3.1-3.4: v- and w-Knots, the Basics

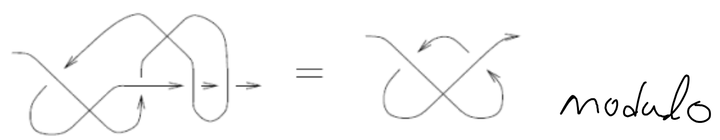
February-21-12

Section 3. knots are the wrong objects for study in knot Theory!

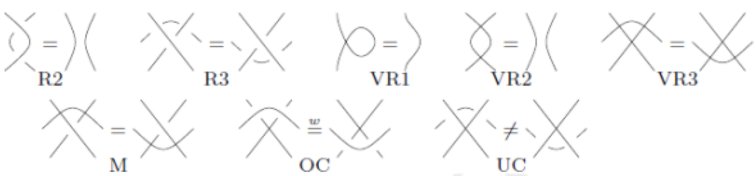
* Not nicely presented.
* No interesting ops.

3.1 v-knots & w-knots.

Definition: v-knots are



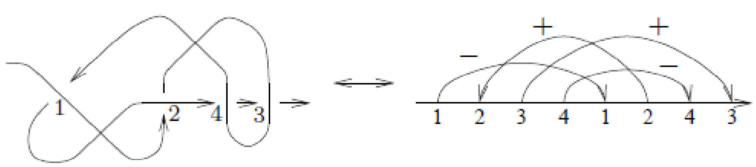
modulo



Warning 3.2 $K^v(\uparrow) \not\cong K^v(\bigcirc)$

Warning 3.3 $K^v(\uparrow)$ is a monoid, <u>not</u> Abelian.

Remark 3.4. Gauss diagrams:



Remark 3.5. We have a split-exact sequence

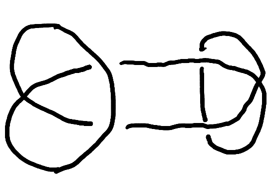
$$0 \longrightarrow \mathbb{Z}^2 \longrightarrow \left\{\begin{matrix}\text{long}\\ \text{v-knots}\end{matrix}\right\} \longrightarrow \left\{\begin{matrix}\text{unframed}\\ \text{long}\\ \text{v-knots}\end{matrix}\right\} \longrightarrow 1$$

with splitting

$$sl = (sl_L, sl_R): \left\{\begin{matrix}\text{long}\\ \text{v-knots}\end{matrix}\right\} \longrightarrow \mathbb{Z}^2$$

by signed counting of left/right arrows.

Remark 3.7 $K^u \hookrightarrow K^w$, yet



Kishino's knot is trivial in K^w & $(K^w)^{op}$.

Important Leftover:

2.5.5 In the lovely diagram on the right,

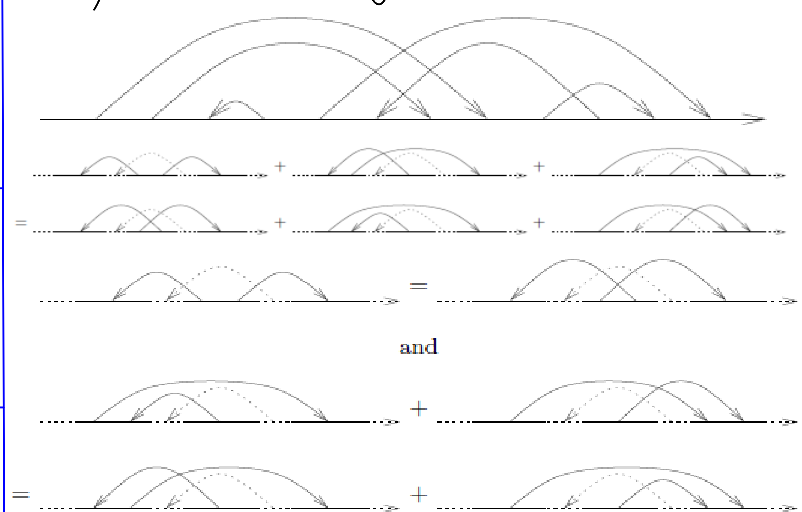
$$\begin{array}{ccc} PuB_n & \xrightarrow{Z^u_c} & \mathcal{A}^u_n \\ \downarrow a & & \downarrow \alpha \\ PwB_n & \xrightarrow{Z^w} & \mathcal{A}^w_n \end{array}$$

$\alpha: \vert\!-\!\vert \mapsto \vert\!\to\!\vert + \vert\!\leftarrow\!\vert$, and it never commutes!!

Remark. I don't fully understand the topology of w-knots as surfaces in $\mathbb{R}^4$!

3.2 Finite Type Invariants

Def,... Arrow diagrams, ... relations:



Proposition 3.9. $\mathcal{A}^v$ & $\mathcal{A}^w$ are graded co-commutative bi-algebras & The Milnor-Moore Theorem applies.

<table>
<tr><th>m</th><th></th><th>0</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th></tr>
<tr><td rowspan="2">$\dim \mathcal{G}_m\mathcal{A}^-(\uparrow)$</td><td>u | v</td><td>1 | 1</td><td>1 | 2</td><td>2 | 7</td><td>3 | 27</td><td>6 | 139</td><td>10 |?</td><td>19 |?</td><td>33 |?</td></tr>
<tr><td>w</td><td>1</td><td>2</td><td>4</td><td>7</td><td>12</td><td>19</td><td>30</td><td>45</td></tr>
<tr><td rowspan="2">$\dim \mathcal{G}_m\mathcal{L}ie^-(\uparrow)$</td><td>u | v</td><td>1 | 1</td><td>1 | 2</td><td>2 | 7</td><td>3 | 27</td><td>6 | ≥128</td><td>10 |?</td><td>19 |?</td><td>33 |?</td></tr>
<tr><td>w</td><td>1</td><td>2</td><td>4</td><td>7</td><td>12</td><td>19</td><td>30</td><td>45</td></tr>
<tr><td rowspan="2">$\dim \mathcal{G}_m\mathcal{A}^{r-}(\uparrow)$</td><td>u | v</td><td>1 | 1</td><td>0 | 0</td><td>1 | 2</td><td>1 | 7</td><td>3 | 42</td><td>4 |?</td><td>9 |?</td><td>14 |?</td></tr>
<tr><td>w</td><td>1</td><td>0</td><td>1</td><td>1</td><td>2</td><td>2</td><td>4</td><td>4</td></tr>
<tr><td rowspan="2">$\dim \mathcal{G}_m\mathcal{P}^-(\uparrow)$</td><td>u | v</td><td>0 | 0</td><td>1 | 2</td><td>1 | 4</td><td>1 | 15</td><td>2 | 82</td><td>3 |?</td><td>5 |?</td><td>8 |?</td></tr>
<tr><td>w</td><td>0</td><td>2</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td rowspan="2">$\dim \mathcal{G}_m\mathcal{A}^-(\bigcirc)$</td><td>u | v</td><td>1 | 1</td><td>1 | 1</td><td>2 | 2</td><td>3 | 5</td><td>6 | 19</td><td>10 | 77</td><td>19 |?</td><td>33 |?</td></tr>
<tr><td>w</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td rowspan="2">$\dim \mathcal{G}_m\mathcal{A}^{r-}(\bigcirc)$</td><td>u | v</td><td>1 | 1</td><td>0 | 0</td><td>1 | 0</td><td>1 | 1</td><td>3 | 4</td><td>4 | 17</td><td>9 |?</td><td>14 |?</td></tr>
<tr><td>w</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
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Section 3.4 – Theorem 3.11.

There exists an expansion

$$Z: \{\text{w-knots}\} \longrightarrow \mathcal{A}^w(\uparrow)$$