Homework Assignment 16

Due: Wednesday March 17, 2021 11:59 PM (Eastern Daylight Time)

Assignment description

Solve and submit your solutions of the following problems. Note also that the late policy remains strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (10 points)

(a) Show that if a chain b is the boundary of another chain c, namely if $b = \partial c$, then the boundary of b is zero, namely $\partial b = 0$.

(b) Use Stokes' theorem to show that the 1-cube $b(t) = (\cos 2\pi t, \sin 2\pi t)$ in $\mathbb{R}^2 \setminus \{0\}$ has $\partial b = 0$, yet it is not the boundary of any 2-chain $c \in C_2(\mathbb{R}^2 \setminus \{0\})$.

Q2 (10 points)

Spivak's 4-29 (modified, hint in text). Show that if $\omega = f dx \in \Omega^1([0,1])$ where f is smooth and $f(0) = f(1)$, then there is a unique real number λ so that $\omega - \lambda dx$ is dg for some smooth g: $[0,1] \to \mathbb{R}$ for which $g(0) = g(1)$.

Q3 (10 points)

Spivak's 4-30 (modified, hint in text) Prove that if $\omega \in \Omega^1(\mathbb{R}^2 \setminus \{0\})$ is closed, then there is a unique real number λ such that $\omega - \lambda\eta$ is exact, where $\eta := \frac{-y dx + x dy}{x^2+y^2}$.

Q4 (10 points)

Spivak's 5-6 (modified). If $f\colon \mathbb{R}^n \to \mathbb{R}^m$ is a function, the graph of f is defined to be $\Gamma_f = \{(x, y) : y = f(x)\}$. Prove that Γ_f is a smooth n-manifold if f is smooth.

Note. An earlier version of this question had "iff" instead of the second "if". But the "only if" side here is false! For example, the function $f(x) = \sqrt[3]{x}$ is not smooth at 0, yet its graph is a manifold.

Q5 (10 points)

Spivak's 5-8(a) (modified). Prove any 12-dimensional manifold M in $\mathbb{R}^{22}$ is of measure 0 in $\mathbb{R}^{22}$.