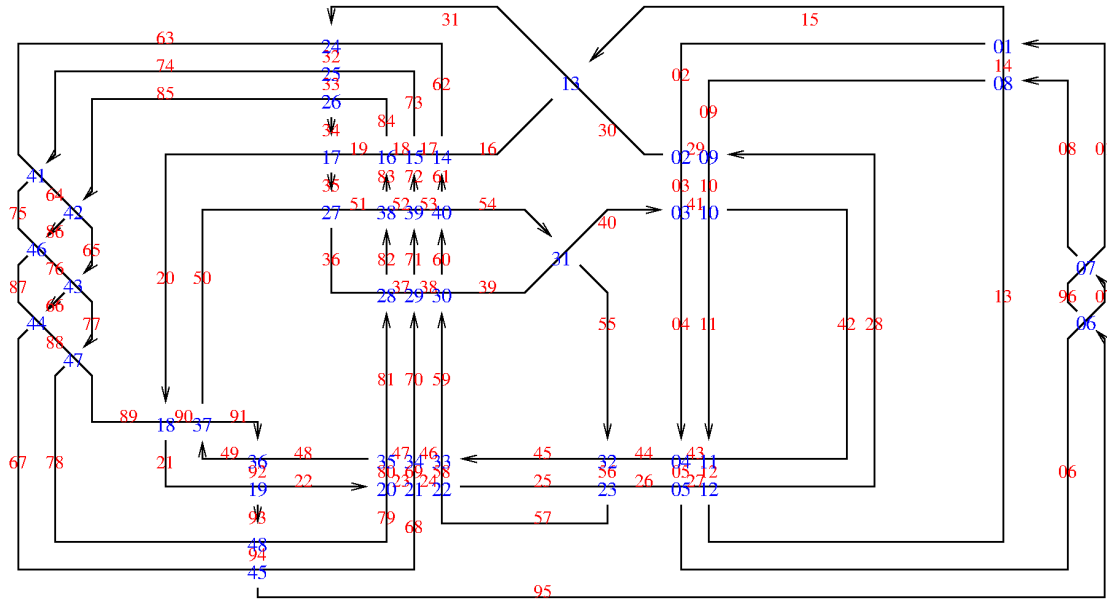


Pensieve header: Notebook for Wednesday September 16: A faster Jones program.

Warning: Hidden cell below:

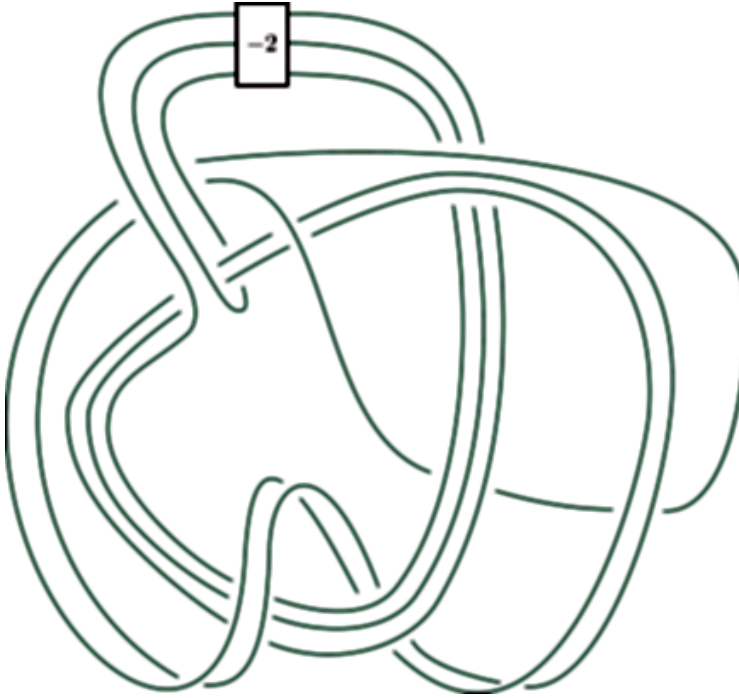
The Gompf-Scharlemann-Thompson 48-crossing

GST48:

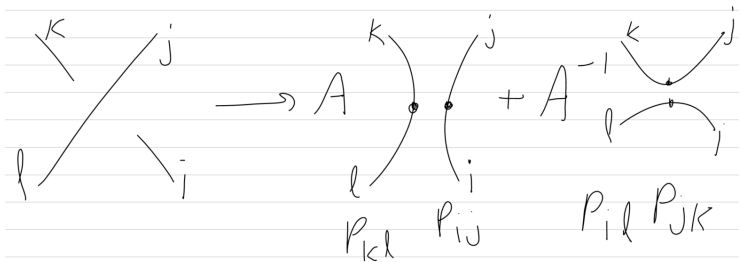


```
In[ ]:= GST48 = PD[X[1, 15, 2, 14], X[29, 2, 30, 3], X[40, 4, 41, 3],
  X[4, 44, 5, 43], X[5, 26, 6, 27], X[95, 7, 96, 6], X[7, 1, 8, 96], X[8, 14, 9, 13],
  X[28, 9, 29, 10], X[41, 11, 42, 10], X[11, 43, 12, 42], X[12, 27, 13, 28],
  X[15, 31, 16, 30], X[61, 16, 62, 17], X[72, 17, 73, 18], X[83, 18, 84, 19],
  X[34, 20, 35, 19], X[20, 89, 21, 90], X[92, 21, 93, 22], X[22, 79, 23, 80],
  X[23, 68, 24, 69], X[24, 57, 25, 58], X[56, 25, 57, 26], X[31, 63, 32, 62],
  X[32, 74, 33, 73], X[33, 85, 34, 84], X[35, 50, 36, 51], X[81, 37, 82, 36],
  X[70, 38, 71, 37], X[59, 39, 60, 38], X[54, 39, 55, 40], X[55, 45, 56, 44],
  X[45, 59, 46, 58], X[46, 70, 47, 69], X[47, 81, 48, 80], X[91, 49, 92, 48],
  X[49, 91, 50, 90], X[82, 52, 83, 51], X[71, 53, 72, 52], X[60, 54, 61, 53],
  X[74, 63, 75, 64], X[85, 64, 86, 65], X[65, 76, 66, 77], X[66, 87, 67, 88],
  X[94, 67, 95, 68], X[86, 75, 87, 76], X[77, 88, 78, 89], X[93, 78, 94, 79]];
```

The 55-crossing Piccirillo's Knot, used to prove that "The Conway Knot is not Slice" (arXiv:1808.02923): _____



Our blackboard from September 14:



```
In[ ]:= Knot [3, 1]
```

```
In[ ]:= KB[pd_PD] := Module[{p, t1, t2, t3, t4, B, d},
  SetAttributes[p, Orderless];
  t1 = pd /. X[i_, j_, k_, l_] -> A * p[i, j] * p[k, l] + B * p[i, l] * p[j, k];
  t2 = Expand[t1 /. PD -> Times];
  t3 = t2 /. {p[i_, j_] p[j_, k_] -> p[i, k]};
  t4 = t3 /. {p[i_, i_] -> d, p[i_, j_]^2 -> d};
  Expand[t4 /. {B -> 1/A, d -> -A^2 - 1/A^2}]
]
```

```
In[ ]:= Knot [8, 17]
```

```
Out[ ]:= PD[X[6, 2, 7, 1], X[14, 8, 15, 7], X[8, 3, 9, 4], X[2, 13, 3, 14],
  X[12, 5, 13, 6], X[4, 9, 5, 10], X[16, 12, 1, 11], X[10, 16, 11, 15]]
```

```
In[ ]:= KB[Knot [8, 17]]
```

```
Out[ ]:= -1/A^18 + 2/A^14 - 2/A^10 + 1/A^6 - 1/A^2 - A^2 + A^6 - 2 A^10 + 2 A^14 - A^18
```

```

In[ ]:= KBon10 = Table[
  Echo[Timing[ {k, KB[Knot[10, k]]} ]],
  {k, 165}
]

» {0.625, {1, -1/A^28 - 1/A^20 + A^4 - A^16}}
» {0.734375, {2, -1 - 1/A^16 - 1/A^8 + A^16 + A^24 - A^28}}
» {0.765625, {3, 1 - 1/A^24 - 1/A^16 + 1/A^12 - 1/A^8 + A^8 - A^12 - A^20}}
» {0.875, {4, -1/A^16 - 1/A^8 + 1/A^4 + A^4 - A^12 - A^20 + A^24 - A^28}}
» {0.828125, {5, 1/A^26 - 1/A^22 + 1/A^18 - 1/A^14 + 1/A^10 - 1/A^2 - 2A^6 - A^14 + A^18}}
» {0.71875, {6, 1 - 1/A^20 - 2/A^12 + 1/A^8 - 1/A^4 - A^8 + 2A^12 - A^16 + A^20 - A^24}}
» {0.734375, {7, 1 - 1/A^24 + 1/A^20 - 2/A^16 + 1/A^12 - 2/A^8 - A^4 + 2A^8 - A^12 + A^16 - A^20}}
» {0.859375, {8, -1 - 1/A^16 - 1/A^8 + 1/A^4 + A^16 - A^20 + A^24 - A^28}}
» {0.75, {9, -1/A^24 + 1/A^20 - 1/A^16 + 2/A^12 - 1/A^8 - 2A^4 + A^8 - A^12 + A^16 - A^20}}
» {0.953125, {10, 1/A^18 - 1/A^14 + 1/A^10 - 2/A^6 + 1/A^2 - A^2 + A^10 - 2A^14 + A^18 - 2A^22 + A^26}}
» {0.921875, {11, 1 - 1/A^20 - 2/A^12 + 2/A^8 - 1/A^4 - A^8 + 2A^12 - 2A^16 + A^20 - A^24}}
» {0.765625, {12, 1/A^22 - 1/A^18 + 2/A^14 - 2/A^10 + 1/A^6 - 1/A^2 - A^2 + A^6 - 3A^10 + A^14 - A^18 + A^22}}
» {0.625, {13, 1 - 1/A^24 + 1/A^20 - 3/A^16 + 2/A^12 - 1/A^8 + 1/A^4 - 2A^4 + 2A^8 - 2A^12 + A^16 - A^20}}
» {0.75, {14, -1/A^20 + 1/A^16 - 2/A^12 + 2/A^8 - 3/A^4 - A^8 + 3A^12 - 2A^16 + 2A^20 - A^24}}
» {0.765625, {15, 1/A^26 - 1/A^22 + 2/A^18 - 2/A^14 - 1/A^6 - 1/A^2 + A^2 - 2A^6 + A^10 - A^14 + A^18}}
» {0.734375, {16, 1 - 1/A^24 + 1/A^20 - 2/A^16 + 2/A^12 - 1/A^8 + 1/A^4 - 2A^4 + A^8 - 2A^12 + A^16 - A^20}}
» {0.8125, {17, 1/A^22 - 1/A^18 + 1/A^14 - 2/A^10 + 1/A^6 - 1/A^2 - A^2 + A^6 - 2A^10 + A^14 - A^18 + A^22}}
» {0.640625, {18, 1 - 1/A^20 + 1/A^16 - 2/A^12 + 2/A^8 - 2/A^4 - 2A^8 + 2A^12 - 2A^16 + 2A^20 - A^24}}
» {0.75, {19, 1/A^18 - 1/A^14 + 1/A^10 - 3/A^6 + 1/A^2 - A^2 + A^10 - 2A^14 + 2A^18 - 2A^22 + A^26}}
» {0.921875, {20, 1 - 1/A^24 - 2/A^16 + 1/A^12 - 1/A^8 + 1/A^4 - A^4 + A^8 - A^12 + A^16 - A^20}}
» {0.875, {21, -1/A^20 + 1/A^16 - 2/A^12 + 1/A^8 - 2/A^4 - A^8 + 3A^12 - A^16 + A^20 - A^24}}

```

- » $\left\{ 0.71875, \left\{ 22, 1 - \frac{1}{A^{20}} + \frac{1}{A^{16}} - \frac{2}{A^{12}} + \frac{2}{A^8} - \frac{1}{A^4} - 2A^8 + 2A^{12} - 2A^{16} + A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.828125, \left\{ 23, \frac{1}{A^{22}} - \frac{1}{A^{18}} + \frac{2}{A^{14}} - \frac{3}{A^{10}} + \frac{2}{A^6} - \frac{1}{A^2} - A^2 + A^6 - 3A^{10} + 2A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.890625, \left\{ 24, 1 - \frac{1}{A^{24}} + \frac{1}{A^{20}} - \frac{3}{A^{16}} + \frac{2}{A^{12}} - \frac{2}{A^8} - A^4 + 3A^8 - 2A^{12} + A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 1.6875, \left\{ 25, 1 - \frac{1}{A^{20}} + \frac{1}{A^{16}} - \frac{3}{A^{12}} + \frac{2}{A^8} - \frac{3}{A^4} + A^4 - A^8 + 3A^{12} - 3A^{16} + 2A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.71875, \left\{ 26, 1 - \frac{1}{A^{20}} + \frac{1}{A^{16}} - \frac{2}{A^{12}} + \frac{3}{A^8} - \frac{2}{A^4} - 2A^8 + 2A^{12} - 3A^{16} + 2A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 1.40625, \left\{ 27, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{2}{A^{14}} - \frac{4}{A^{10}} + \frac{2}{A^6} - \frac{1}{A^2} - A^2 + 2A^6 - 3A^{10} + 3A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 1.04688, \left\{ 28, \frac{1}{A^{18}} - \frac{1}{A^{14}} + \frac{2}{A^{10}} - \frac{2}{A^6} + \frac{1}{A^2} - 2A^2 - A^6 + A^{10} - 2A^{14} + 2A^{18} - 2A^{22} + A^{26} \right\} \right\}$
- » $\left\{ 1.07813, \left\{ 29, 2 - \frac{1}{A^{20}} + \frac{1}{A^{16}} - \frac{3}{A^{12}} + \frac{2}{A^8} - \frac{2}{A^4} + A^4 - 2A^8 + 2A^{12} - 3A^{16} + 2A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 1.07813, \left\{ 30, 1 - \frac{1}{A^{24}} + \frac{2}{A^{20}} - \frac{3}{A^{16}} + \frac{2}{A^{12}} - \frac{3}{A^8} - 2A^4 + 3A^8 - 2A^{12} + 2A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.703125, \left\{ 31, \frac{1}{A^{22}} - \frac{1}{A^{18}} + \frac{2}{A^{14}} - \frac{3}{A^{10}} + \frac{1}{A^6} - \frac{2}{A^2} - A^2 + 2A^6 - 2A^{10} + 2A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 1.07813, \left\{ 32, -\frac{1}{A^{24}} + \frac{2}{A^{20}} - \frac{3}{A^{16}} + \frac{3}{A^{12}} - \frac{2}{A^8} - 3A^4 + 3A^8 - 2A^{12} + 2A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.71875, \left\{ 33, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{2}{A^{14}} - \frac{3}{A^{10}} + \frac{2}{A^6} - \frac{1}{A^2} - A^2 + 2A^6 - 3A^{10} + 2A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.640625, \left\{ 34, \frac{1}{A^{18}} - \frac{1}{A^{14}} + \frac{1}{A^{10}} - \frac{1}{A^6} + \frac{1}{A^2} - A^2 - A^6 - 2A^{14} + A^{18} - A^{22} + A^{26} \right\} \right\}$
- » $\left\{ 0.625, \left\{ 35, 1 - \frac{1}{A^{20}} + \frac{1}{A^{16}} - \frac{2}{A^{12}} + \frac{2}{A^8} - \frac{1}{A^4} - 2A^8 + 2A^{12} - 2A^{16} + A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.671875, \left\{ 36, -\frac{1}{A^{24}} + \frac{1}{A^{20}} - \frac{2}{A^{16}} + \frac{2}{A^{12}} - \frac{2}{A^8} - 2A^4 + 2A^8 - A^{12} + 2A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.6875, \left\{ 37, \frac{1}{A^{22}} - \frac{1}{A^{18}} + \frac{2}{A^{14}} - \frac{3}{A^{10}} + \frac{1}{A^6} - \frac{1}{A^2} - A^2 + A^6 - 3A^{10} + 2A^{14} - A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.6875, \left\{ 38, 1 - \frac{1}{A^{24}} + \frac{1}{A^{20}} - \frac{3}{A^{16}} + \frac{2}{A^{12}} - \frac{2}{A^8} + \frac{1}{A^4} - 2A^4 + 2A^8 - 2A^{12} + 2A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.71875, \left\{ 39, 1 - \frac{1}{A^{20}} + \frac{1}{A^{16}} - \frac{3}{A^{12}} + \frac{2}{A^8} - \frac{2}{A^4} - 2A^8 + 3A^{12} - 2A^{16} + 2A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.78125, \left\{ 40, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{3}{A^{14}} - \frac{3}{A^{10}} + \frac{3}{A^6} - \frac{1}{A^2} - 2A^2 + A^6 - 4A^{10} + 3A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.671875, \left\{ 41, 1 - \frac{1}{A^{20}} + \frac{2}{A^{16}} - \frac{3}{A^{12}} + \frac{2}{A^8} - \frac{3}{A^4} + A^4 - 2A^8 + 3A^{12} - 3A^{16} + 2A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.640625, \left\{ 42, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{3}{A^{14}} - \frac{4}{A^{10}} + \frac{2}{A^6} - \frac{2}{A^2} - A^2 + 3A^6 - 3A^{10} + 3A^{14} - 3A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.8125, \left\{ 43, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{3}{A^{14}} - \frac{3}{A^{10}} + \frac{2}{A^6} - \frac{2}{A^2} - 2A^2 + 2A^6 - 3A^{10} + 3A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.828125, \left\{ 44, 1 - \frac{1}{A^{20}} + \frac{2}{A^{16}} - \frac{3}{A^{12}} + \frac{3}{A^8} - \frac{3}{A^4} - 3A^8 + 3A^{12} - 3A^{16} + 3A^{20} - A^{24} \right\} \right\}$

- » $\left\{ 0.84375, \left\{ 45, \frac{1}{A^{22}} - \frac{3}{A^{18}} + \frac{3}{A^{14}} - \frac{4}{A^{10}} + \frac{3}{A^6} - \frac{1}{A^2} - A^2 + 3A^6 - 4A^{10} + 3A^{14} - 3A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 1.23438, \left\{ 46, -1 - \frac{1}{A^{28}} + \frac{1}{A^{24}} - \frac{1}{A^{20}} + \frac{1}{A^{16}} + \frac{1}{A^8} + \frac{1}{A^4} - 2A^8 - A^{16} \right\} \right\}$
- » $\left\{ 0.9375, \left\{ 47, \frac{1}{A^{26}} - \frac{1}{A^{22}} + \frac{2}{A^{18}} - \frac{1}{A^{14}} + \frac{1}{A^{10}} - \frac{1}{A^6} - \frac{2}{A^2} - 2A^6 + A^{10} - A^{14} + A^{18} \right\} \right\}$
- » $\left\{ 1.07813, \left\{ 48, \frac{1}{A^{22}} - \frac{1}{A^{18}} + \frac{2}{A^{14}} - \frac{2}{A^{10}} + \frac{1}{A^6} - \frac{2}{A^2} - 2A^2 + A^6 - 2A^{10} + 2A^{14} - A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.96875, \left\{ 49, 1 - \frac{1}{A^{20}} + \frac{1}{A^{16}} - \frac{3}{A^{12}} + \frac{1}{A^8} - \frac{3}{A^4} + A^4 - A^8 + 3A^{12} - 2A^{16} + 2A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 1.07813, \left\{ 50, 1 - \frac{1}{A^{24}} + \frac{1}{A^{20}} - \frac{2}{A^{16}} + \frac{3}{A^{12}} - \frac{1}{A^8} + \frac{1}{A^4} - 2A^4 + A^8 - 3A^{12} + A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.875, \left\{ 51, \frac{1}{A^{22}} - \frac{1}{A^{18}} + \frac{3}{A^{14}} - \frac{3}{A^{10}} + \frac{2}{A^6} - \frac{2}{A^2} - 2A^2 + A^6 - 3A^{10} + 3A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 1.57813, \left\{ 52, \frac{1}{A^{26}} - \frac{2}{A^{22}} + \frac{3}{A^{18}} - \frac{2}{A^{14}} + \frac{1}{A^{10}} - \frac{1}{A^6} - \frac{2}{A^2} + A^2 - 3A^6 + 2A^{10} - A^{14} + A^{18} \right\} \right\}$
- » $\left\{ 0.59375, \left\{ 53, 1 - \frac{1}{A^{24}} + \frac{2}{A^{20}} - \frac{4}{A^{16}} + \frac{2}{A^{12}} - \frac{3}{A^8} - 2A^4 + 4A^8 - 2A^{12} + 2A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.671875, \left\{ 54, \frac{1}{A^{26}} - \frac{1}{A^{22}} + \frac{2}{A^{18}} - \frac{2}{A^{14}} + \frac{1}{A^{10}} - \frac{1}{A^6} - \frac{2}{A^2} - 2A^6 + 2A^{10} - A^{14} + A^{18} \right\} \right\}$
- » $\left\{ 0.625, \left\{ 55, 1 - \frac{1}{A^{24}} + \frac{1}{A^{20}} - \frac{3}{A^{16}} + \frac{2}{A^{12}} - \frac{3}{A^8} - A^4 + 3A^8 - 2A^{12} + 2A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.609375, \left\{ 56, 1 - \frac{1}{A^{24}} + \frac{2}{A^{20}} - \frac{3}{A^{16}} + \frac{3}{A^{12}} - \frac{1}{A^8} + \frac{1}{A^4} - 3A^4 + 2A^8 - 3A^{12} + A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.625, \left\{ 57, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{4}{A^{14}} - \frac{3}{A^{10}} + \frac{2}{A^6} - \frac{2}{A^2} - 2A^2 + 2A^6 - 4A^{10} + 3A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.65625, \left\{ 58, 1 - \frac{1}{A^{24}} + \frac{1}{A^{20}} - \frac{3}{A^{16}} + \frac{3}{A^{12}} - \frac{2}{A^8} + \frac{1}{A^4} - 2A^4 + 2A^8 - 3A^{12} + 2A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.609375, \left\{ 59, -\frac{1}{A^{24}} + \frac{2}{A^{20}} - \frac{3}{A^{16}} + \frac{4}{A^{12}} - \frac{2}{A^8} - 3A^4 + 3A^8 - 3A^{12} + 2A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.625, \left\{ 60, 1 - \frac{1}{A^{24}} + \frac{3}{A^{20}} - \frac{4}{A^{16}} + \frac{3}{A^{12}} - \frac{3}{A^8} - 3A^4 + 4A^8 - 3A^{12} + 2A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.921875, \left\{ 61, -\frac{1}{A^{28}} + \frac{1}{A^{24}} - \frac{1}{A^{20}} + \frac{1}{A^{16}} - \frac{1}{A^{12}} + \frac{1}{A^4} + A^4 - 2A^8 - A^{16} \right\} \right\}$
- » $\left\{ 0.9375, \left\{ 62, \frac{1}{A^{26}} - \frac{1}{A^{22}} + \frac{2}{A^{18}} - \frac{2}{A^{14}} + \frac{1}{A^{10}} - \frac{1}{A^2} - 3A^6 + A^{10} - A^{14} + A^{18} \right\} \right\}$
- » $\left\{ 0.796875, \left\{ 63, -\frac{1}{A^{24}} + \frac{1}{A^{20}} - \frac{3}{A^{16}} + \frac{2}{A^{12}} - \frac{2}{A^8} - 2A^4 + 3A^8 - A^{12} + 2A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.96875, \left\{ 64, -\frac{1}{A^{24}} + \frac{1}{A^{20}} - \frac{2}{A^{16}} + \frac{3}{A^{12}} - \frac{1}{A^8} - 2A^4 + 2A^8 - 2A^{12} + A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 1., \left\{ 65, \frac{1}{A^{22}} - \frac{1}{A^{18}} + \frac{3}{A^{14}} - \frac{3}{A^{10}} + \frac{1}{A^6} - \frac{2}{A^2} - A^2 + 2A^6 - 3A^{10} + 2A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.75, \left\{ 66, 2 - \frac{1}{A^{20}} + \frac{1}{A^{16}} - \frac{4}{A^{12}} + \frac{2}{A^8} - \frac{3}{A^4} + A^4 - 2A^8 + 3A^{12} - 3A^{16} + 3A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.71875, \left\{ 67, -\frac{1}{A^{24}} + \frac{1}{A^{20}} - \frac{3}{A^{16}} + \frac{3}{A^{12}} - \frac{2}{A^8} - 2A^4 + 3A^8 - 2A^{12} + 2A^{16} - A^{20} \right\} \right\}$

- » $\left\{ 0.671875, \left\{ 68, \frac{1}{A^{26}} - \frac{2}{A^{22}} + \frac{2}{A^{18}} - \frac{3}{A^{14}} + \frac{1}{A^{10}} - \frac{1}{A^2} + A^2 - 3A^6 + 2A^{10} - A^{14} + A^{18} \right\} \right\}$
- » $\left\{ 0.890625, \left\{ 69, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{4}{A^{14}} - \frac{4}{A^{10}} + \frac{2}{A^6} - \frac{2}{A^2} - A^2 + 3A^6 - 4A^{10} + 3A^{14} - 3A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.65625, \left\{ 70, 1 - \frac{1}{A^{24}} + \frac{2}{A^{20}} - \frac{3}{A^{16}} + \frac{3}{A^{12}} - \frac{2}{A^8} - 2A^4 + 3A^8 - 3A^{12} + A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.515625, \left\{ 71, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{3}{A^{14}} - \frac{4}{A^{10}} + \frac{2}{A^6} - \frac{1}{A^2} - A^2 + 2A^6 - 4A^{10} + 3A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.59375, \left\{ 72, 1 - \frac{1}{A^{24}} + \frac{3}{A^{20}} - \frac{3}{A^{16}} + \frac{3}{A^{12}} - \frac{2}{A^8} - 3A^4 + 3A^8 - 3A^{12} + A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.875, \left\{ 73, \frac{1}{A^{22}} - \frac{3}{A^{18}} + \frac{3}{A^{14}} - \frac{4}{A^{10}} + \frac{2}{A^6} - \frac{1}{A^2} - A^2 + 3A^6 - 4A^{10} + 3A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.65625, \left\{ 74, 1 - \frac{1}{A^{24}} + \frac{2}{A^{20}} - \frac{3}{A^{16}} + \frac{2}{A^{12}} - \frac{3}{A^8} - \frac{1}{A^4} - A^4 + 4A^8 - 2A^{12} + A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.65625, \left\{ 75, 1 - \frac{1}{A^{20}} + \frac{2}{A^{16}} - \frac{3}{A^{12}} + \frac{4}{A^8} - \frac{2}{A^4} - A^4 - 4A^8 + 3A^{12} - 3A^{16} + 3A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.671875, \left\{ 76, 2 - \frac{1}{A^{24}} + \frac{2}{A^{20}} - \frac{3}{A^{16}} + \frac{2}{A^{12}} - \frac{1}{A^8} + \frac{1}{A^4} - 2A^4 + 2A^8 - 3A^{12} - A^{20} \right\} \right\}$
- » $\left\{ 0.84375, \left\{ 77, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{3}{A^{14}} - \frac{2}{A^{10}} + \frac{2}{A^6} - \frac{1}{A^2} - 2A^2 + A^6 - 4A^{10} + 2A^{14} - A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.59375, \left\{ 78, -\frac{1}{A^{20}} + \frac{2}{A^{16}} - \frac{3}{A^{12}} + \frac{2}{A^8} - \frac{3}{A^4} - 2A^8 + 4A^{12} - 2A^{16} + 2A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.625, \left\{ 79, \frac{1}{A^{22}} - \frac{1}{A^{18}} + \frac{3}{A^{14}} - \frac{3}{A^{10}} + \frac{1}{A^6} - \frac{2}{A^2} - 2A^2 + A^6 - 3A^{10} + 3A^{14} - A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.6875, \left\{ 80, 1 - \frac{1}{A^{20}} + \frac{1}{A^{16}} - \frac{4}{A^{12}} + \frac{2}{A^8} - \frac{3}{A^4} + A^4 - A^8 + 4A^{12} - 3A^{16} + 2A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 1.01563, \left\{ 81, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{4}{A^{14}} - \frac{4}{A^{10}} + \frac{2}{A^6} - \frac{2}{A^2} - 2A^2 + 2A^6 - 4A^{10} + 4A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 1.125, \left\{ 82, -\frac{1}{A^{20}} + \frac{2}{A^{16}} - \frac{2}{A^{12}} + \frac{2}{A^8} - \frac{3}{A^4} - 2A^8 + 3A^{12} - 2A^{16} + 2A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.953125, \left\{ 83, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{3}{A^{14}} - \frac{4}{A^{10}} + \frac{3}{A^6} - \frac{1}{A^2} - A^2 + 2A^6 - 4A^{10} + 3A^{14} - 3A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 1.32813, \left\{ 84, \frac{1}{A^{22}} - \frac{3}{A^{18}} + \frac{4}{A^{14}} - \frac{3}{A^{10}} + \frac{3}{A^6} - \frac{1}{A^2} - 2A^2 + 2A^6 - 5A^{10} + 3A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 1.65625, \left\{ 85, \frac{1}{A^{18}} - \frac{2}{A^{14}} + \frac{1}{A^{10}} - \frac{3}{A^6} + \frac{1}{A^2} - A^2 + 2A^{10} - 2A^{14} + 2A^{18} - 2A^{22} + A^{26} \right\} \right\}$
- » $\left\{ 0.8125, \left\{ 86, 1 - \frac{1}{A^{20}} + \frac{2}{A^{16}} - \frac{3}{A^{12}} + \frac{4}{A^8} - \frac{3}{A^4} - 3A^8 + 3A^{12} - 4A^{16} + 3A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.78125, \left\{ 87, -\frac{1}{A^{20}} + \frac{3}{A^{16}} - \frac{3}{A^{12}} + \frac{3}{A^8} - \frac{3}{A^4} - 3A^8 + 4A^{12} - 3A^{16} + 2A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.875, \left\{ 88, \frac{1}{A^{22}} - \frac{3}{A^{18}} + \frac{4}{A^{14}} - \frac{5}{A^{10}} + \frac{3}{A^6} - \frac{1}{A^2} - A^2 + 3A^6 - 5A^{10} + 4A^{14} - 3A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.578125, \left\{ 89, \frac{1}{A^{22}} - \frac{4}{A^{18}} + \frac{4}{A^{14}} - \frac{4}{A^{10}} + \frac{3}{A^6} - \frac{1}{A^2} - 2A^2 + 3A^6 - 5A^{10} + 4A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 1.09375, \left\{ 90, 1 - \frac{1}{A^{20}} + \frac{2}{A^{16}} - \frac{3}{A^{12}} + \frac{3}{A^8} - \frac{3}{A^4} + A^4 - 2A^8 + 3A^{12} - 4A^{16} + 2A^{20} - A^{24} \right\} \right\}$

- » $\left\{ 0.5625, \left\{ 91, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{3}{A^{14}} - \frac{3}{A^{10}} + \frac{2}{A^6} - \frac{2}{A^2} - 2A^2 + 2A^6 - 3A^{10} + 3A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.640625, \left\{ 92, 1 - \frac{1}{A^{24}} + \frac{3}{A^{20}} - \frac{4}{A^{16}} + \frac{4}{A^{12}} - \frac{2}{A^8} + \frac{1}{A^4} - 4A^4 + 3A^8 - 4A^{12} + 2A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.734375, \left\{ 93, \frac{1}{A^{18}} - \frac{2}{A^{14}} + \frac{2}{A^{10}} - \frac{3}{A^6} + \frac{2}{A^2} - A^2 - A^6 + A^{10} - 3A^{14} + 3A^{18} - 2A^{22} + A^{26} \right\} \right\}$
- » $\left\{ 0.90625, \left\{ 94, 1 - \frac{1}{A^{24}} + \frac{2}{A^{20}} - \frac{3}{A^{16}} + \frac{3}{A^{12}} - \frac{2}{A^8} + \frac{1}{A^4} - 3A^4 + 2A^8 - 3A^{12} + 2A^{16} - A^{20} \right\} \right\}$
- » $\left\{ 0.84375, \left\{ 95, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{4}{A^{14}} - \frac{4}{A^{10}} + \frac{3}{A^6} - \frac{2}{A^2} - 2A^2 + 2A^6 - 4A^{10} + 4A^{14} - 3A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.703125, \left\{ 96, 2 - \frac{1}{A^{20}} + \frac{2}{A^{16}} - \frac{4}{A^{12}} + \frac{4}{A^8} - \frac{3}{A^4} + A^4 - 3A^8 + 3A^{12} - 5A^{16} + 3A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.5625, \left\{ 97, -\frac{1}{A^{20}} + \frac{3}{A^{16}} - \frac{3}{A^{12}} + \frac{4}{A^8} - \frac{3}{A^4} - 3A^8 + 4A^{12} - 4A^{16} + 2A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.96875, \left\{ 98, 1 - \frac{1}{A^{20}} + \frac{2}{A^{16}} - \frac{4}{A^{12}} + \frac{2}{A^8} - \frac{4}{A^4} + 2A^4 - A^8 + 4A^{12} - 4A^{16} + 2A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.703125, \left\{ 99, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{4}{A^{14}} - \frac{3}{A^{10}} + \frac{2}{A^6} - \frac{3}{A^2} - 3A^2 + 2A^6 - 3A^{10} + 4A^{14} - 2A^{18} + A^{22} \right\} \right\}$
- » $\left\{ 0.875, \left\{ 100, \frac{1}{A^{18}} - \frac{2}{A^{14}} + \frac{2}{A^{10}} - \frac{3}{A^6} + \frac{1}{A^2} - 2A^2 - A^6 + 2A^{10} - 2A^{14} + 3A^{18} - 2A^{22} + A^{26} \right\} \right\}$
- » $\left\{ 0.671875, \left\{ 101, 1 - \frac{1}{A^{20}} + \frac{3}{A^{16}} - \frac{3}{A^{12}} + \frac{4}{A^8} - \frac{2}{A^4} - 4A^8 + 3A^{12} - 4A^{16} + 2A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.8125, \left\{ 102, 1 - \frac{1}{A^{20}} + \frac{2}{A^{16}} - \frac{3}{A^{12}} + \frac{3}{A^8} - \frac{2}{A^4} - 3A^8 + 3A^{12} - 3A^{16} + 2A^{20} - A^{24} \right\} \right\}$
- » $\left\{ 0.8125, \left\{ 103, \frac{1}{A^{22}} - \frac{2}{A^{18}} + \frac{3}{A^{14}} - \frac{4}{A^{10}} + \frac{1}{A^6} - \frac{2}{A^2} - A^2 + 3A^6 - 3A^{10} + 3A^{14} - 2A^{18} + A^{22} \right\} \right\}$

Out[]:= \$Aborted

Plus @@ KBon10

In[]:= **KB[GST48]**

Out[]:= \$Aborted

```
In[ ]:= FKB[pd_PD] := Module[{p, t1, t2, t3, t4, B, d, KB, todo},
  SetAttributes[p, Orderless];
  KB = 1;
  todo = pd;
  While[Length[todo] > 0,
    x = First[todo];
    todo = DeleteCases[todo, x];
    t1 = KB (x /. X[i_, j_, k_, l_] -> A * p[i, j] * p[k, l] + B * p[i, l] * p[j, k]);
    t2 = Expand[t1];
    t3 = t2 /. {p[i_, j_] * p[j_, k_] -> p[i, k]};
    t4 = t3 /. {p[i_, i_] -> d, p[i_, j_]^2 -> d};
    KB = Expand[t4 /. {B -> 1/A, d -> -A^2 - 1/A^2}];
  ];
  KB
]
```

In[]:= **FKB[Knot[8, 17]]**

$$\text{Out[]} = -\frac{1}{A^{18}} + \frac{2}{A^{14}} - \frac{2}{A^{10}} + \frac{1}{A^6} - \frac{1}{A^2} - A^2 + A^6 - 2A^{10} + 2A^{14} - A^{18}$$

In[]:= **Short[FKBon10 = Table[
 Timing[{k, FKB[Knot[10, k]]}],
 {k, 165}
]]**

Out[]//Short= {{0.046875, {1, <<1>>}}, <<164>>}

In[]:= **Plus @@ FKBon10**

$$\text{Out[]} = \left\{ 12.9219, \left\{ 13695, 53 - \frac{3}{A^{28}} + \frac{8}{A^{26}} - \frac{31}{A^{24}} + \frac{28}{A^{22}} + \frac{18}{A^{20}} - \frac{59}{A^{18}} - \frac{49}{A^{16}} + \frac{112}{A^{14}} - \frac{5}{A^{12}} - \frac{141}{A^{10}} + \frac{12}{A^8} + \frac{69}{A^6} - \frac{80}{A^4} - \frac{88}{A^2} - 76A^2 - 70A^4 + 62A^6 + 7A^8 - 138A^{10} + 18A^{12} + 112A^{14} - 52A^{16} - 72A^{18} + 30A^{20} + 36A^{22} - 35A^{24} + 7A^{26} - 3A^{28} \right\} \right\}$$

```
In[ ]:= EFKB[pd_PD] := Module[{p, t1, t2, t3, t4, B, d, KB, todo, front, x, v},
    SetAttributes[p, Orderless];
    KB = 1;
    todo = List @@ pd;
    front = {};
    v[x_X] := Length[front ∩ (List @@ x)];
    While[Length[todo] > 0,
        x = RandomChoice[MaximalBy[todo, v]];
        todo = DeleteCases[todo, x];
        t1 = KB (x /. X[i_, j_, k_, l_] → A * p[i, j] * p[k, l] + B * p[i, l] * p[j, k]);
        t2 = Expand[t1];
        t3 = t2 /. {p[i_, j_] * p[j_, k_] → p[i, k]};
        t4 = t3 /. {p[i_, i_] → d, p[i_, j_]^2 → d};
        KB = Expand[t4 /. {B → 1/A, d → -A^2 - 1/A^2}];
        front = Complement[front ∪ (List @@ x), front ∩ (List @@ x)];
    ];
    KB
]
```

In[]:= **Short[EFKBon10 = Table[
 Timing[{k, EFKB[Knot[10, k]]}],
 {k, 165}
]]**
Plus @@ EFKBon10

Out[]//Short= {{0.015625, {1, <<1>>}}, <<164>>}

$$\text{Out[]} = \left\{ 4.25, \left\{ 13695, 53 - \frac{3}{A^{28}} + \frac{8}{A^{26}} - \frac{31}{A^{24}} + \frac{28}{A^{22}} + \frac{18}{A^{20}} - \frac{59}{A^{18}} - \frac{49}{A^{16}} + \frac{112}{A^{14}} - \frac{5}{A^{12}} - \frac{141}{A^{10}} + \frac{12}{A^8} + \frac{69}{A^6} - \frac{80}{A^4} - \frac{88}{A^2} - 76A^2 - 70A^4 + 62A^6 + 7A^8 - 138A^{10} + 18A^{12} + 112A^{14} - 52A^{16} - 72A^{18} + 30A^{20} + 36A^{22} - 35A^{24} + 7A^{26} - 3A^{28} \right\} \right\}$$

In[]:= **Timing**[**EFKB**[**GST48**]]

$$\text{Out[]:= } \left\{ 6.04688, \frac{1}{A^{60}} - \frac{2}{A^{56}} + \frac{1}{A^{52}} + \frac{1}{A^{48}} - \frac{2}{A^{44}} - \frac{1}{A^{24}} + \frac{2}{A^{20}} + \frac{1}{A^{16}} - \frac{2}{A^{12}} + \frac{4}{A^8} - \frac{2}{A^4} - 2A^8 - 2A^{16} + A^{20} - A^{28} + A^{36} \right\}$$