Q1 $f \in V^*$, $f \neq 0 \iff \exists v \in V: f(v) \neq 0$.

Lemma: If $v \in V$, $v \neq 0$, $\exists f \in V^*$; $f \neq 0$, $f(v) \neq 0$.

Lemma: $V = \langle e_1, \dots e_n \rangle$ Def: f_i are dual to e_i: $f_i(e_j) = \delta_{ij} = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases}$ (1)

$V^* = \langle f_1, \dots f_n \rangle$

Then $\forall f \in V^*$ $f = \underbrace{f(e_1)}_{\mathbb{R}} \cdot \underbrace{f_1}_{V^*} + f(e_2) \cdot f_2 + \dots + f(e_n) \cdot f_n$

dual basis.

Q2: $V = \langle \overset{e_1}{1}, \overset{e_2}{x}, \overset{e_3}{x^2} \rangle$ - 3-dim real vector space

$\varphi_{-1} : V \to \mathbb{R}$
$\varphi_0 : V \to \mathbb{R}$
$\varphi_1 : V \to \mathbb{R}$

$\varphi_{-1}(1 + 2x) = -1$

$\underline{\varphi_{-1}, \varphi_0, \varphi_1} \in V^*$.

to find a dual basis to φ_i. use (1) to construct a lin syst. of eq.

e_1, e_2, e_3 basis

produce the dual

f_1, f_2, f_3 (Rem: $f_i \neq \varphi_i$)

$\varphi_{-1} = c_{11} f_1 + c_{12} f_2 + c_{13} f_3$

V - real v. space.

$T^k(V) = \{$ set of multilin. maps $V^k \to \mathbb{R}\}$

1) $f(v_1, v_2, .. v_i + w_i, ... v_k) = f(v_1, .. v_i, .. v_k) + f(w_i)$ $\quad v_i \in V$

2) $f(v_1, .. \lambda v_i .. v_k) = \lambda f(v_i)$ $\quad \forall i \in \{1, .. k\}$ $\quad \lambda \in \mathbb{R}$

$\|$

W

W is a vector space

a) want to show that $B: W^2 \to \mathbb{R}$ is a bilinear fun.

$B(T_1 + S_1, T_2) = B(T_1, T_2) + B(S_1, T_2)$

$B(T_1, S_2 + T_2) = B(T_1, S_2) + B(T_1, T_2)$ $\quad$ check this! (by expanding RHS)

$B(aT, S) = aB(T, S)$

$B(T, aS) = aB(T, S)$.

b) $B(T, T) > 0$, $\quad B(T, T) =$ sum of squares of real num's.

Term test 2 debriefing

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} r\cos\varphi\cos\theta \\ r\cos\phi\sin\theta \\ r\sin\phi \end{pmatrix}$

Q3: Fubini + spherical

$$\int_{\mathbb{R}^3} g(x,y,z)\,dx\,dy\,dz = \int_0^\infty \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} g(r)\cdot r^2 |\cos\varphi|\,d\varphi\,d\theta\,dr =$$

g only depends on the dist to the 0.

$$= \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^\infty g(r) r^2 |\cos\varphi|\,dr\,d\varphi\,d\theta =$$

f is comp. supp

$\Downarrow$

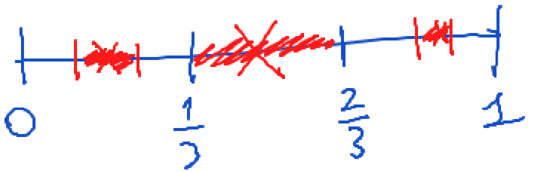
g is comp. supp

$$= \underline{\int_0^\infty g(r) r^2\,dr} \cdot \underline{\left(2\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos\varphi|\,d\varphi\right)}$$

$$\int_{\mathbb{R}^3} g\,dx\,dy\,dz \leq \mathrm{Vol}(\mathrm{supp}\, g)\cdot \|g\|_\infty.$$

cont. fn-ns on compact sets are bdd.

Q4.



$C_0 = [0, 1]$

$C_1 = C_0 \setminus [\frac{1}{3}, \frac{2}{3}]$

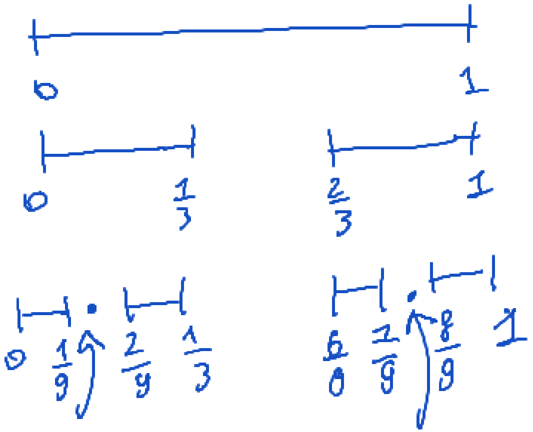
$C_2 = C_1 \setminus [\frac{1}{9}, \frac{2}{9}] \cup [\frac{7}{9}, \frac{8}{9}]$

Lemma: C_n does not contain num's with 1's in first n places.

$C_0 =$

$C_1 =$

$C_2 =$



0,02

0,21...

Cantor set

$\boxed{C = \{0, \overline{a_1 a_2 a_3 \ldots}, \; a_i \neq 1 \text{ in base-3}\}}$

$$C = \bigcap_{i=0}^{\infty} C_i$$

huge! small! set at the same time

the Cantor set

1) C has cardinality of $2^{\mathbb{N}}$ (huge!)

2) $m(C) = 0$

Proof: 2) C_i are, essent., the cover C due to def.

$\mathrm{Vol}(C_0) = 1$, $\mathrm{Vol}(C_1) = \frac{2}{3}$, $\mathrm{Vol}(C_2) = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$

$\mathrm{Vol}(C_n) = \left(\frac{2}{3}\right)^n \to 0.$

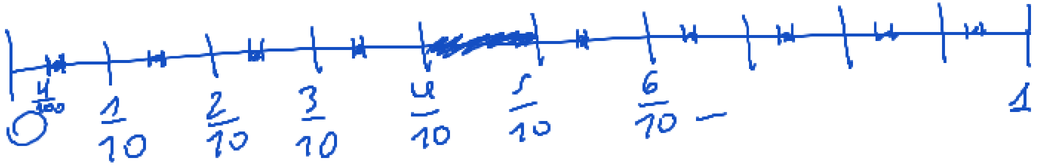
Proof: 1) Consider using base-3,

0,0... ✓

0,1... ✗ ← does not lie in C_1.

0,2... ✓

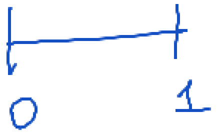
Q4



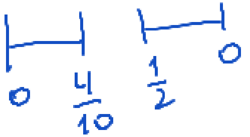
$\mathrm{Vol} = 1$

HW for you people.

"thick Cantor set"

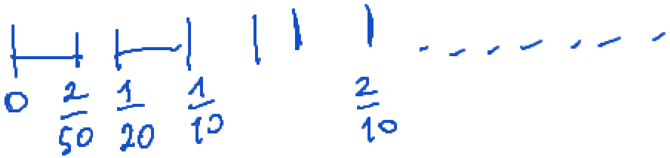
$C_0 =$ 

$\mathrm{Vol} = \frac{9}{10}$

$C_1 =$ 

fourth interval in the smaller $[\frac{1}{10}, \frac{1}{5}]$...

$\mathrm{Vol} = \left(\frac{9}{10}\right)^2$

$C_2 =$ 

$\vdots$

$\mathrm{Vol}(C_n) = \left(\frac{9}{10}\right)^n \longrightarrow 0.$

base-3 (stand) Cantor set
base-10 Cantor set are the same!