Homework Assignment 17

Due: Wednesday March 24, 2021 11:59 PM (Eastern Daylight Time)

Assignment description

Solve and submit your solutions of the following problems. Note also that the late policy remains strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (20 points)

(a) Show that if $M^k \subset \mathbb{R}^m$ and $N^l \subset \mathbb{R}^n$ are manifolds without boundary, then $M \times N$ is a $(k+l)$-dimensional manifold without boundary in $\mathbb{R}^{m+n}$.

(b) Show that if $M^k \subset \mathbb{R}^m$ and $N^l \subset \mathbb{R}^n$ are manifolds with boundary, then $M \times N$ can be presented as the disjoint union of a $(k+l)$-manifold with boundary and a $(k+l-2)$-manifold without boundary.

Hint. Start by understanding the case of $M = N = [0,1]$.

Q2 (20 points)

Show that $O(3) := \{A \in M_{3\times 3}(\mathbb{R}) : A^T A = I\} \subset M_{3\times 3}(\mathbb{R}) = \mathbb{R}^9$ is a manifold. What is its dimension $\dim O(3)$?

If you have a problem visualizing $O(3)$, that's okay. So do I.

Hint. It is best to present $O(3)$ as the zero set of some function g and then show that the appropriate condition on the differential of g is satisfied.

Q3 (20 points)

Show that an $(n-1)$-dimensional manifold M in $\mathbb{R}^n$ is orientable if and only if one may find a consistent non-zero normal field ν to M in $\mathbb{R}^n$. Precisely, ν should be a smooth function on M which maps every $p \in M$ to a non-zero vector in $T_p\mathbb{R}^n$ such that for every p the vector $\nu(p)$ is perpendicular to T_pM.

(This exercise explains the relationship between "being orientable" and "having two sides". Don't write about this, but make sure that you understand this relationship).