Homework Assignment 9

Due: Friday December 4, 2020 11:59 PM (Eastern Standard Time)

Assignment description

Solve and submit your solutions of the following problems. Note also that the late policy remains strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (30 points)

Spivak's 3-38, modified. A function $f\colon \mathbb{R} \to \mathbb{R}$ vanishes for $x < 1$ and all on intervals of the form $(n - 1/3, n + 1/3)$, where n is a positive integer, yet for every such n, $\int_{n+1/3}^{n+2/3} f = (-1)^n/n$.

(a) Prove that such a function f exists and draw an approximate plot thereof.

(b) Show that $\int_{\mathbb{R}} f$ does not exist.

(c) Find two partitions of unity Φ and Ψ of $\mathbb{R}$, such that the sums $\sum_{\phi\in\Phi} \int \phi f$ and $\sum_{\psi\in\Psi} \int \psi f$ both absolutely converge, yet to different values.

Q2 (20 points)

Let A be an arbitrary subset of $\mathbb{R}^n$, and let $f\colon A \to \mathbb{R}$ be given. We say that "f is smooth at $a \in A$" if there is an open set U containing a and a smooth (C^∞) function $g\colon U \to \mathbb{R}$ such that f and g agree on $A \cap U$.

Prove that if f is smooth at every $a \in A$ then it can be extended to a smooth function on some open set $V \supset A$.

Warning. The function g from the definition of "smooth" may depend on a!

Hint. PO1.