

$$\begin{aligned} \text{In}[*]:= & \mathbf{R}_{i,j} := \text{Expand} \left[T^{-1/2} \left(a_i a_j + d_i a_j + T a_i d_j - (1 - T) k c_i k b_j - T d_i d_j \right) \right]; \\ & \bar{\mathbf{R}}_{i,j} := \text{Expand} \left[T^{1/2} \left(a_i a_j + d_i a_j + T^{-1} a_i d_j - (1 - T^{-1}) k c_i k b_j - T^{-1} d_i d_j \right) \right]; \\ & \mathbf{C}_{i,j} := \text{Expand} \left[T^{-1/2} (k a_i + k d_i) \right]; \quad \bar{\mathbf{C}}_{i,j} := \text{Expand} \left[T^{1/2} (k a_i + k d_i) \right]; \end{aligned}$$

$$\text{In}[*]:= \mathbf{R}_{i,j} (k a_1 - k d_1) // \mathbf{m}_{j,1 \rightarrow j}$$

$$\text{Out}[*]:= \frac{a_i k a_j}{\sqrt{T}} + \frac{d_i k a_j}{\sqrt{T}} - \frac{b_j k c_i}{\sqrt{T}} + \sqrt{T} b_j k c_i - \sqrt{T} a_i k d_j + \sqrt{T} d_i k d_j$$

$$\text{In}[*]:= \bar{\mathbf{R}}_{i,j} (k a_1 - k d_1) // \mathbf{m}_{j,1 \rightarrow j}$$

$$\text{Out}[*]:= \sqrt{T} a_i k a_j + \sqrt{T} d_i k a_j + \frac{b_j k c_i}{\sqrt{T}} - \sqrt{T} b_j k c_i - \frac{a_i k d_j}{\sqrt{T}} + \frac{d_i k d_j}{\sqrt{T}}$$

$$\text{In}[*]:= \text{Factor} \left[\frac{a_i k a_j}{\sqrt{T}} + \frac{d_i k a_j}{\sqrt{T}} - \frac{b_j k c_i}{\sqrt{T}} + \sqrt{T} b_j k c_i - \sqrt{T} a_i k d_j + \sqrt{T} d_i k d_j \right]$$

$$\text{Out}[*]:= - \frac{- a_i k a_j - d_i k a_j + b_j k c_i - T b_j k c_i + T a_i k d_j - T d_i k d_j}{\sqrt{T}}$$

$$\text{In}[*]:= \text{Expand} \left[T^{1/2} \left(\frac{a_i k a_j}{\sqrt{T}} + \frac{d_i k a_j}{\sqrt{T}} - \frac{b_j k c_i}{\sqrt{T}} + \sqrt{T} b_j k c_i - \sqrt{T} a_i k d_j + \sqrt{T} d_i k d_j \right) \right]$$

$$\text{Out}[*]:= a_i k a_j + d_i k a_j - b_j k c_i + T b_j k c_i - T a_i k d_j + T d_i k d_j$$

$$\text{In}[*]:= \text{Expand} \left[T^{-1/2} \left(\sqrt{T} a_i k a_j + \sqrt{T} d_i k a_j + \frac{b_j k c_i}{\sqrt{T}} - \sqrt{T} b_j k c_i - \frac{a_i k d_j}{\sqrt{T}} + \frac{d_i k d_j}{\sqrt{T}} \right) \right]$$

$$\text{Out}[*]:= a_i k a_j + d_i k a_j - b_j k c_i + \frac{b_j k c_i}{T} - \frac{a_i k d_j}{T} + \frac{d_i k d_j}{T}$$

$$\text{In}[*]:= \mathbf{R}_{i,j} := \text{Expand} \left[\frac{a_i k a_j}{\sqrt{T}} + \frac{d_i k a_j}{\sqrt{T}} - \frac{b_j k c_i}{\sqrt{T}} + \sqrt{T} b_j k c_i - \sqrt{T} a_i k d_j + \sqrt{T} d_i k d_j \right];$$

$$\bar{\mathbf{R}}_{i,j} := \text{Expand} \left[\sqrt{T} a_i k a_j + \sqrt{T} d_i k a_j + \frac{b_j k c_i}{\sqrt{T}} - \sqrt{T} b_j k c_i - \frac{a_i k d_j}{\sqrt{T}} + \frac{d_i k d_j}{\sqrt{T}} \right];$$

$$\mathbf{C}_{i,j} := \text{Expand} \left[T^{-1/2} (k a_i + k d_i) \right]; \quad \bar{\mathbf{C}}_{i,j} := \text{Expand} \left[T^{1/2} (k a_i + k d_i) \right];$$

$$\text{In}[*]:= \text{Simplify} \left[\mathbf{R}_{i,j} == T^{-1/2} \left(a_i k a_j - T a_i k d_j + (T - 1) b_j k c_i + d_i k a_j + T d_i k d_j \right) \right]$$

Out[*]= True

$$\text{In}[*]:= \text{Simplify} \left[\bar{\mathbf{R}}_{i,j} == T^{1/2} \left(a_i k a_j - T^{-1} a_i k d_j + (T^{-1} - 1) b_j k c_i + d_i k a_j + T^{-1} d_i k d_j \right) \right]$$

Out[*]= True

Some bad news:

In[*]:= $\varphi_1 = \{a_1 \rightarrow 1/2, b_1 \rightarrow 0, c_1 \rightarrow 0, d_1 \rightarrow 1/2, ka_1 \rightarrow 0, kb_1 \rightarrow 0, kc_1 \rightarrow 0, kd_1 \rightarrow 0\};$
Simplify[{R_{1,2}, R_{1,2} R_{4,3} // m_{1,4→1}} /. φ_1]

$$\text{Out[*]} = \left\{ \frac{ka_2}{\sqrt{T}}, \frac{ka_2 ka_3}{T} + T kd_2 kd_3 \right\}$$

In[*]:= $\varphi_3 = \{a_3 \rightarrow 1/2, b_3 \rightarrow 0, c_3 \rightarrow 0, d_3 \rightarrow 1/(2T), ka_3 \rightarrow 0, kb_3 \rightarrow 0, kc_3 \rightarrow 0, kd_3 \rightarrow 0\};$
Simplify[{R_{1,3}, R_{2,3} R_{1,4} // m_{3,4→3}} /. φ_3]

$$\text{Out[*]} = \left\{ 0, \frac{a_1 ((1+T) a_2 - (-1+T) d_2) + d_1 (-((-1+T) a_2) + (1+T) d_2)}{2T} \right\}$$