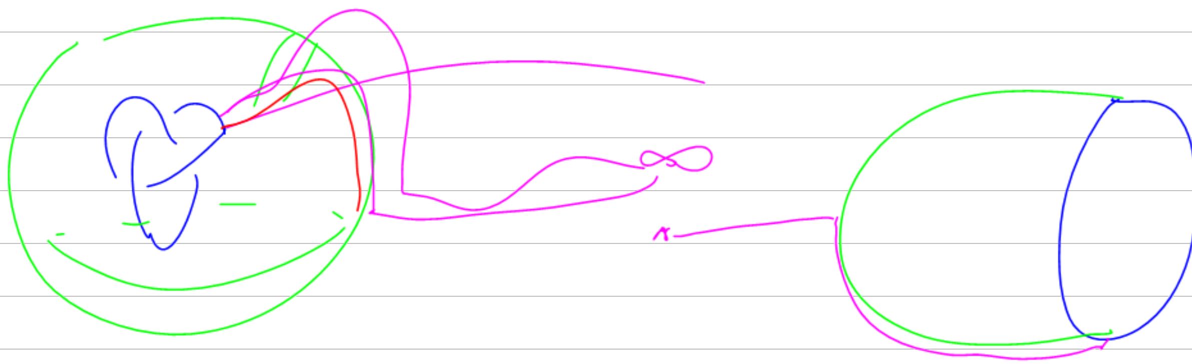
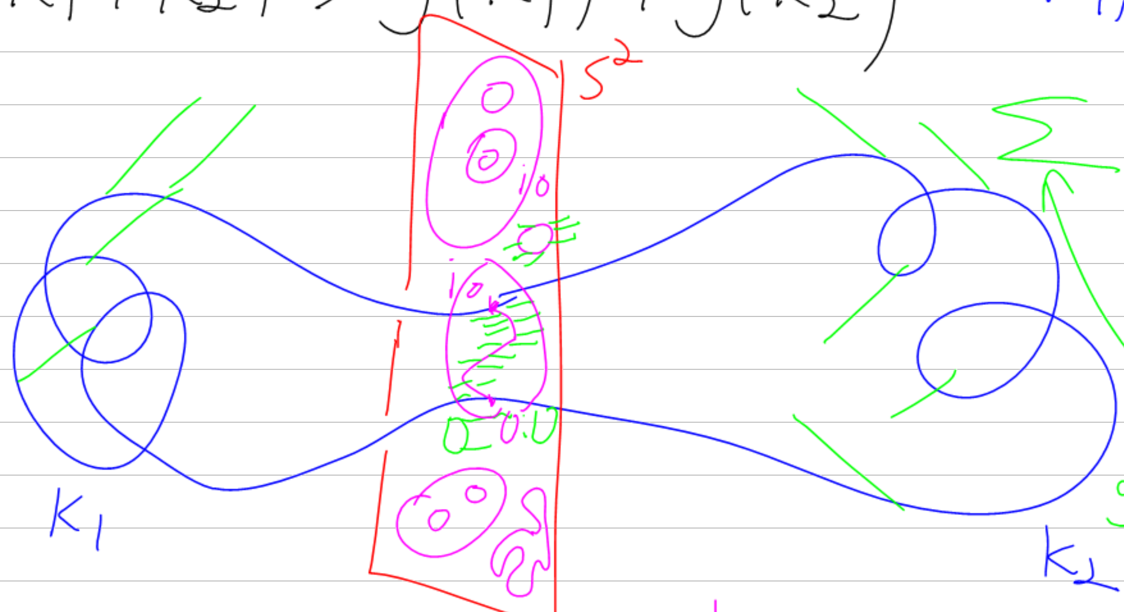
Thm $g(K_1+K_2)=g(K_1)+g(K_2)$

Pf of thm (Modulo all about diff geom & the topology of $\mathbb{R}^2$)

$g(K_1+K_2)\le g(K_1)+g(K_2)$ [Easy, yet...]



$g(K_1+K_2)\ge g(K_1)+g(K_2)$ $K_{1,2}\subset S^3$

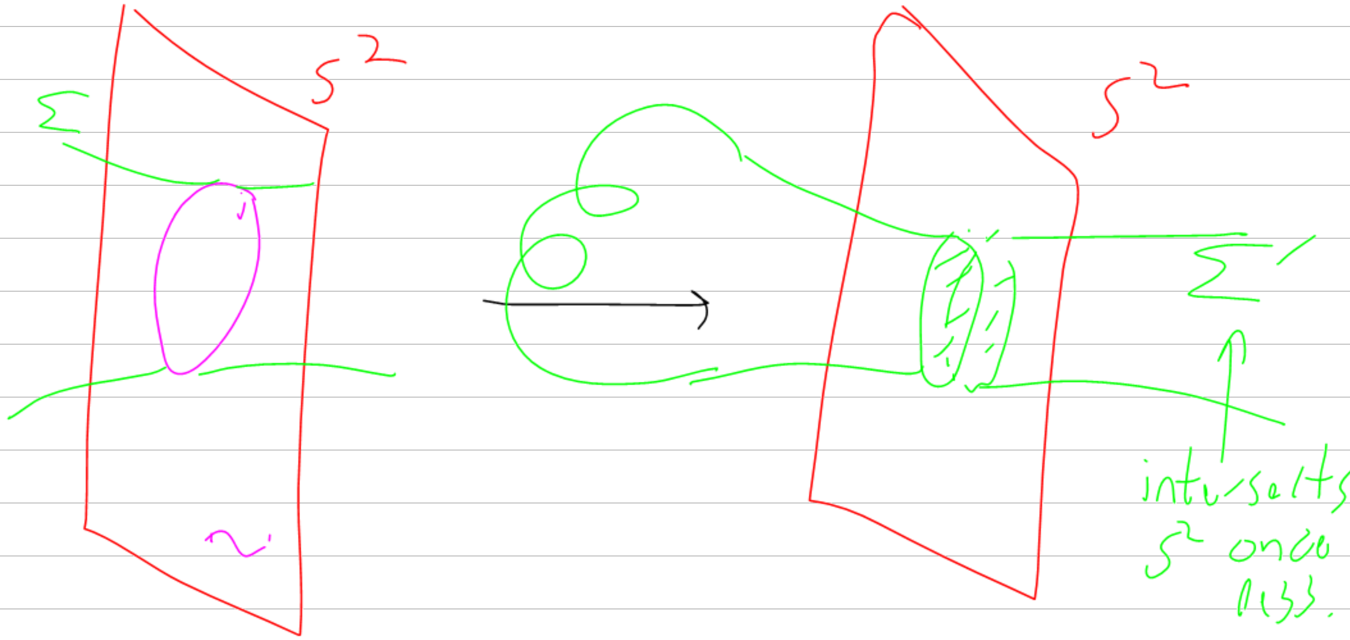


Σ a s.s. for K_1+K_2

WLOG, $\Sigma \pitchfork S^2$

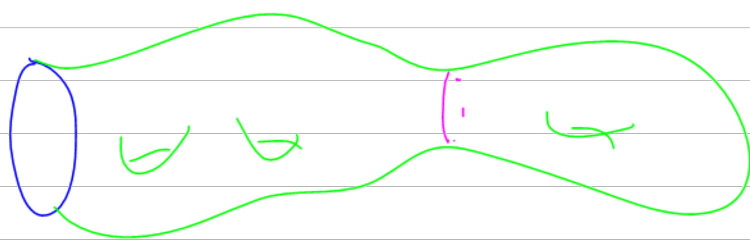
genus minimize $g(\Sigma)=g(K_1+K_2)$

Pick an inner most C in $\Sigma\cap S^2$



<u>Case 1</u> Σ' is no longer connected

$\to$ Keep only the component of Σ' that touches $K_1 + K_2$, call it Σ''

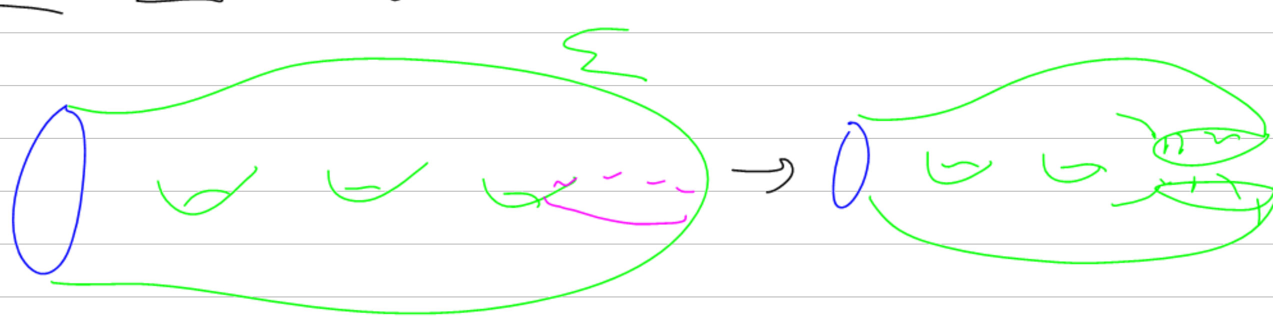


$g(\Sigma'') \leq g(\Sigma)$

So as Σ was gen-min,

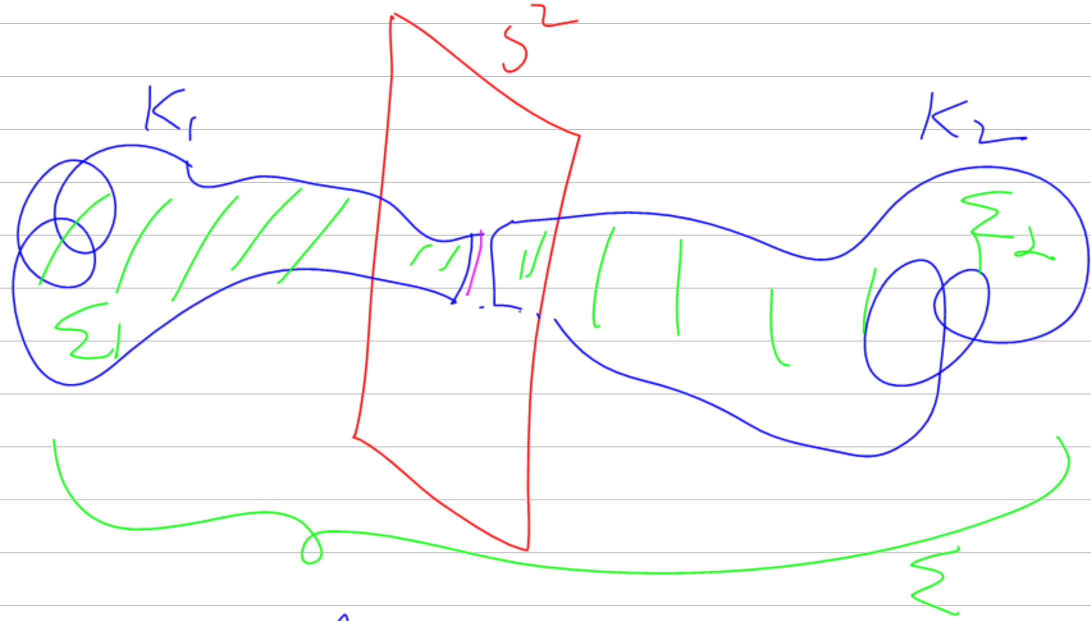
$g(\Sigma'') = g(\Sigma)$

<u>Case 2</u> Σ' is connected



$g(\Sigma') < g(\Sigma) \Rightarrow\Leftarrow$

Iterate until there are no circles in $\Sigma \cap S^2$.



Found Σ_1 & Σ_2 w/ $\partial \Sigma_i = K_i$

$$g(K_1) + g(K_2) \leq g(\Sigma_1) + g(\Sigma_2) = g(\Sigma) = g(K_1 + K_2)$$

__Thm__ Suppose $P + Q = \underbrace{K_1}_{P+L} + K_2$ where P is prime. Then,

either $\exists L$ s.t. $P + L = K_1$ & $Q = L + K_2$

or $\exists L$ s.t. $P + L = K_2$ & $Q = L + K_1$

__Cor 1__ If $P + Q_1 = \underbrace{P}_{K_1} + \underbrace{Q_2}_{K_2}$ then $Q_1 = Q_2$

__pf__ By Thm either

$\exists L$ $\underbrace{P + L = P}$ & $Q_1 = L + Q_2$

$L = 0 \Rightarrow Q_1 = 0 + Q_2 = Q_2$

or $\exists L \quad P+L=Q_2$ & $Q_1=L+P$

$$\Rightarrow Q_1=Q_2 \qquad \square.$$

<u>Cor 2</u> If $p_1+\dots+p_n=p_1'+\dots+p_{n'}'$ where p_i & p_i' are primes, $n\leq n'$ then $n=n'$ & (p_i) are a perm of the (p_i').

<u>Pf</u> By induction on n.

If $n=0$,

$$0=p_1'+\dots p_{n'}' \Rightarrow n'=0,$$

otherwise

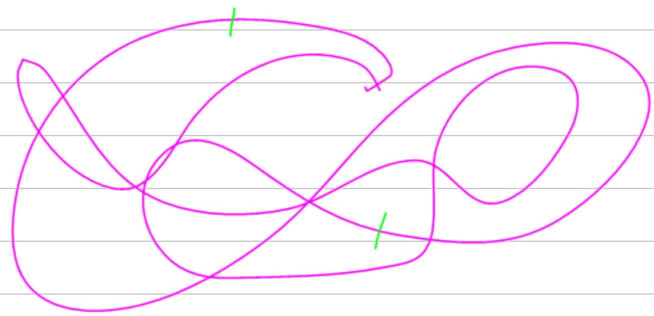
$$\underbrace{p_1}+\dots+p_n=p_1'+\dots+p_{n'}'$$

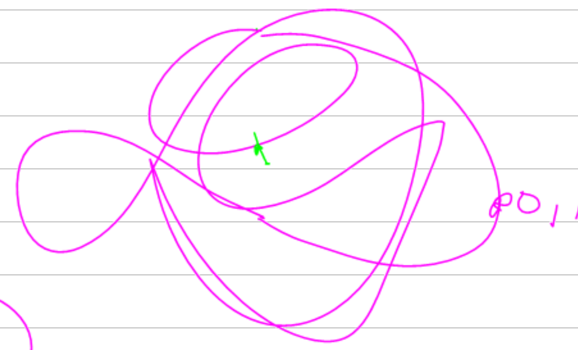
p_1 divides p_i' for some i, so $p_1=p_i'$,

cancel p_1 & p_i' from the two sides & use induction.

$\square$.


P
Q
0
K_1
K_2
1




$\rho_{0,1}$

Hour 30, Wednesday November 25: Funny things in R^3, decomposition into primes.

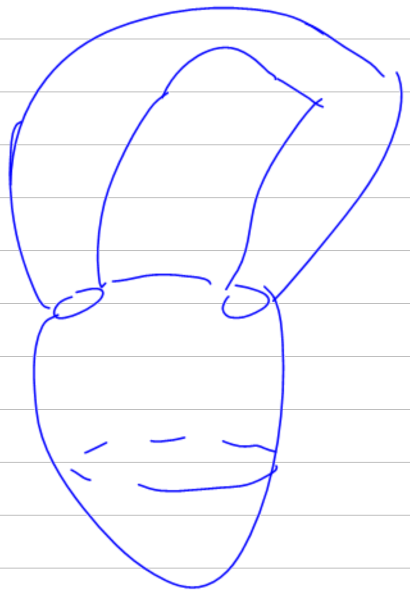
Thm If $P+Q = K_1 + K_2$ w/ P prime then

either $K_1 = P+L$ & $Q = L + K_2$

or $K_2 = P+L$ & $Q = L + K_1$

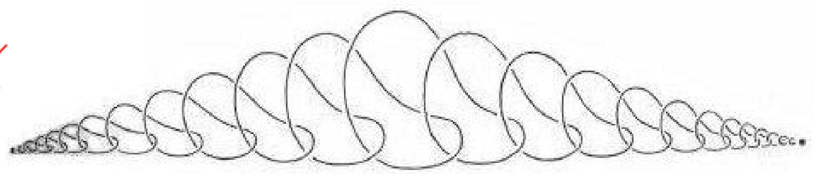
We started class all wrong to prove this!!

Get a feel for funny $\mathbb{R}^3$.

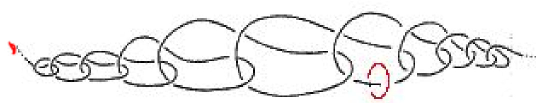


Topological Pathologies in R^3

An embedding of an interval in R^3 whose complement is not simply connected:

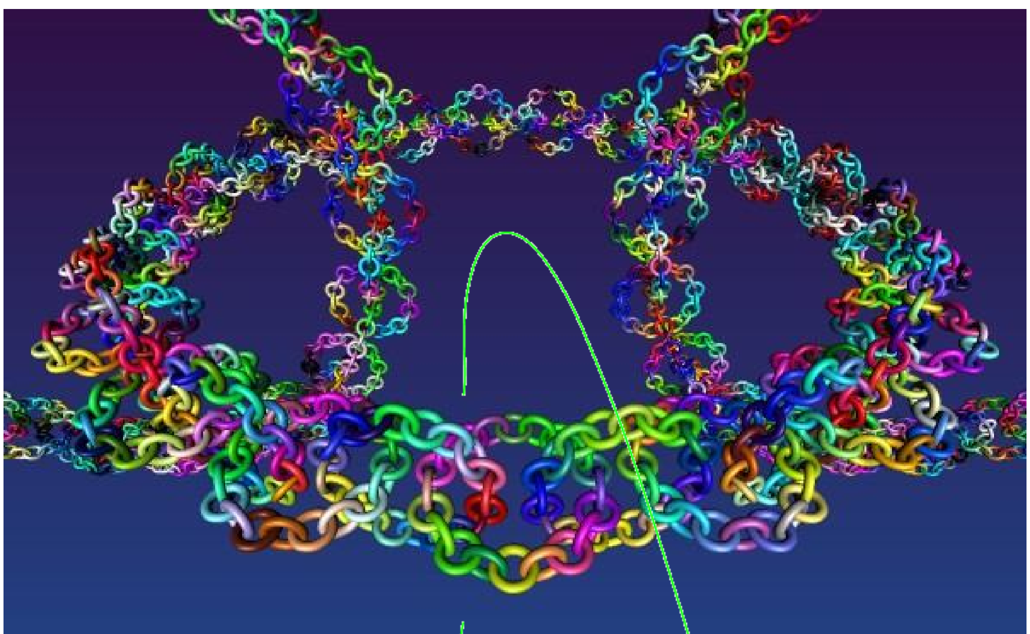


See Hocking and Young's *Topology* pp. 176-177.



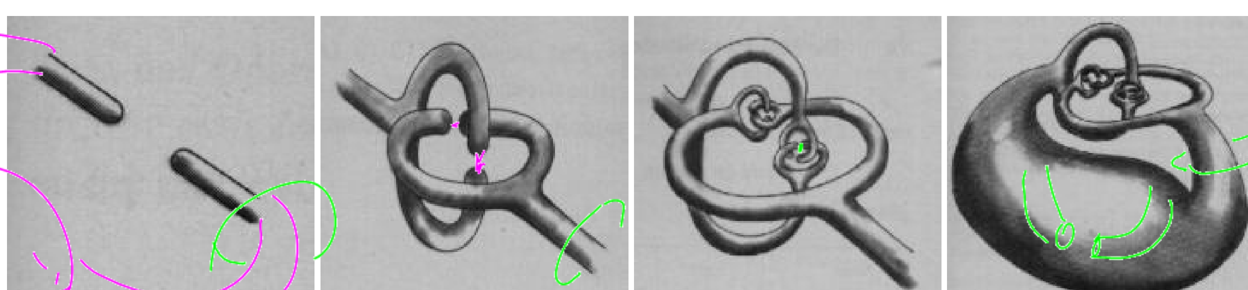
See http://www.math.ohio-state.edu/~fiedorow/math655/Jordan.html.

Antoine's necklace - an embedding of a Cantor set in R^3 whose complement is not simply connected:



See http://www.cs.ubc.ca/nest/imager/contributions/scharein/various/AntoinesNecklace.html.

The Alexander horned sphere - a continuous embedding of a ball in R^3 whose complement is not simply connected:



See http://users.math.uni-potsdam.de/~oeitner/EIGENES/RAEUME/hornsph.htm.

Yet in $\mathbb{R}^2$ if γ is a closed cont. simple loop, then $S^2 \setminus \gamma$ has two connected comps both homeomorphic B^2

Alexander-Schöflies thm a smooth S^2 in S^3 divides S^3 into two components each diffeomorphic to a ball.

pf: 2. Allan Hatcher: "3 manifolds ..."

1. DBN $\longrightarrow$ Students $\longrightarrow$ Morton-Ferguson

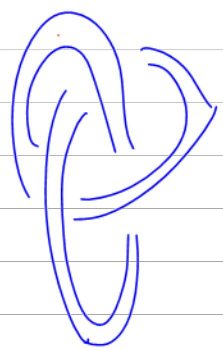
Cor There are no knotted smooth S^2 in S^3, i.e. if S is an S^2 in ~~$\mathbb{R}^3$~~ S^3 and if S' is equatorial S^2 of $\mathbb{R}^3$ then

$$(S^3, S) \overset{\phi}{\sim} (S^3, S')$$

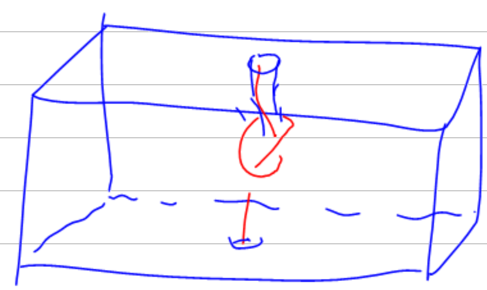
$S^1 \times S^1 = T^2$

Yet there are knotted tori in S^3

A smooth T^2 in $\mathbb{R}^3$ still divides $\mathbb{R}^3$
into two connected comps one containing
∞ "~~out~~ outside" & one that doesn't
"inside"



inside is a ~~std~~ homeo to $B^2 \times S^1$
outside is not homeo to $B^2 \times S^1$



If T is a smooth T^2 in S^3
then either the inside or the outside
is $B^2 \times S^1$.

---

4 equiv. def'n of knot in $\mathbb{R}^3$.
A smooth simple curve in $\mathbb{R}^3$ modulo
a. Smooth homotopies of such.
b. "Ambient isotopies"
$\gamma_0 \sim \gamma_1$ if $\exists\, H_t : \mathbb{R}^3 \to \mathbb{R}^3$
of diffeomorphisms

s.t. $H_0 = Id$

$$H_1 \circ \gamma_1 = \gamma_0$$

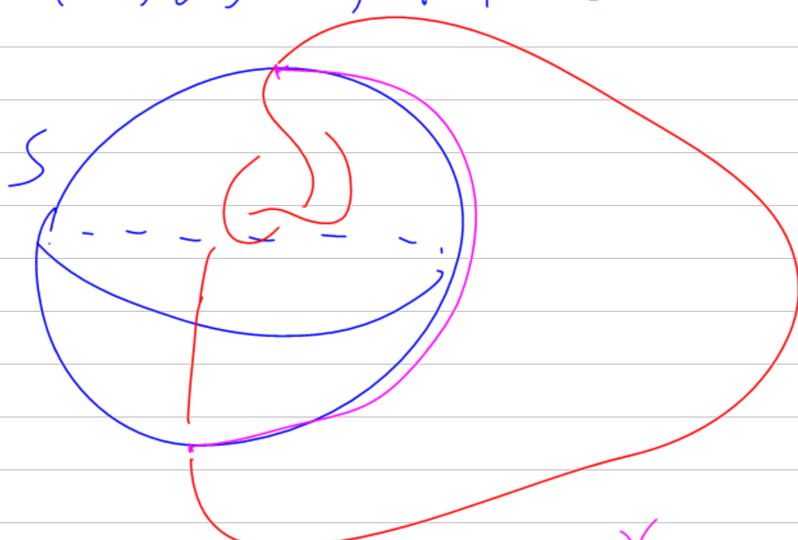
c. $\gamma_0 \sim \gamma_1$ if $\exists\ H : \mathbb{R}^3 \circlearrowleft$ s.t. $H \circ \gamma_1 = \gamma_0$

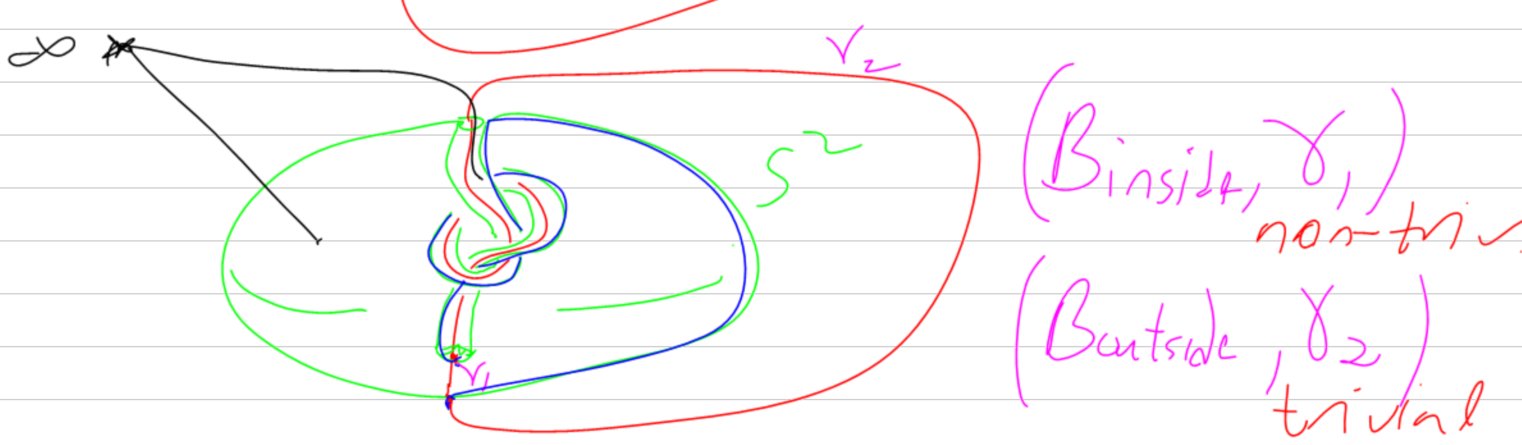
orientation pres. diffeomorphism.

4. Planar curves / R123.

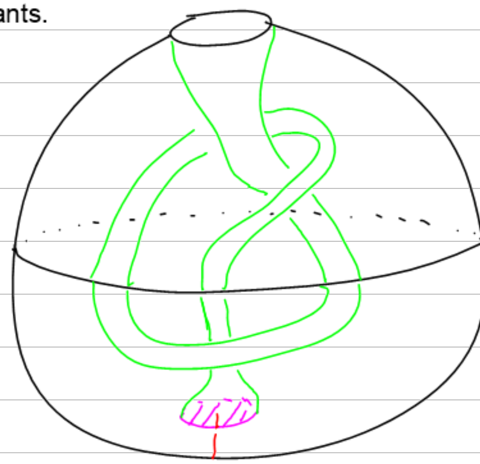
<u>Thm</u> These are all equiv.

<u>Def</u> γ is prime if whenever an S^2 S intersect γ exactly twice, then one of the sides is trivial it is non-trivial





Hour 31, Friday November 27: Decomposition into primes, end. Next: Braids and variants.
Should we have a "HW discussion" class?
HW9 is online!



Thm If $P+Q = K_1+K_2$ w/ P prime then

either $K_1 = P+L$ & $Q = L+K_2$

or $K_2 = P+L$ & $Q = L+K_1$

Pf γ The curve

B a ball separating P from Q

$S = \partial B$ P is in B & Q in B^c

Σ a sphere separating K_1 & K_2

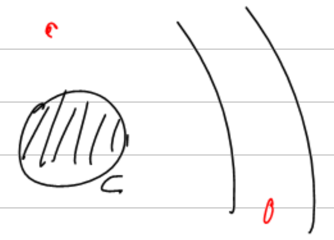
If S & Σ are disjoint, thm is proven

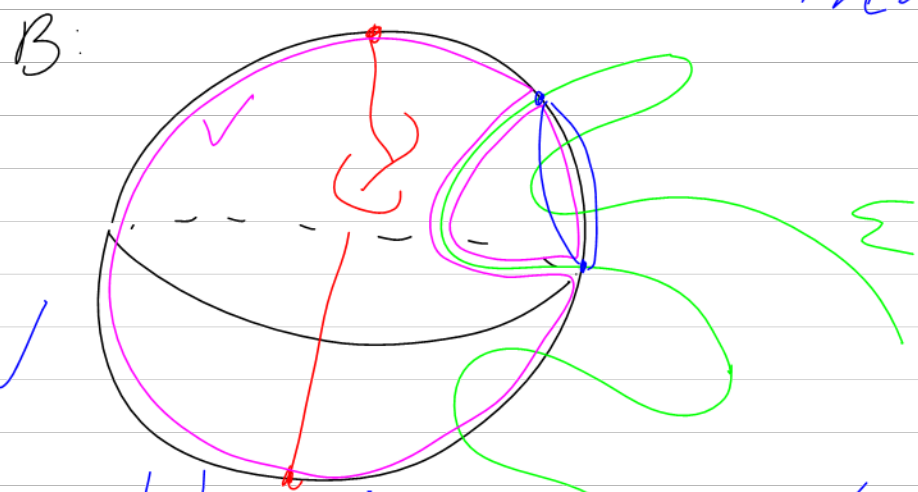
otherwise Σ wears a pajama:

Colour Σ black where it is inside B & white otherwise, Bndries are $\Sigma \cap S$

$\Sigma \cap S$ is a finite collection of smooth closed curves on Σ.

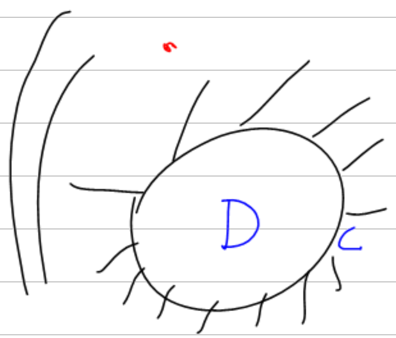
If there's a black island on Σ, w/o red dot.

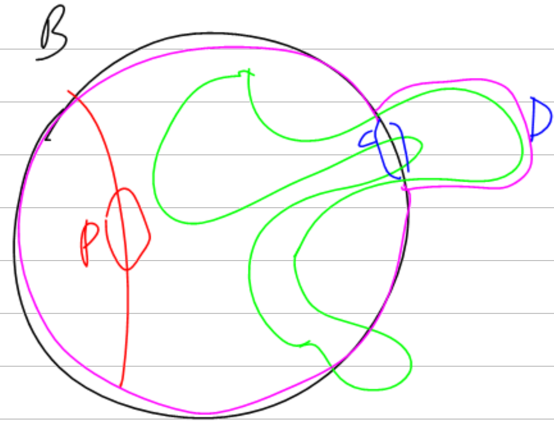


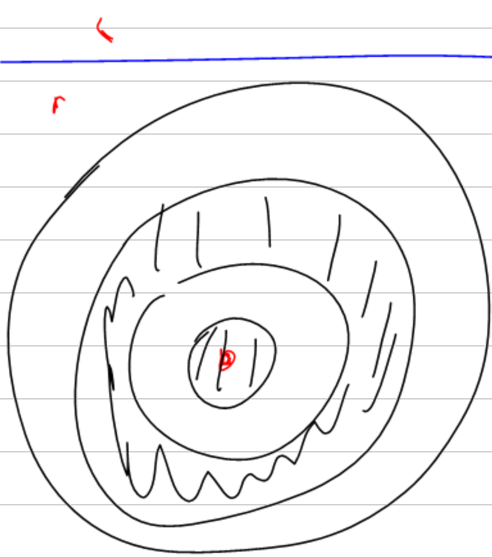


Replace B by V

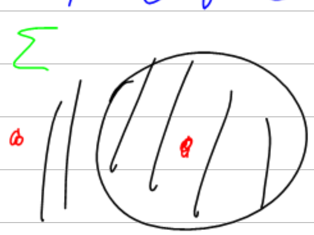
If there's a white disk on Σ, w/o red.



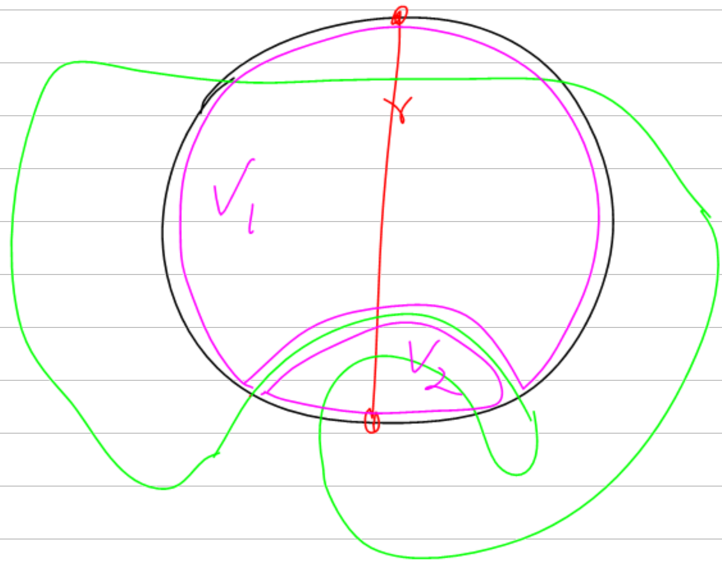




Next case: A black disk w/ a red dot inside:

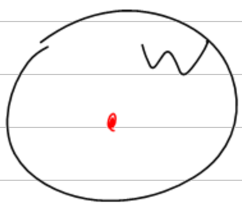


B:

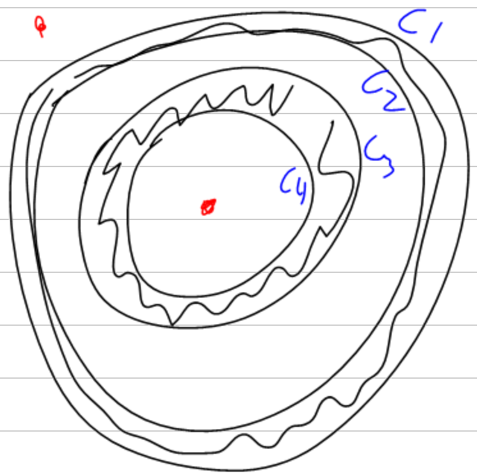


p lies in V_1 or V_2

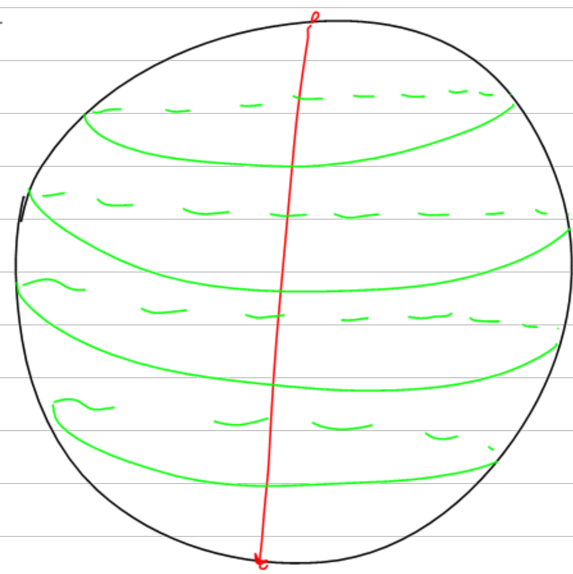
If p is in V_i replace B by V_i

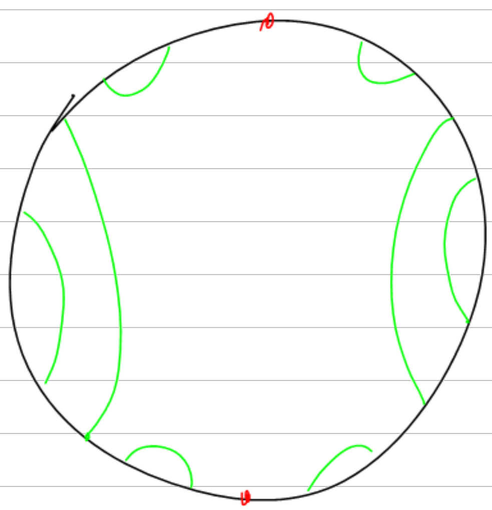


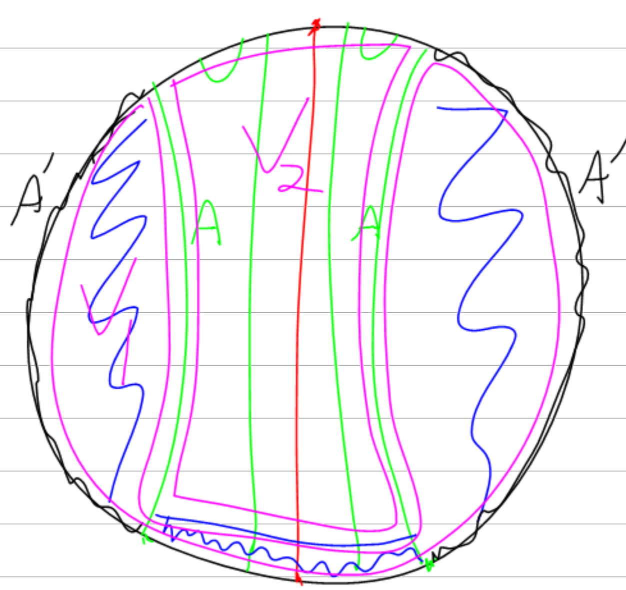
I dunno.



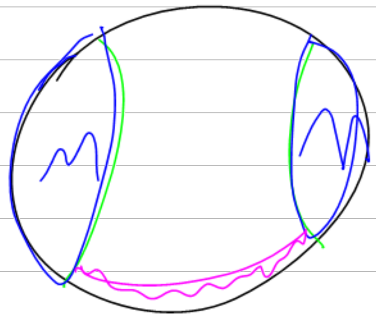
B:



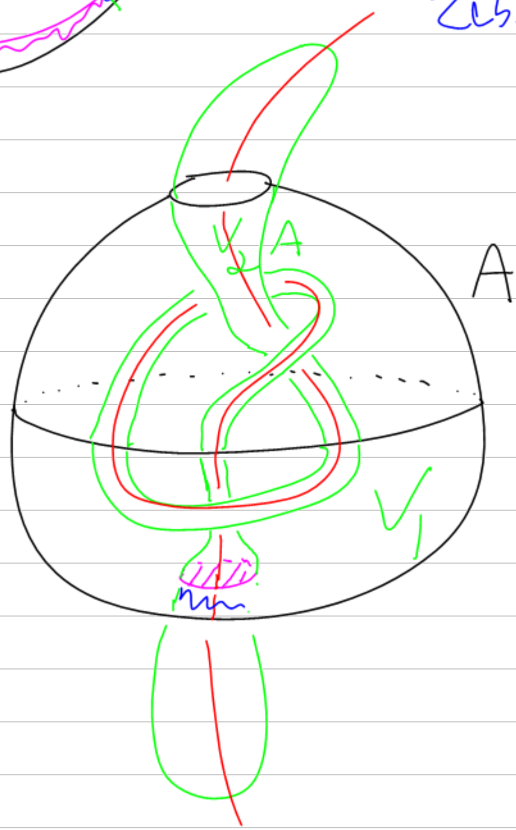




One of the V_i contains p replace



B by that V_1, and we're down one zebra stripe.



See Lickorish p 19–21

□