This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.

Term Test 2

Due: Tuesday January 19, 2021 7:20 PM (Eastern Standard Time)

Assignment description

Solve all 5 problems on this test, and do Task 6.

Each problem is worth 20 points.

You have two hours to write this test, and another 20 minutes for Task 6 and for uploading.

Allowed material. Open book(s), open notes, but you can only use the internet (during the exam) to read the exam, to submit the exam, and to connect with the instructor/TAs to ask clarification questions. No contact allowed with other students or with any external advisors, online or in person.

Neatness counts! Language counts! The *ideal* written solution to a problem looks like a page from a textbook; neat and clean and consisting of complete and grammatical sentences. Definitely phrases like "there exists" or "for every" cannot be skipped. Lectures are mostly made of spoken words, and so the blackboard part of proofs given during lectures often omits or shortens key phrases. The ideal written solution to a problem does not do that.

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (20 points)

Show that "the volume under the graph of a function is equal to the integral of that function". Precisely, show that if $f\colon A \to \mathbb{R}$ is continuous, bounded, and non-negative, where $A \subset \mathbb{R}^n$ is a rectangle, and if $G \subset \mathbb{R}^{n+1} = \mathbb{R}^n_x \times \mathbb{R}_y$ is defined by $G := \{(x,y)\colon 0 \leq y \leq f(x)\}$, then $\int_A f = v(G)$. (Recall that the volume of a set is the integral of its characteristic function).

Tip. Don't start working! Read the whole test first. You may wish to start with the questions that are easiest for you.

Q2 (20 points)

Show that every open set U in $\mathbb{R}^n$ can be presented as the union of a sequence of compact sets C_1, C_2, C_3, ..., satisfying $C_k \subset \operatorname{int} C_{k+1}$ for all $k \geq 1$.

Tip. In math exams, "show" means "prove".

Q3 (20 points)

Let $f\colon \mathbb{R}_{>0} \to \mathbb{R}$ be a continuous function whose support $\operatorname{supp} f$ is a compact subset of $\mathbb{R}_{>0}$, where $\mathbb{R}_{>0}$ denotes the positive real numbers. Define a function $g\colon \mathbb{R}^3 \to \mathbb{R}$ by $g(x) = f(|x|)$. Show that $\int_{\mathbb{R}^3} g = 4\pi \int_0^\infty r^2 f(r) dr$.

Q4 (20 points)

For reasons unknown to me, my apartment building has no floors whose number contains the digit 4. Prove that the set of real numbers between 0 and 1 whose decimal expansion does not contain the digit 4 is of measure 0.

Hint. What's the length of the set C_1 of real numbers between 0 and 1 whose first digit after the decimal point isn't 4?

Q5 (20 points)

Let $g\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a continuously differentiable function, and assume that at some point $a \in \mathbb{R}^2$ we have that $D_2 g_2(a) \neq 0$, where g_2 is the second component of g. Prove that near a the function g can be written as a composition of two continuously differentiable functions defined on some open sets in $\mathbb{R}^2$ and taking values in some open sets in $\mathbb{R}^2$, and such that one of those functions preserves the first coordinate and the other one preserves the second coordinate.

Tip. Once you have finished writing a test, if you have time left, it is always a good idea to go back and re-read and improve everything you have written, and perhaps even completely rewrite any parts that came out messy.

Task 6 (0 points)

Please copy in your own handwriting, fill in the missing details, and sign the statement below, and then submit it along with a photo of your student ID card to complete this test. (See a sample submission below)

By signing this statement, I am attesting to the fact that I, [name], [student number], have abided fully to the Code of Behaviour on Academic Matters. I have not committed academic misconduct, and I am aware of the penalties that may be imposed if I have committed an academic offence.

Signature:

By signing this statement, I am attesting to the fact that I, Dror Bar-Natan, 123456789, have abided fully to the Code of Behaviour on Academic Matters. I have not committed academic misconduct, and I am aware of the penalties that may be imposed if I have committed an academic offence.

Dror Bar-Natan