In Progress

Finite Type Invariants of W-Knotted Objects: From Alexander to Kashiwara and Vergne

Joint with Zsuzsanna Dancso

Download WKO.pdf: last updated ≥ March 3, 2012. first edition: not yet.

Abstract. w-Knots, and more generally, w-knotted objects (w-braids, w-tangles, etc.) make a class of knotted objects which is wider but weaker than their "usual" counterparts. To get (say) w-knots from u-knots, one has to allow non-planar "virtual" knot diagrams, hence enlarging the the base set of knots. But then one imposes a new relation, the "overcrossings commute" relation, further beyond the ordinary collection of Reidemeister moves, making w-knotted objects a bit weaker once again.

The group of w-braids was studied (under the name "welded braids") by Fenn, Rimanyi and Rourke [FRR] and was shown to be isomorphic to the McCool group [Mc] of "basis-conjugating" automorphisms of a free group F_n - the smallest subgroup of $\mathrm{Aut}(F_n)$ that contains both braids and permutations. Brendle and Hatcher [BH], in work that traces back to Goldsmith [Gol], have shown this group to be a group of movies of flying rings in $\mathbb{R}^3$. Satoh [Sa] studied several classes of w-knotted objects (under the name "weakly-virtual") and has shown them to be closely related to certain classes of knotted surfaces in $\mathbb{R}^4$. So w-knotted objects are algebraically and topologically interesting.

In this article we study finite type invariants of several classes of w-knotted objects. Following Berceanu and Papadima [BP], we construct a homomorphic universal finite type invariant of w-braids, and hence show that the McCool group of automorphisms is "1-formal". We also construct a homomorphic universal finite type invariant of w-tangles. We find that the universal finite type invariant of w-knots is more or less the Alexander polynomial (details inside).

Much as the spaces $\mathcal{A}$ of chord diagrams for ordinary knotted objects are related to metrized Lie algebras, we find that the spaces $\mathcal{A}^w$ of "arrow diagrams" for w-knotted objects are related to not-necessarily-metrized Lie algebras. Many questions concerning w-knotted objects turn out to be equivalent to questions about Lie algebras. Most notably we find that a homomorphic universal finite type invariant of w-knotted trivalent graphs is essentially the same as a solution of the Kashiwara-Vergne [KV] conjecture and much of the Alekseev-Torrosian [AT] work on Drinfel'd associators and Kashiwara-Vergne can be re-interpreted as a study of w-knotted trivalent graphs.

The true value of w-knots, though, is likely to emerge later, for we expect them to serve as a warmup example for what we expect will be even more interesting - the study of virtual knots, or v-knots. We expect v-knotted objects to provide the global context whose projectivization (or "associated graded structure") will be the Etingof-Kazhdan theory of deformation quantization of Lie bialgebras [EK].

Wideo Companion

The **wClips Seminar** is a series of weekly wideotaped meetings at the University of Toronto, systematically going over the content of the WKO paper section by section.

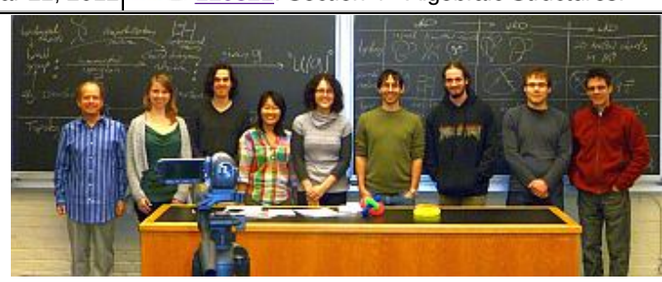
Next Meetings. On Wednesday March 28 we will have an out-of-sequence not-on-video meeting to watch and discuss the video of my talk at George Washington University (see Talks: GWU-1203). We will meet at 12 at my office (not the usual place!), and start watching the video shortly after that (less socializing!). On Wednesday April 4, 2012, 12-2, at Bahen 4010 we will return to the main sequence and talk about Section 3.7, "the Alexander polynomial".

Announcements. small circle, wide circle, UofT, LDT Blog (also here). Email Dror to **join our mailing list!**

Resources. How to use this site, Dror's notebook, blackboard shots.

The wClips

Date	Links
Jan 11, 2012	120111-1: Introduction. 120111-2: Section 2.1 - v-Braids.
Jan 18, 2012	120118-1: An introduction to this web site. 120118-2: Section 2.2 - w-Braids by generators and relations and as flying rings. 120118-3: Section 2.2 - w-Braids - other drawing conventions, "wens".
Jan 25, 2012	120125-1: Section 2.2.3 - basis conjugating automorphisms of F_n. 120125-2: A very quick introduction to finite type invariants in the "u" case.
Feb 1, 2012	120201: Section 2.3 - finite type invariants of v- and w-braids, arrow diagrams, 6T, TC and 4T relations, expansions / universal finite type invariants.
Feb 8, 2012	120208: Review of u,v, and w braids and of Section 2.3.
Feb 15, 2012	120215: Section 2.5 - mostly compatibilities of Z^w, also injectivity and uniqueness of Z^w.
Feb 22, 2012	120222: Section 2.5.5, $\alpha : \mathcal{A}^u \to \mathcal{A}^v$, and Section 3.1 (partially), the definition of v- and w-knots.
Feb 29, 2012	120229: Sections 3.1-3.4: v-Knots and w-Knots: Definitions, framings, finite type invariants, dimensions, and the expansion in the w case.
Mar 7, 2012	120307: Section 3.5: Jacobi diagrams and the bracket-rise theorem.
Mar 14, 2012	120314: Section 3.6 - the relation with Lie algebras.
Mar 21, 2012	120321: Section 4 - Algebraic Structures.



Group photo on January 11, 2012: DBN, ZD, Stephen Morgan, Lucy Zhang, Iva Halacheva, David Li-Bland, Sam Selmani, Oleg Chterental, Peter Lee.