Term Test 1

Due: Tuesday November 3, 2020 7:15 PM (Eastern Standard Time)

Assignment description

Solve all 5 problems on this test, and do Task 6.

Each problem is worth 20 points.

You have two hours to write this test, and another 15 minutes for Task 6 and for uploading.

Allowed material. Open book(s), open notes, but you can only use the internet (during the exam) to read the exam, to submit the exam, and to connect with the instructor/TAs to ask clarification questions. No contact allowed with other students or with any external advisors, online or in person.

Neatness counts! Language counts! The *ideal* written solution to a problem looks like a page from a textbook; neat and clean and consisting of complete and grammatical sentences. Definitely phrases like "there exists" or "for every" cannot be skipped. Lectures are mostly made of spoken words, and so the blackboard part of proofs given during lectures often omits or shortens key phrases. The ideal written solution to a problem does not do that.

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (20 points)

(a) Let (x_k) be a Cauchy sequence in $\mathbb{R}^n$. Prove that either $A = \{x_k\}$ (the set whose members are all the x_k's) is compact, or it is possible to find one extra point $a \in \mathbb{R}^n$ such that $A \cup \{a\}$ is compact.

(b) Find an example of a non-Cauchy sequence for which the above is not true.

Tip. Don't start working! Read the whole test first. You may wish to start with the questions that are easiest for you.

Q2 (20 points)

We say that a function $\lambda\colon \mathbb{R}^n \to \mathbb{R}^n$ is "small" if there is a constant C such that for small enough x we have that $|f(x)| \leq C|x|$, and we say that a function $\mu\colon \mathbb{R}^n \to \mathbb{R}^n$ is "tiny" if $\mu(0) = 0$ and $\lim_{x\to 0} \frac{\mu(x)}{|x|} = 0$. Prove that if λ is small and μ is tiny, then both $\mu \circ \lambda$ and $\lambda \circ \mu$ are tiny.

Tip. Neatness, cleanliness and organization count, here and everywhere else!

Q3 (20 points)

We will say that a function $f\colon \mathbb{R}^n \to \mathbb{R}^n$ has "all scale fidelity" is there is some $\epsilon > 0$ so that

$$\forall x_1, x_2 \in \mathbb{R}^n, \qquad |(x_1 - x_2) - (f(x_1) - f(x_2))| \leq \epsilon |x_1 - x_2|.$$

Show that if an invertible function $f\colon \mathbb{R}^n \to \mathbb{R}^n$ has all scale fidelity with $\epsilon = \frac{1}{10}$, then the function f^{-1} also has all scale fidelity (perhaps with a different value for ϵ).

Tip. In math tests, "show" means "prove".

Q4 (20 points)

It is given that function $f\colon \mathbb{R}^n \to \mathbb{R}$ has bounded partial derivatives near 0 (but it is not given that these derivatives are continuous). Prove that f is continuous at 0.

Q5 (20 points)

The functions $f\colon \mathbb{R}^3 \to \mathbb{R}$ and $g\colon \mathbb{R}^2 \to \mathbb{R}$ are differentiable, and a function $h\colon \mathbb{R}^2 \to \mathbb{R}$ is defined by $h(x,y) = f(x,y,g(x,y))$.

(a) Compute h' in terms of the partial derivatives of f and of g.

(b) If it is known that $h(x,y) = 0$ for all x and y, compute as best as you can the partial derivatives of g in terms of those of f.

Tip. Once you have finished writing a test, if you have time left, it is always a good idea to go back and re-read and improve everything you have written, and perhaps even completely rewrite any parts that came out messy.

Task 6 (0 points)

Please copy in your own handwriting, fill in the missing details, and sign the statement below, and then submit it along with a photo of your student ID card to complete this test. (See a sample submission below)

By signing this statement, I am attesting to the fact that I, [name], [student number], have abided fully to the Code of Behaviour on Academic Matters. I have not committed academic misconduct, and I am aware of the penalties that may be imposed if I have committed an academic offence.

Signature: