1. Is it true that any closed subset of $\mathbb{R}$ is the closure of its interior? (it is true for $[a,b] \subseteq \mathbb{R}$)

Claim: $\mathbb{Q}$ is a counterexample. $Int(\mathbb{Q}) = \phi$.

$\{a\}$ is a counterex $\forall a \in \mathbb{R}$, $Int(\{a\}) = \phi$.

no singleton is open

Counterex is not enough but justific. might be really simple!

$Int([a,b]) = (a,b)$ (a, b are finite)

Two equiv. def of the closure:

$X \subseteq \mathbb{R}^n$.

1) $\overline{X} = \bigcap$ all closed subsets, containing X $= \bigcap_{\substack{A \supseteq X \\ A \text{ is closed}}} A$, 2) $\overline{X} = X \cup$ (all limit points of X)

1') $\overline{X}$ is the smallest closed subset of $\mathbb{R}^n$, cont. X.

1) $\Leftrightarrow$ 2) equivalent!

2) Lemma: f is cont, $f: \mathbb{R}^n \to \mathbb{R}^m$, $\forall K \subseteq \mathbb{R}^n$ (comp) $f(K)$ is also compact.

(f(compact) = compact)

$f^{-1}(\mathcal{U}_1) \cup .. \cup f^{-1}(\mathcal{U}_k) \supseteq K$.

$f(f^{-1}(\mathcal{U}_1) \cup .. \cup f^{-1}(\mathcal{U}_k)) \supseteq f(K)$

$\mathcal{U}_1 \cup .. \cup \mathcal{U}_k \supseteq f(K)$.

Proof: Let $(\mathcal{U}_\alpha)_\alpha$ be a cover of $f(K)$. Then $(f^{-1}(\mathcal{U}_\alpha))_\alpha$ is a cover of K.

Idea: construct a cont. fn-n $F: \mathbb{R}^n \to \mathbb{R}^n \times \mathbb{R}^m$

$F(x) = (x, f(x))$. , F is continuous

C is comp $\overset{\text{Lemma}}{\Rightarrow}$ $F(C)$ is comp.

$F(C) = \Gamma(f)|_C$.

$A = (a_{ij})$

3) $f(x) = \sum a_{ij} x_i x_j$

The diff of f is a linear map.

$Df_y : x \longmapsto 2\langle Ax, y\rangle$ or $2\langle x, Ay\rangle$ (I just forget!)

5. $f = o(h) \Rightarrow f(0) = 0$.

$f(h) = o(h)$

Recall that f is diff'able at $0 \iff \exists A$: (linear)

$f(h) - f(0) = Ah + o(h)$. Set $A = 0$.

$\underset{f(h)}{\overset{\parallel}{f(h) - f(0)}} = \underset{o(h).}{\overset{\parallel}{Ah + o(h).}}$

Invertible matrices form an open subset of $\mathbb{R}^{n^2}$.

4. For some small ϵ, $\forall x_1, x_2$ $\underbrace{|\overset{\text{inv.}}{\lambda}(x_1 - x_2) - (f(x_1) - f(x_2))| \leq \epsilon |x_1 - x_2|}$

$\forall y \in \mathbb{R}^n \ \exists x \in \mathbb{R}^n: f(x) = y$. (We need to prove that f is invertible.)

You might want to apply the IFT.

We claim that $Df(x)$ is really close to λ. $\forall x$.

You can prove that f is 1-1.

6. a) $Df = \begin{pmatrix} e^x & e^y \\ e^x & -e^{-y} \end{pmatrix}$, det $= -e$

b) chain rule of memor. IFT.

7. Use the Implicit FT. $z = g(x,y)$, look at $\frac{\partial}{\partial z}$.

$1 - \underset{0\ \ 0}{\overset{\parallel\ \parallel}{x\,y}} \underset{1}{\overset{\parallel}{\cos(xyz)}}$

7) $\frac{\partial}{\partial z}\; 1 - \underset{0}{\underset{\parallel}{x}}\,\underset{0}{\underset{\parallel}{y}}\,\underset{1}{\underset{\parallel}{\cos(xyz)}} = 1 \neq 0.$ Use IFT.

8) a) was in the assignment for $\mathbb{R}$ (this is the 2-dim case)

expect the $\text{Int} = \phi$.

b)

$\text{Int } B = B$

$\text{Bnd } B = \{x^2+y^2=1\} \cup \{0\}.$

c)

d)

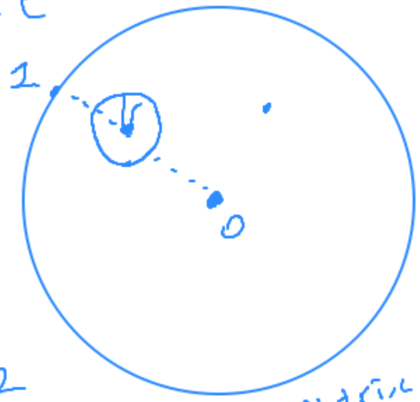
$\text{Bnd}\, C = \{x^2 = y\}$

$\parallel$

$\text{Bnd}\, D$

9) similar to the fact that $|x|$ is nondiffable
but you might want to look at $\frac{|xy|}{\sqrt{x^2+y^2}}$

10) Chain rule



$\boxed{r \le \min(x^2+y^2,\ 1-x^2-y^2)}$

closed set contains all limit points
closed = compl. to open
$cl = \mathbb{R}^n \setminus$ open

Hessian = matrix consis. of second derivatives

Constant rank thm: $f:\mathbb{R}^n \to \mathbb{R}^m$
if x is a point with a nbhd: $U_x \ni x$ $\forall y \in U_x$ $rk(Df_y) = n-k$ $\Rightarrow$ there is a change of coord. s.t. $f(x_1, \dots, x_n) = (x_1, \dots, x_{n-k}, 0, \dots, 0)$

$xy - z\log y + e^{xz} = 1$

$z = f(x,y)$ or $y = g(x,z)$ exist at $(0,1,1)$

$f : \mathbb{R}^{n+k} \to \mathbb{R}^n$

$f(0) = 0$

$\operatorname{rk} Df(0) = n$

column-vectors

$$Df(0) = \left(\frac{\partial f_1}{\partial x}, \frac{\partial f_2}{\partial x}, \dots \frac{\partial f_{n+k}}{\partial x} \right)$$

choose n lin. indep. columns.

WLOG assume, that the first n col are indep.

Consider $g(x_1 \dots x_n) = f(\underset{\sim}{x_1} \dots \underset{\sim}{x_n}, \underbrace{a_{n+1} \dots a_{n+k}}_{\text{comp. of } a.})$

inverse.

g satisfies IFT