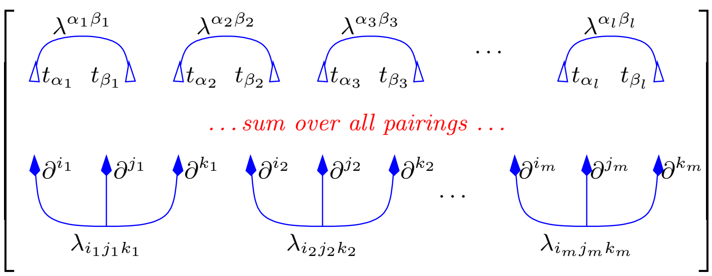
Di dah Di dah

Gaussian Integration. (λ_{ij}) is a symmetric positive definite matrix and (λ^{ij}) is its inverse, and (λ_{ijk}) are the coefficients of some cubic form. Denote by $(x^i)_{i=1}^n$ the coordinates of $\mathbb{R}^n$, let $(t_i)_{i=1}^n$ be a set of "dual" variables, and let ∂^i denote $\frac{\partial}{\partial t_i}$. Also let $C \coloneqq \frac{(2\pi)^{n/2}}{\det(\lambda_{ij})}$. Then

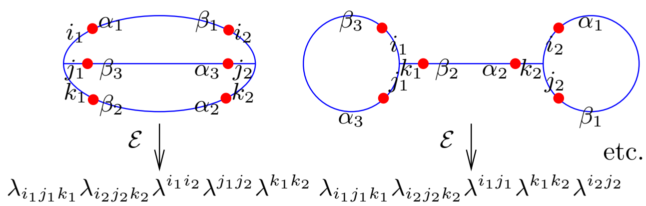
$$\int_{\mathbb{R}^n} e^{-\frac{1}{2}\lambda_{ij}x^ix^j+\frac{\epsilon}{6}\lambda_{ijk}x^ix^jx^k} = \sum_{m\geq 0}\frac{\epsilon^m}{6^m m!}\int_{\mathbb{R}^n}(\lambda_{ijk}x^ix^jx^k)^m e^{-\frac{1}{2}\lambda_{ij}x^ix^j}$$

$$= \sum_{m\geq 0}\frac{C\epsilon^m}{6^m m!}(\lambda_{ijk}\partial^i\partial^j\partial^k)^m e^{\frac{1}{2}\lambda^{\alpha\beta}t_\alpha t_\beta}\bigg|_{t_\alpha=0} = \sum_{\substack{m,l\geq 0\\ 3m=2l}}\frac{C\epsilon^m}{6^m m! 2^l l!}\left(\lambda_{ijk}\partial^i\partial^j\partial^k\right)^m\left(\lambda^{\alpha\beta}t_\alpha t_\beta\right)^l$$

$$= \sum_{\substack{m,l\geq 0\\ 3m=2l}}\frac{C\epsilon^m}{6^m m! 2^l l!}$$



$$= \sum_{\substack{m,l\geq 0\\ 3m=2l}}\frac{C\epsilon^m}{6^m m! 2^l l!}\sum_{\substack{m\text{-vertex fully marked}\\ \text{Feynman diagrams } D}}\mathcal{E}(D)$$



$$= C\sum_{\substack{\text{unmarked Feynman}\\ \text{diagrams } D}}\frac{\epsilon^{m(D)}\mathcal{E}(D)}{|\mathrm{Aut}(D)|}.$$

Claim. The number of pairings that produce a given unmarked Feynman diagram D is $\frac{6^m m! 2^l l!}{|\mathrm{Aut}(D)|}$.

Proof of the Claim. The group $G_{m,l} \coloneqq [(S_3)^m \rtimes S_m] \times [(S_2)^l \rtimes S_l]$ acts on the set of pairings, the action is transitive on the set of pairings P that produce a given D, and the stabilizer of any given P is $\mathrm{Aut}(D)$. $\square$