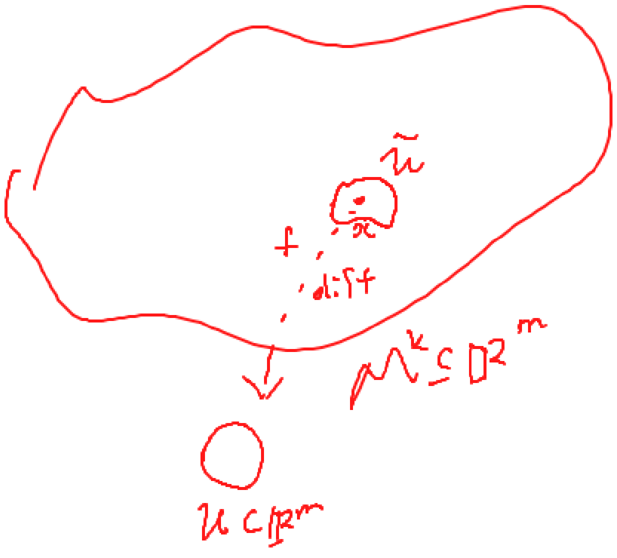
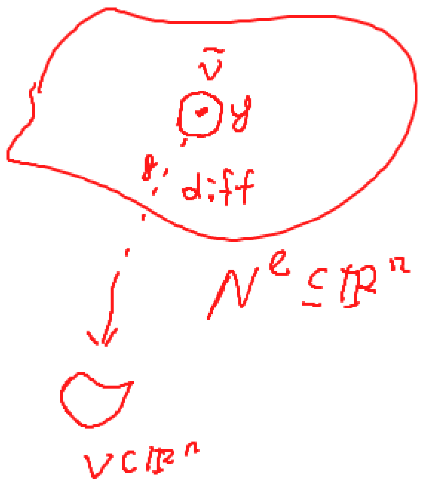
Q1a)





$f : \tilde{U} \to U$

$g : \tilde{V} \to V$

$f \times g : \tilde{U} \times \tilde{V} \to U \times V$

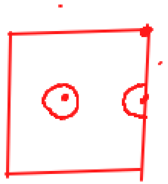
product of mnfds w/out bound. = mnfd w/out bound

1b) $M = N = [0,1]$ ← mnfd w/ boundary

1) corners might be an issue (maybe even the sides?)

2) are there other issues?

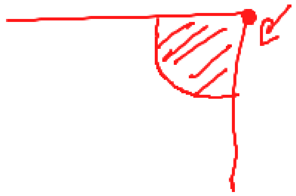
$M \times N = [0,1]^2 =$



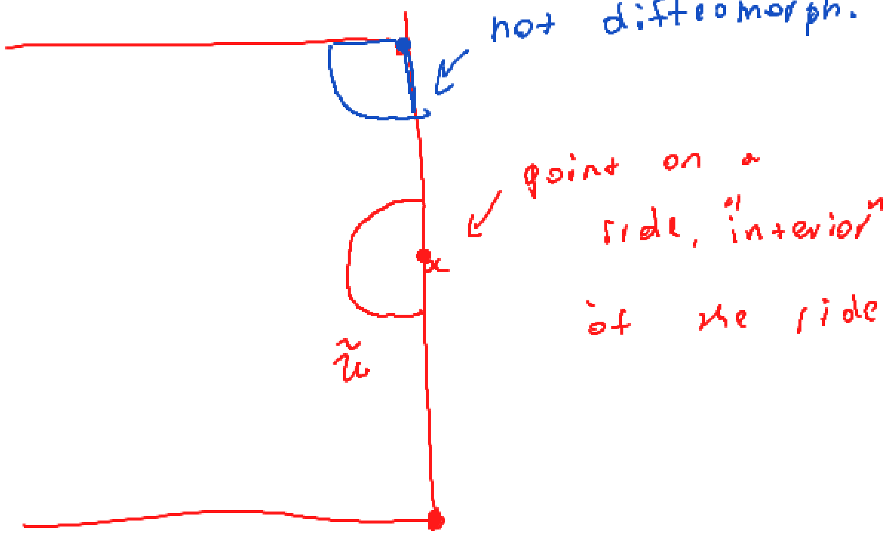
an example of a mfld w/ corners

interior is ok!

sides is ok! (except corners)



← $\boxed{\text{not}}$ diff. to an open subset of $\mathbb{H}^2$



Want to find an open nbhd $\tilde{u}$ of x (in the interior topology of $[0,1]^2$) such that $\tilde{u} \underset{\text{diff}}{\cong} U \subset \mathbb{H}^2$

half-plane

Direct products of manifolds with bdries = manifolds with corners

"model space" =  corner in $\mathbb{R}^n$

Idea: consider $[0,1]^2 \setminus \{(0,0),(0,1),(1,0),(1,1)\} \sqcup \{(0,0),(0,1),(1,0),(1,1)\} = M \times N$

a 0-dim mfld

(throw away the "corners")

top bdry

$$\partial^{\text{top}}(M \times N) \overset{\text{homeom.}}{\cong} (\partial M \times N) \sqcup (M \times \partial N)$$

"Leibniz rule"

orientation

(1)

a "consistent" choice of orient. on the tgt spaces

(at each point we have a basis, depends contin. w.r.t pt)

(2)

a nowhere vanishing volume form

not a full proof without checking consistency!

orient $\rightsquigarrow$ find $\overset{\text{vector}}{w_x}$: $\det(v_{1_x}, \dots v_{n-1_x}, w_x) = 1$, for $x \in M$

both are non-vanishing!

orient $\Rightarrow$ normal field

w is the normal field $\Rightarrow$ $\overset{\text{diff. form}}{\omega_x} : \Lambda^{n-1}(T_x M) \to \mathbb{R}$

$(v_1, \dots v_{n-1}) \to \det(v_{1_x} \dots v_{n-1_x}, w_x)$

Q2: We consider $Mat_3(\mathbb{R}) \to Mat_3(\mathbb{R})$

$$A \to A^TA - I$$

(just check the rank of the Jacobian!)

$$\dim O(3) = 3$$

1) We can go from $O(3)$ to $SO(3)$ just by noticing that $A^TA = 1$ forces $\det A = \pm 1$. , take $\det = 1$.

$T \in SO(3)$ is a orient. preserv linear isometry $\mathbb{R}^3 \to \mathbb{R}^3$

thus spherical coords

the angle of the rotation

2) Try to compute the $T_{Id}(SO(3))$ by "diff" $A^TA = Id$

$d(A^TA) = 0 \Rightarrow A^T = -A$

skew-symm. matr.