;
Simplify[lhs - rhs]
```

pdf

Out[∘]= $2 \times (-1 + T)\, e_{1,0,1}[2]\, e_{1,1,0}[1]$

pdf

In[∘]:=
```
lhs = R[1, 4] R̄[5, 2] γ[3] // Expand // m[2, 4 → 2] // m[1, 3 → 1] // m[1, 5 → 1]
rhs = γ[1] × η[2] // Expand;
lhs == rhs
```

pdf

Out[∘]= $e_{0,0,0}[2]\, e_{1,0,0}[1] + e_{0,1,1}[2]\, e_{1,0,0}[1] + e_{0,0,0}[2]\, e_{1,1,1}[1] + e_{0,1,1}[2]\, e_{1,1,1}[1]$

pdf

Out[∘]= True

pdf

RVK and Z

pdf

RVK, rot, Z modified from 2016-09/OneSmidgen.nb. See also in AP/Projects/SL2Invariant/.

pdf

Some details of the code below are at http://drorbn.net/bbs/show?shot=Dror-160920-151350.jpg.

pdf

In[∘]:=
```
RVK::usage =
  "RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings
    xs and a length 2n list of rotation numbers rots. Crossing
    sites are indexed 1 through 2n, and rots⟦k⟧ is the rotation
    between site k-1 and site k. RVK is also a casting operator
    converting to the RVK presentation from other knot presentations.";
```

pdf

In[∘]:=
```
RVK[pd_PD] := Module[{n, xs, x, rots, front = {0}, k},
   n = Length@pd; rots = Table[0, {2 n}];
   xs = Cases[pd, x_X :→ { Xp[x⟦4⟧, x⟦1⟧]  PositiveQ@x
                          { Xm[x⟦2⟧, x⟦1⟧]  True        ];
   For[k = 0, k < 2 n, ++k, If[k == 0 ∨ FreeQ[front, -k],
     front = Flatten@Replace[front, k → (xs /. {
              Xp[k + 1, l_] | Xm[l_, k + 1] :→ {l, k + 1, 1 - l},
              Xp[l_, k + 1] | Xm[k + 1, l_] :→ (++rots⟦l⟧; {1 - l, k + 1, l}),
              _Xp | _Xm :→ {}
             }), {1}],
     Cases[front, k | -k] /. {k, -k} :→ --rots⟦k + 1⟧;
    ]];
   RVK[xs, rots] ];
RVK[K_] := RVK[PD[K]];
```

pdf

In[∘]:=
```
Kink[i_] := Kink[i] = R[1, 3] × γ[2] // m[1, 2 → 1] // m[1, 3 → i];
K̄ink[i_] := K̄ink[i] = R̄[1, 3] γ[2] // m[1, 2 → 1] // m[1, 3 → i];
```

pdf

In[∘]:=
```
rot[i_, n_] := { η[i]  EvenQ[n]
               { ɣ[i]  OddQ[n]
```

pdf

In[∘]:=
```
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := Module[{todo, n, rots, ξ, done, st, cx, ξ1, i, j, k, k1, k2, k3},
  {todo, rots} = List @@ rvk;
  AppendTo[rots, 0];
  n = Length[todo];
  ξ = η[0];
  done = {0};
  st = Range[0, 2 n + 1];
  While[{} =!= todo,
   {cx} = MaximalBy[todo, Length[done ⋂ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
   {i, j} = List @@ cx;
   ξ1 = Switch[Head[cx],
      Xp, (R[i, j] Kink‾[k]) // m[j, k → j],
      Xm, (R‾[i, j] Kink[k]) // m[j, k → j]
     ];
   ξ1 = (rot[k, rots[[i]]] ξ1) // m[k, i → i]; rots[[i]] = 0;
   ξ1 = (ξ1 rot[k, rots[[i + 1]]]) // m[i, k → i]; rots[[i + 1]] = 0;
   ξ1 = (rot[k, rots[[j]]] ξ1) // m[k, j → j]; rots[[j]] = 0;
   ξ1 = (ξ1 rot[k, rots[[j + 1]]]) // m[j, k → j]; rots[[j + 1]] = 0;
   ξ *= ξ1;
   If[MemberQ[done, i], ξ = ξ // m[i, i + 1 → i]; st = st /. st[[i + 2]] → st[[i + 1]]];
   If[MemberQ[done, i - 1], ξ = ξ // m[st[[i]], i → st[[i]]]; st = st /. st[[i + 1]] → st[[i]]];
   If[MemberQ[done, j], ξ = ξ // m[j, j + 1 → j]; st = st /. st[[j + 2]] → st[[j + 1]]];
   If[MemberQ[done, j - 1], ξ = ξ // m[st[[j]], j → st[[j]]]; st = st /. st[[j + 1]] → st[[j]]];
   done = done ⋃ {i - 1, i, j - 1, j};
   todo = DeleteCases[todo, cx]
  ];
  Factor@ξ
 ]
```

pdf

In[∘]:=
```
K = Knot[8, 17]; Alexander[K][T]
Z[K]
```

pdf

Out[∘]= $11 - \frac{1}{T^3} + \frac{4}{T^2} - \frac{8}{T} - 8\,T + 4\,T^2 - T^3$

pdf

Out[∘]= $-\frac{\left(1 - 4\,T + 8\,T^2 - 11\,T^3 + 8\,T^4 - 4\,T^5 + T^6\right)\,\left(e_{0,0,0}[0] + e_{0,1,1}[0]\right)}{T^4}$