Feb 1 - 2.3: Finite Type Invariants

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2.3. F.T.I of v-braids & w-braids.

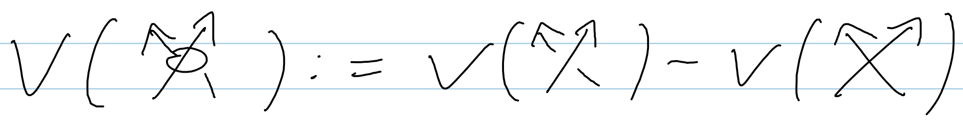
2.3.1 The "classical/pictorial" approach.

1. F.T for knots

done line

Never done line — lecture given differently by Peter Lee

2. semi-virtual xings,



"Type m" $\Leftrightarrow V(\text{anything w/} > m \text{ s.v.}) = 0$

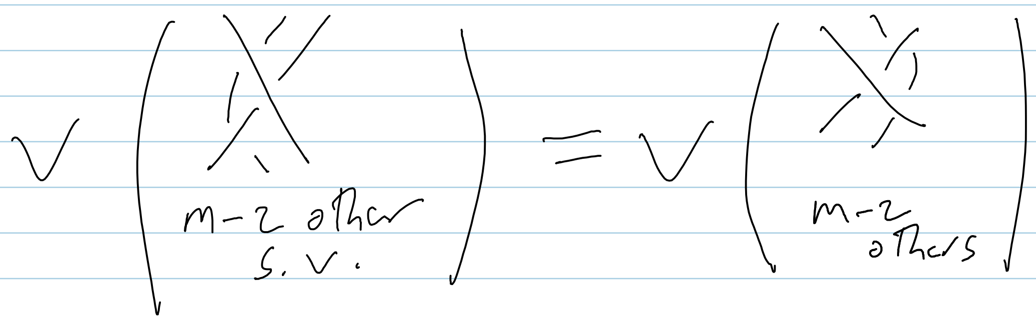
$$W = W_m(V) := V\Big|_{\text{braids w/ exactly } m \text{ s.v.}}$$

$$\mathcal{G}_m \mathcal{D}^v_n := \{m\text{-singular}\} \Big/ \{m+1 \text{ singular}\}$$

$= \text{arrow diagrams}$

Claim. W satisfies the "6T" relation.

Pf.



Now write $\sigma_i = s_i + (\sigma_i - s_i) =$

$= s_i + \overline{\sigma_i} \ldots$

Claim. In the w case, W satisfies OC & $4\vec{T}$. In the u case, ...

Definition. $\mathcal{A}_n^w$, $\mathcal{A}_n^v$, $\mathcal{A}_n^u$

"$\mathcal{A}^{v}_{-}$"

Definition 2.11 An expansion $Z: vB_n \to \mathcal{A}_n^u$

Theorem. An expansion exists iff every W comes from a V.

2.3.2 F.T.I, the "algebraic" approach.

Start from a general group G, define

$$\mathbb{F}G,\ \mathcal{I},\ \mathcal{A}(G),\ \text{expansion}$$

Claim. An invariant of v-braids is of type m iff it vanishes on $\mathcal{I}^{m+1}$

Claim.

$$\mathcal{A}_n^v \twoheadrightarrow \mathcal{A}(vB_n)$$

Def. Expansion $Z: G \to \mathcal{A}(G)$

Claim An $\mathcal{A}_n^v$-expansion implies an expansion & $\mathcal{A}(G) \cong \mathcal{A}_n^v$.

Claim. $\mathcal{A}$ is a functor.