Homework Assignment 19

Due: Monday April 5, 2021 11:59 PM (Eastern Daylight Time)

Assignment description

Solve and submit your solutions of the following problems. Note also that the late policy remains strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (10 points)

Spivak's 5-19&20 (combined and modified). (a) Show that Stokes's Theorem fails if the manifold is not compact (hint in text), yet that it holds again if the form has a compact support.

(b) Show that the integral of an exact form on a compact oriented manifold with no boundary vanishes, and give a counterexample where the manifold is not compact.

Q2 (10 points)

Munkres' 37.4 (modified). Let $M^2 = \{(x, y, z) \colon 4x^2 + y^2 + 4z^2 = 4 \ \&\ y \geq 0\} \subset \mathbb{R}^3_{xyz}$. The map $\alpha(u, v) = (u, 2(1 - u^2 - v^2)^{1/2}, v)$ defined when $u^2 + v^2 < 1$ is a coordinate patch that covers $M \setminus \partial M$. Orient M so that α is orientation preserving, and give ∂M the induced orientation. Let $\omega = y dx + 3x dz$.

(a) At any point $(x, 0, z)$ of ∂M, write a tangent vector that defines the given orientation of ∂M.

(b) Evaluate $\int_{\partial M} \omega$ directly.

(c) Evaluate $\int_M d\omega$ directly.

(d) Repeat until steps (b) and (c) give the same answer.

Q3 (10 points)

Munkres' 37-5 (modified). For $r > 0$ let $D_r^3 = \{x \in \mathbb{R}^3 \colon |x| \leq r\}$ be the 3-disk of radius r oriented with the orientation induced from the standard orientation of $\mathbb{R}^3$, and let $S_r^2 = \partial D_r^3$ be its boundary with its induced orientation. Assume that $\omega \in \Omega^2(\mathbb{R}^3 \setminus \{0\})$ satisfies

$$\int_{S_r^2} \omega = a + (b/r)$$

for some constants a and b and for all $r > 0$.

(a) Given $0 < c < d$, compute $\int_{D_d^3 \setminus (\operatorname{int} D_c^3)} d\omega$.

(b) If ω is closed, what can you say about a and b?

(c) If ω is exact, what can you say about a and b?

Q4 (10 points)

Munkres' 37.6 (modified). Given a compact oriented $(k + l + 1)$ -dimensional manifold without boundary and given $\omega \in \Omega^k(M)$ and $\eta \in \Omega^l(M)$, prove that the following "integration by parts" formula holds,

$$\int_M \omega \wedge d\eta = s \int_M d\omega \wedge \eta,$$

for some sign s. What is s?