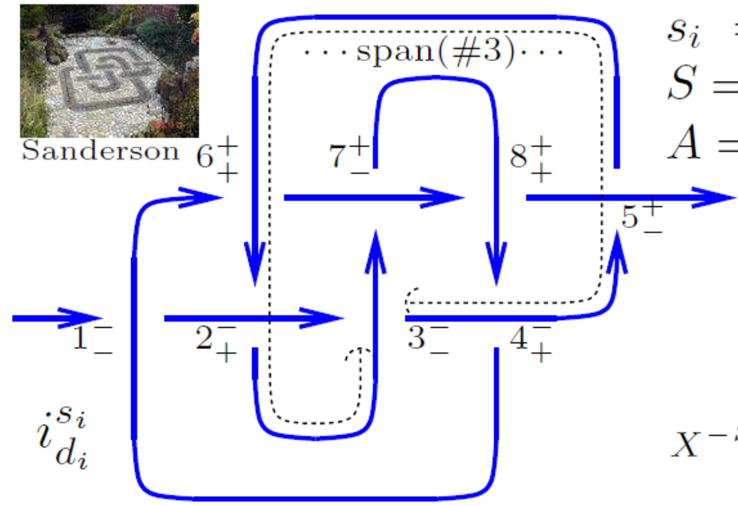
Handout for April 18 and 25

The Alexander Theorem.



$T_{ij} = |\mathrm{low}(\#j) \in \mathrm{span}(\#i)|$,
$s_i = \mathrm{sign}(\#i)$, $d_i = \mathrm{dir}(\#i)$,
$S = \mathrm{diag}(s_i d_i)$,
$A = \det\left(I + T(I - X^{-S})\right)$.

$$T = \begin{pmatrix} 0&1&1&1&1&0&1&0\\ 0&0&1&0&1&0&0&0\\ 0&1&0&0&1&0&0&0\\ 0&1&0&0&1&0&1&0\\ 0&1&0&1&0&1&1&1\\ 0&1&0&1&0&0&1&0\\ 0&0&0&1&0&1&0&0\\ 0&0&0&1&0&1&0&0 \end{pmatrix},$$

$$X^{-S} = \mathrm{diag}\left(\tfrac{1}{X}, X, \tfrac{1}{X}, X, X, \tfrac{1}{X}, X, \tfrac{1}{X}\right).$$

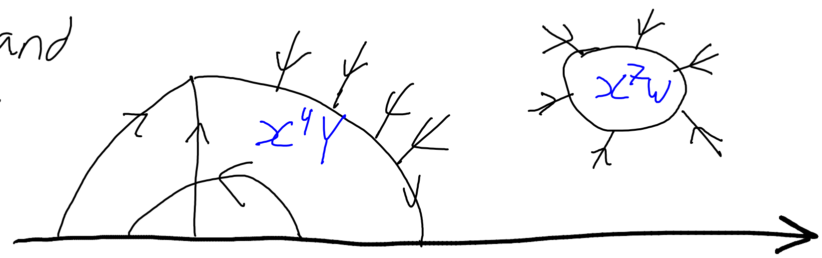
Conjecture. For u-knots, A is the Alexander polynomial.

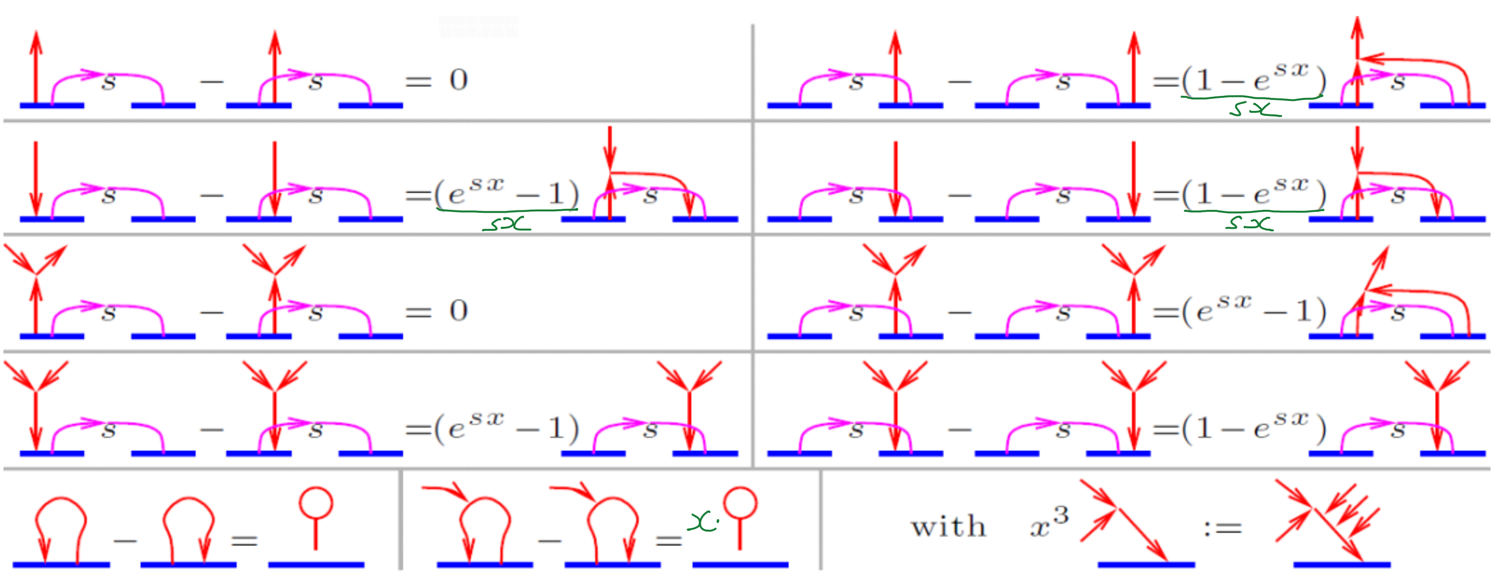
Theorem. With $w : x^k \mapsto w_k = $ (the k-wheel),

$$Z = N \exp_{\mathcal{A}^w}\left(-w\left(\log_{\mathbb{Q}[\![x]\!]} A(e^x)\right)\right)$$

mod $w_k w_l = w_{k+l}$,
$Z = N \cdot A^{-1}(e^x)$

Claim. "Detached wheels" and "hairy Y's" make sense in $\mathcal{A}(\uparrow)$.





Snips from http://www.math.toronto.edu/~drorbn/Talks/Toronto-1005/