Feb 15 - 2.5: Compatibilities, Power, Uniqueness, Wens, u-Braids

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7:04 PM

Recall: $G \rightsquigarrow \mathbb{Q}G \supset I \quad \mathcal{A} = \bigoplus I^n/I^{n+1}$
$\mathcal{A}^Q = T(I/I^2 = V)/\ker(V\otimes V \to I^2/I^3)$ } functors? | on board

An expansion $Z: G \to \mathcal{A}^Q$ also proves $\mathcal{A}^Q \cong \mathcal{A}$

For PwB_n, $\mathcal{A}_n^w = \mathcal{A}_n^{wQ} = \left\langle a_{ij} : \begin{matrix}[a_{ij}+a_{ik}, a_{jk}] = 0 \\ [a_{ij}, a_{ik}] = [a_{ij}, a_{kl}] = 0\end{matrix} \right\rangle$

$= \langle \ldots \rangle / 4T, TC$

$Z: PwB_n \to \mathcal{A}_n^w$ by $\sigma_{ij} \to e^{a_{ij}}$

Are we meeting next week?

2.5.1 Compatibility with Braid operations.

2.5.1.1 Braid inversion. [always $g \mapsto g^{-1}$ induces $(-1)^{\deg}$ on $\mathcal{A}/\mathcal{A}^Q$ & Z is compatible.]

2.5.1.2 "Braid cloning".

Always $\mathcal{A}^Q(G\times G) = \mathcal{A}(G)\otimes\mathcal{A}(G)$ & $g \overset{\square}{\mapsto} (g,g)$ induces
$V \ni a \overset{\square}{\mapsto} a\otimes 1 + 1\otimes a \in V\otimes V$. Z is compatible:

$$\begin{array}{ccc} PwB & \xrightarrow{\square} & PwB \times PwB \\ \downarrow Z & & \downarrow Z\otimes Z \\ \mathcal{A}^w & \xrightarrow{\square} & \mathcal{A}^w \otimes \mathcal{A}^w \end{array}$$

$\square Z(\sigma_{ij}) \overset{?}{=} Z(\sigma_{ij}) \otimes Z(\sigma_{ij})$

$\square(e^a) \overset{?}{=} e^a \otimes e^a$

2.5.1.3 "Strand insertions"

2.5.1.4 "Strand deletions"

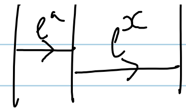
2.5.1.5 Compatibility with the action on F_n:

$$\begin{array}{ccc} \xi \in F_n & \leftleftarrows & PwB_n \ni \beta \\ Z\downarrow & & \downarrow Z \\ FA_n & \leftleftarrows & \mathcal{A}_n^w \end{array}$$

Leib   mult.

$x_j // a_{ij} = [x_i, x_j]$

$$Z(\xi /\!/ \beta) = Z(\xi) /\!/ Z(\beta)$$



$\xi_j /\!/ \sigma_{ij}$: $\xi_i \, \xi_j \, \xi_i^{-1}$ $\quad e^{x_j} /\!/ e^{a_{ij}}$

2.5.1.6 unzipping a strand

$Z(u_i B) \stackrel{?}{=} u_i Z(B)$ with $B = \overset{}{\underset{1 \quad 2}{\diagup\!\!\!\diagdown}}$, true for u_2 but false for u_1

2.5.2 Power and Injectivity

Theorem (Berceanu & Papadima)
$Z : wB_n \longrightarrow \mathcal{A}_n^w$ is injective. [hence FTI sep. braids]

Proof. $Z(B_1) = Z(B_2) \Rightarrow \forall \xi \;\; Z(\xi) /\!/ Z(B_1) = Z(\xi) /\!/ Z(B_2)$
$\Rightarrow \forall \xi \;\; Z(\xi /\!/ B_1) = Z(\xi /\!/ B_2) \Rightarrow \forall \xi \;\; \xi /\!/ B_1 = \xi /\!/ B_2$
$\Rightarrow B_1 = B_2$

Remark unlike the uB case & the wK case, $\mathcal{A}_n^w$ is not understood.

2.5.3 Uniqueness

Prop. If $Z_{1,2} : wB_n \to \mathcal{A}_n^w$ are expansions, then $\exists$ degree increasing map $P : \mathcal{A}_n^w \to \mathcal{A}_n^w$ s.t.
$Z_2 = (1 + P) \circ Z_1$. Review

Prop If $Z_{1,2}$ are homomorphic expansions that commute w/ braid cloning & w/ strand insertion, then $Z_1 = Z_2$. Review

2.5.4 Non-horizontal Flying rings.

No expansion unless w_i are made a part of the skeleton! Review

2.5.5 The relationship w/ u-braids.

$$\begin{tikzcd} PuB \arrow[r, dashed, "Z^u"] \arrow[d, "a"] & \mathcal{A}^u \arrow[d, "\alpha"] \\ PwB \arrow[r, "Z^w"] & \mathcal{A}^w \end{tikzcd}$$

never commutative.

Review.

$$\begin{tikzcd} PuB_n \arrow[r, "Z^u_c"] \arrow[d, "a"] & \mathcal{A}^u_n \arrow[d, "\alpha"] \\ PwB_n \arrow[r, "Z^w"] & \mathcal{A}^w_n \end{tikzcd}$$

"commutes up to conjugation in a bigger space"