Homework Assignment 3

Due: Wednesday October 7, 2020 11:59 PM (Eastern Daylight Time)

Assignment description

Solve and submit your solutions of the following problems. They are mostly taken from Spivak's book, pages 17,18, and 23. Note that I've made the late policy stricter - you will lose 10% for each hour that you are late. In other words, please submit on time!

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (10 points)

Spivak's 2-1. Prove that if $f: \mathbb{R}^n \to \mathbb{R}^m$ is diffable at $a \in \mathbb{R}^n$, then it is continuous at a. (Hint in text).

Q2 (10 points)

Spivak's 2-4. Let g be a continuous real-valued function on the unit circle $S^1 = \{x \in \mathbb{R}^2 : |x| = 1\}$ such that $g(0,1) = g(1,0) = 0$ and $g(-x) = -g(x)$. Define $f: \mathbb{R}^2 \to \mathbb{R}$ by $f(0) = 0$ and by $f(x) = |x| g(x/|x|)$ otherwise.

(a) Show that the restriction of f to any line through the origin is diffable. In other words, if $x \in \mathbb{R}^2$ and $h: \mathbb{R} \to \mathbb{R}$ is defined by $h(t) = f(tx)$, show that h is diffable.

(b) Show that f is not diffable at $(0,0)$ unless $g = 0$. (Hint in text).

Q3 (10 points)

Spivak's 2-7. Let $f: \mathbb{R}^n \to \mathbb{R}$ be a function such that $|f(x)| \leq |x|^2$. Show that f is diffable at 0.

Q4 (10 points)

Spivak's 2-10abg. Use the results from class and from Spivak's book up to page 23 to find f' in the following cases:

(a) $f(x, y, z) = x^y$.

(b) $f(x, y, z) = (x^y, z)$.

(g) $f(x, y, z) = (x + y)^z$.

Q5 (10 points)

Spivak's 2-12. A function $f\colon \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$ is **bilinear** if it is linear as a function of the first $\mathbb{R}^n$ input when the second is held constant, and of the second $\mathbb{R}^m$ input when the first is held constant (details in text).

(a) Prove that if f is bilinear, then

$$\lim_{(h,k)\to 0} \frac{|f(h, k)|}{|(h, k)|} = 0.$$

(b) Prove that $Df(a, b)(x, y) = f(a, y) + f(x, b)$.

(c) Show that the formula for the differential of a product that we've seen in class (also in Spivak's Theorem 2-3) is a special case of (b).

Q6 (10 points)

Spivak's 2-14 and 2-15 cut short. The space of 2×2 matrices with entries in $\mathbb{R}$ is equivalent to $\mathbb{R}^4$ via $(a, b, c, d) \leftrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Is the determinant function on 2×2 matrices, regarded as a function $f\colon \mathbb{R}^4 \to \mathbb{R}$, diffable? What is its differential?