pdf

In[∘]:=
```
Once[<< KnotTheory`];
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, ■, ■]];
```

pdf

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

pdf

In[∘]:=

MT =

□	a	b	c	d	ka	kb	kc	kd
a	a	b	0	0	ka	kb	0	0
b	0	0	a	b	0	0	-ka	-kb
c	c	d	0	0	-kc	-kd	0	0
d	0	0	c	d	0	0	kc	kd
ka	ka	kb	0	0	a	b	0	0
kb	0	0	ka	kb	0	0	-a	-b
kc	kc	kd	0	0	-c	-d	0	0
kd	0	0	kc	kd	0	0	c	d

;

```
MRule_{i_,j_→k_} := Flatten@Table[MT[[α, 1]]_i MT[[1, β]]_j →
    (MT[[α, β]] /. v : (a | b | c | d | ka | kb | kc | kd) :→ v_k), {α, 2, 9}, {β, 2, 9}];
ℰ_ // m_{i_,j_→k_} := Expand[ℰ] /. MRule_{i,j→k}
```

In[∘]:= MRule$_{1,2\to3}$

Out[∘]= $\{a_1 a_2 \to a_3,\ a_1 b_2 \to b_3,\ a_1 c_2 \to 0,\ a_1 d_2 \to 0,\ a_1 ka_2 \to ka_3,\ a_1 kb_2 \to kb_3,\ a_1 kc_2 \to 0,$
$a_1 kd_2 \to 0,\ a_2 b_1 \to 0,\ b_1 b_2 \to 0,\ b_1 c_2 \to a_3,\ b_1 d_2 \to b_3,\ b_1 ka_2 \to 0,\ b_1 kb_2 \to 0,\ b_1 kc_2 \to -ka_3,$
$b_1 kd_2 \to -kb_3,\ a_2 c_1 \to c_3,\ b_2 c_1 \to d_3,\ c_1 c_2 \to 0,\ c_1 d_2 \to 0,\ c_1 ka_2 \to -kc_3,\ c_1 kb_2 \to -kd_3,$
$c_1 kc_2 \to 0,\ c_1 kd_2 \to 0,\ a_2 d_1 \to 0,\ b_2 d_1 \to 0,\ c_2 d_1 \to c_3,\ d_1 d_2 \to d_3,\ d_1 ka_2 \to 0,\ d_1 kb_2 \to 0,$
$d_1 kc_2 \to kc_3,\ d_1 kd_2 \to kd_3,\ a_2 ka_1 \to ka_3,\ b_2 ka_1 \to kb_3,\ c_2 ka_1 \to 0,\ d_2 ka_1 \to 0,\ ka_1 ka_2 \to a_3,$
$ka_1 kb_2 \to b_3,\ ka_1 kc_2 \to 0,\ ka_1 kd_2 \to 0,\ a_2 kb_1 \to 0,\ b_2 kb_1 \to 0,\ c_2 kb_1 \to ka_3,\ d_2 kb_1 \to kb_3,$
$ka_2 kb_1 \to 0,\ kb_1 kb_2 \to 0,\ kb_1 kc_2 \to -a_3,\ kb_1 kd_2 \to -b_3,\ a_2 kc_1 \to kc_3,\ b_2 kc_1 \to kd_3,$
$c_2 kc_1 \to 0,\ d_2 kc_1 \to 0,\ ka_2 kc_1 \to -c_3,\ kb_2 kc_1 \to -d_3,\ kc_1 kc_2 \to 0,\ kc_1 kd_2 \to 0,\ a_2 kd_1 \to 0,$
$b_2 kd_1 \to 0,\ c_2 kd_1 \to kc_3,\ d_2 kd_1 \to kd_3,\ ka_2 kd_1 \to 0,\ kb_2 kd_1 \to 0,\ kc_2 kd_1 \to c_3,\ kd_1 kd_2 \to d_3\}$

pdf

In[∘]:=
```
KBasis[{i_}] := {a_i, b_i, c_i, d_i, ka_i, kb_i, kc_i, kd_i};
KBasis[{i_, is__}] := Flatten@Outer[Times, KBasis[{i}], KBasis[{is}]]
```

pdf

In[∘]:=
```
η_{i_} := a_i + d_i;
γ_{i_} := ka_i + kd_i;
```

pdf

In[∘]:=
```
lhs = η_1 KBasis[{2}] // m_{1,2→1};
HL[lhs == KBasis[{1}]]
```

pdf

Out[∘]= True

pdf

In[∘]:=
```
lhs = η1 KBasis[{2}] // Expand // m1,2→1;
HL[lhs == KBasis[{1}]]
```

pdf

Out[∘]= True

pdf

In[∘]:=
```
Short[lhs = KBasis[{1, 2, 3}] // m1,2→1 // m1,3→1]
rhs = KBasis[{1, 2, 3}] // m2,3→2 // m1,2→1!;
lhs == rhs // HL
```

pdf

Out[∘]//Short= $\{a_1, b_1, 0, 0, ka_1, kb_1, 0, 0, 0, 0, a_1, b_1, 0, 0, -ka_1, \ll 482 \gg, d_1, 0, 0, -kc_1, -kd_1, 0, 0, 0, 0, c_1, d_1, 0, 0, kc_1, kd_1\}$

pdf

Out[∘]= True

pdf

In[∘]:= $R_{i\_,j\_} := a_i\, a_j + d_i\, a_j + T\, a_i\, d_j - (1 - T)\, kc_i\, kb_j - T\, d_i\, d_j$

pdf

In[∘]:=
```
Short[lhs = R1,2 R4,3 R5,6 // m1,4→1 // m2,5→2 // m3,6→3];
rhs = R2,3 R1,4 R5,6 // m1,5→1 // m2,6→2 // m3,4→3;
lhs == rhs // HL
```

pdf

Out[∘]= True

pdf

In[∘]:= $\overline{R}_{i\_,j\_} := a_i\, a_j + d_i\, a_j + T^{-1}\, a_i\, d_j - \left(1 - T^{-1}\right) kc_i\, kb_j - T^{-1}\, d_i\, d_j$

pdf

In[∘]:=
```
Short[lhs = R1,2 R̄3,4 // m1,3→1 // m2,4→2]
rhs = η1 η2 // Expand;
lhs == rhs // HL
```

pdf

Out[∘]//Short= $a_1\, a_2 + a_2\, d_1 + a_1\, d_2 + d_1\, d_2$

pdf

Out[∘]= True

pdf

In[∘]:=
```
Short[lhs = R1,2 R̄3,4 // m1,3→1 // m4,2→2]
rhs = η1 η2 // Expand;
Simplify[lhs - rhs]
```

pdf

Out[∘]//Short= $a_1\, a_2 + a_2\, d_1 + a_1\, d_2 + d_1\, d_2 - 2\, kb_2\, kc_1 + 2\, T\, kb_2\, kc_1$

pdf

Out[∘]= $2 \times (-1 + T)\, kb_2\, kc_1$

pdf

In[◦]:=
```
lhs = R1,4 R̄5,2 γ3 // m2,4→2 // m1,3→1 // m1,5→1
rhs = γ1 η2 // Expand;
lhs == rhs // HL
```

pdf

Out[◦]= $a_2\, ka_1 + d_2\, ka_1 + a_2\, kd_1 + d_2\, kd_1$

pdf

Out[◦]= True

pdf

RVK and Z

pdf

RVK, rot, Z modified from 2016-09/OneSmidgen.nb. See also in AP/Projects/SL2Invariant/.

pdf

Some details of the code below are at http://drorbn.net/bbs/show?shot=Dror-160920-151350.jpg.

pdf

In[◦]:=
```
RVK::usage =
  "RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings
    xs and a length 2n list of rotation numbers rots. Crossing
    sites are indexed 1 through 2n, and rots⟦k⟧ is the rotation
    between site k-1 and site k. RVK is also a casting operator
    converting to the RVK presentation from other knot presentations.";
```

pdf

In[◦]:=
```
RVK[pd_PD] := Module[{n, xs, x, rots, front = {0}, k},
   n = Length@pd; rots = Table[0, {2 n}];
   xs = Cases[pd, x_X :> { Xp[x⟦4⟧, x⟦1⟧]  PositiveQ@x
                           Xm[x⟦2⟧, x⟦1⟧]  True       ];
   For[k = 0, k < 2 n, ++k, If[k == 0 ∨ FreeQ[front, -k],
     front = Flatten@Replace[front, k → (xs /. {
            Xp[k + 1, l_] | Xm[l_, k + 1] :> {l, k + 1, 1 - l},
            Xp[l_, k + 1] | Xm[k + 1, l_] :> (++rots⟦l⟧; {1 - l, k + 1, l}),
            _Xp | _Xm :> {}
           }), {1}],
     Cases[front, k | -k] /. {k, -k} :> --rots⟦k + 1⟧;
    ]];
   RVK[xs, rots] ];
RVK[K_] := RVK[PD[K]];
```

pdf

In[◦]:=
```
Kink_i_ := Kink_i = R1,3 γ2 // m1,2→1 // m1,3→i;
Kink̄_i_ := Kink̄_i = R̄1,3 γ2 // m1,2→1 // m1,3→i;
```

pdf

In[∘]:=

```
rot_i_[n_] := { η_i  EvenQ[n]
              { γ_i  OddQ[n]
```

pdf

In[∘]:=

```
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := Module[{todo, n, rots, ξ, done, st, cx, ξ1, i, j, k, k1, k2, k3},
  {todo, rots} = List @@ rvk;
  AppendTo[rots, 0];
  n = Length[todo];
  ξ = η_0;
  done = {0};
  st = Range[0, 2 n + 1];
  While[{} =!= todo,
   {cx} = MaximalBy[todo, Length[done ⋂ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
   {i, j} = List @@ cx;
   ξ1 = Switch[Head[cx],
      Xp, (R_{i,j} OverBar[Kink]_k) // m_{j,k→j},
      Xm, (OverBar[R]_{i,j} Kink_k) // m_{j,k→j}
     ];
   ξ1 = (rot_k[rots[[i]]] ξ1) // m_{k,i→i}; rots[[i]] = 0;
   ξ1 = (ξ1 rot_k[rots[[i + 1]]]) // m_{i,k→i}; rots[[i + 1]] = 0;
   ξ1 = (rot_k[rots[[j]]] ξ1) // m_{k,j→j}; rots[[j]] = 0;
   ξ1 = (ξ1 rot_k[rots[[j + 1]]]) // m_{j,k→j}; rots[[j + 1]] = 0;
   ξ *= ξ1;
   If[MemberQ[done, i], ξ = ξ // m_{i,i+1→i}; st = st /. st[[i + 2]] → st[[i + 1]]];
   If[MemberQ[done, i - 1], ξ = ξ // m_{st[[i]],i→st[[i]]}; st = st /. st[[i + 1]] → st[[i]]];
   If[MemberQ[done, j], ξ = ξ // m_{j,j+1→j}; st = st /. st[[j + 2]] → st[[j + 1]]];
   If[MemberQ[done, j - 1], ξ = ξ // m_{st[[j]],j→st[[j]]}; st = st /. st[[j + 1]] → st[[j]]];
   done = done ⋃ {i - 1, i, j - 1, j};
   todo = DeleteCases[todo, cx]
  ];
  Factor@ξ
 ]
```

pdf

In[ ]:=
```
K = Knot[8, 17]; Alexander[K][T]
Z[K]
```

pdf

KnotTheory: Loading precomputed data in PD4Knots`.

pdf

Out[ ]= $11 - \frac{1}{T^3} + \frac{4}{T^2} - \frac{8}{T} - 8\,T + 4\,T^2 - T^3$

pdf

Out[ ]= $-\frac{\left(1 - 4\,T + 8\,T^2 - 11\,T^3 + 8\,T^4 - 4\,T^5 + T^6\right)\,(a_0 + d_0)}{T^4}$