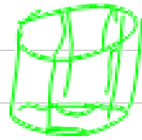
Hour 3, Monday Sep 14



$$\langle \times \rangle = A\langle)(\rangle + A^{-1}\langle \asymp \rangle \qquad \langle \bigcirc^{k}\rangle = (-A^2-A^{-2})^k$$

invt under R2 & R3 but $\langle \rho \rangle = -A^3\langle | \rangle$ $\langle \rho' \rangle = -A^{-3}\langle | \rangle$

$$W(D) = \sum_{\substack{x\ \text{xing} \\ \text{in } D}} \operatorname{sign}(x) \qquad \operatorname{sign}(\times_+) = +1 \quad \operatorname{sign}(\times_-) = -1$$

invt under R2 & R3 but $w(\rho) = w(|)+1$ $w(\rho') = w(|)-1$.

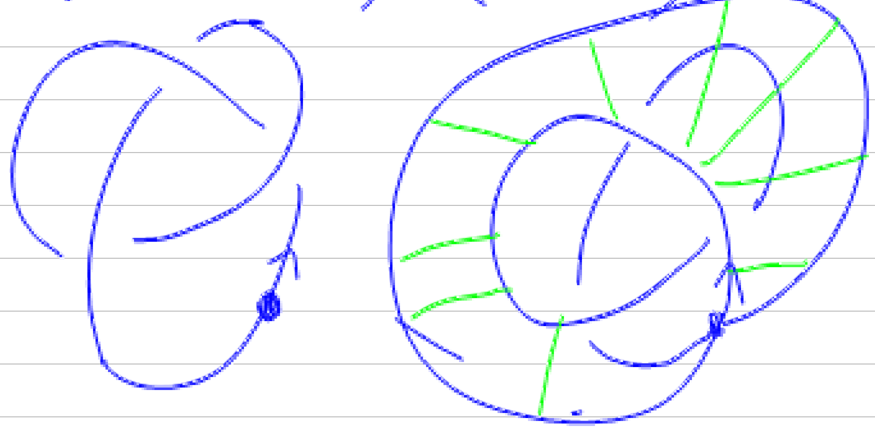
$$J(K) := (-A^3)^{-w(K)}\frac{\langle K\rangle}{d} \;/.\; A \to q^{-1/4}$$

The Jones skein relation:

$$J(\times_+) = -q^{3/4}\left(q^{-1/4}J()() + q^{1/4}J(\asymp)\right)$$

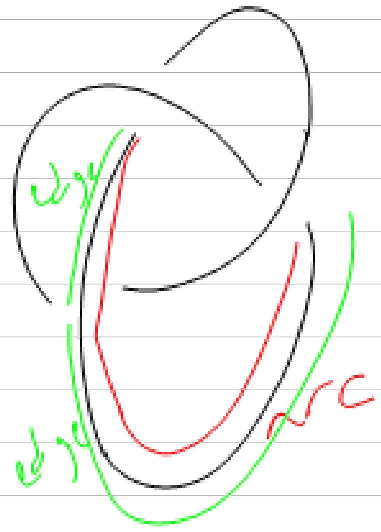
$$J(\times_-) = -q^{-3/4}\left(q^{-1/4}\langle \asymp \rangle + q^{1/4}\langle)(\rangle\right)$$

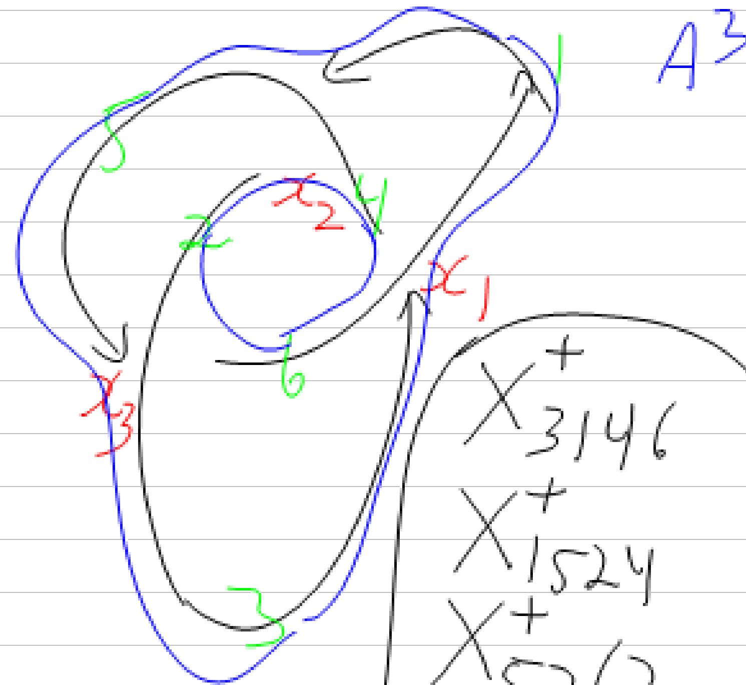
$$\Longrightarrow \quad q^{-1}J(\times_+) - q\,J(\times_-) = (q^{1/2}-q^{-1/2})\,J(\uparrow\uparrow)$$



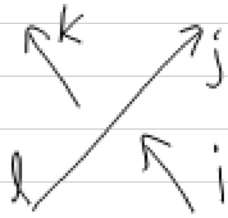
$$J(\bigcirc^{k}) = \left(-q^{1/2}-q^{-1/2}\right)^k$$

PD - notation



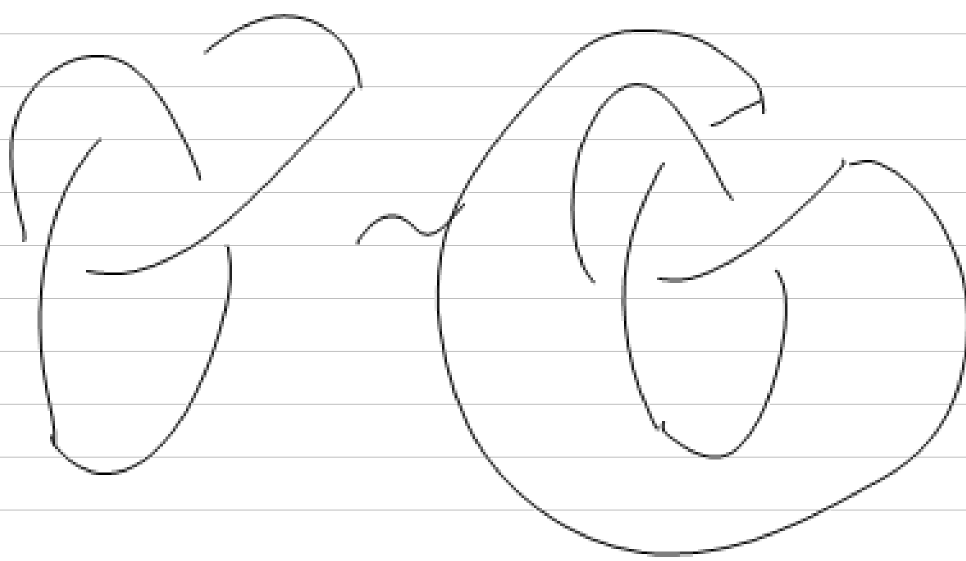


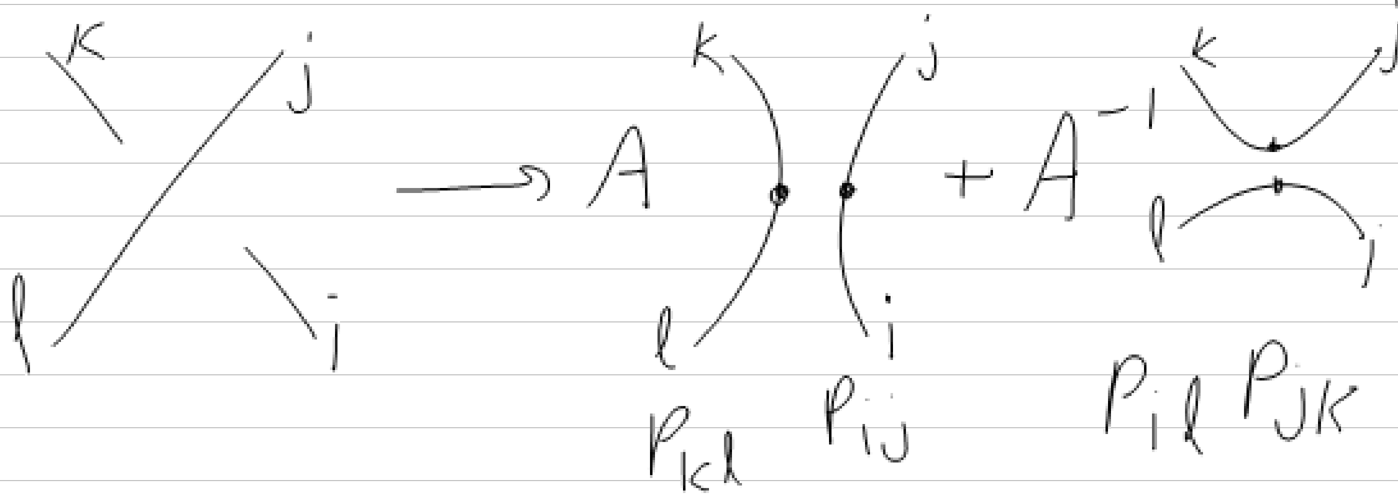
X^+_{3146}
X^+_{1524}
X^+_{5362}



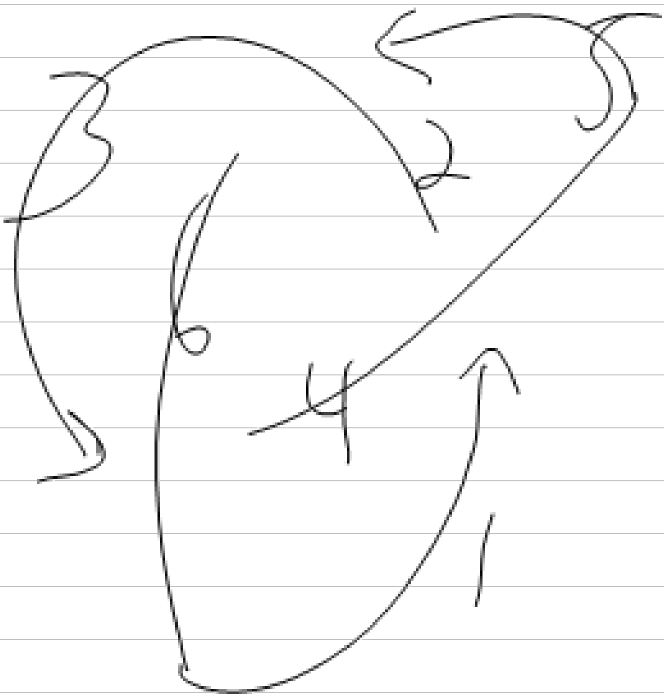
$\longrightarrow X^s_{ijkl}$

Exercise: This list of xing info determines D as a diagram on S^2




k j l i
A
k j l i
P_{kl} P_{ij}
+ A^{-1}
k j l i
P_{il} P_{JK}

Hour 4, Wednesday Sep 16
HW 1 on web by midnight!

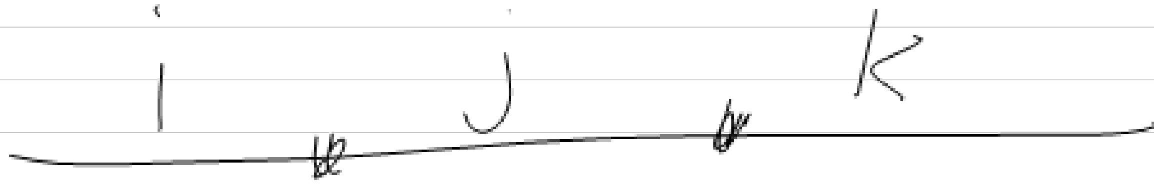


X_{1524}

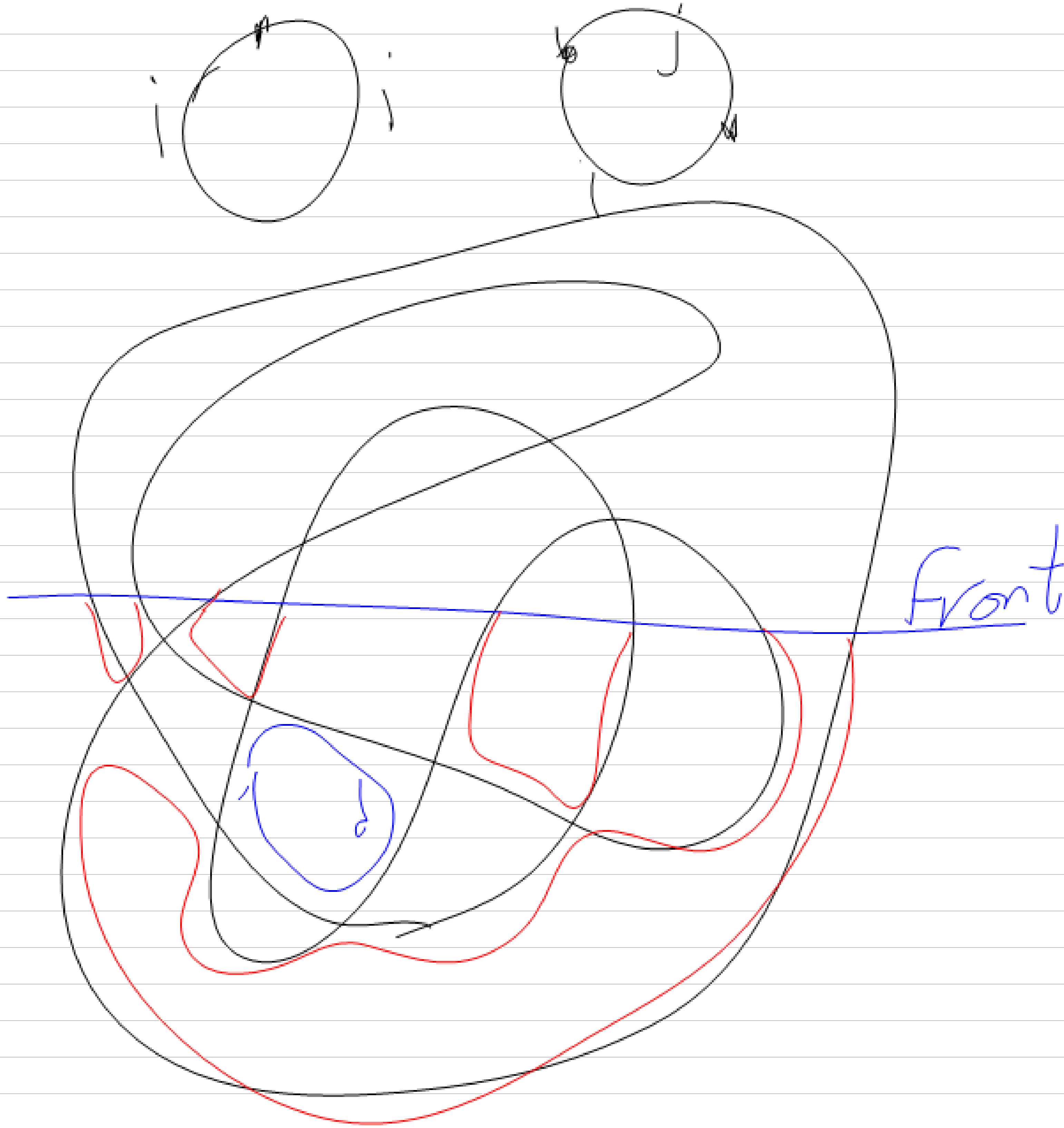
X_{5362}

n_K 3_1 4_1

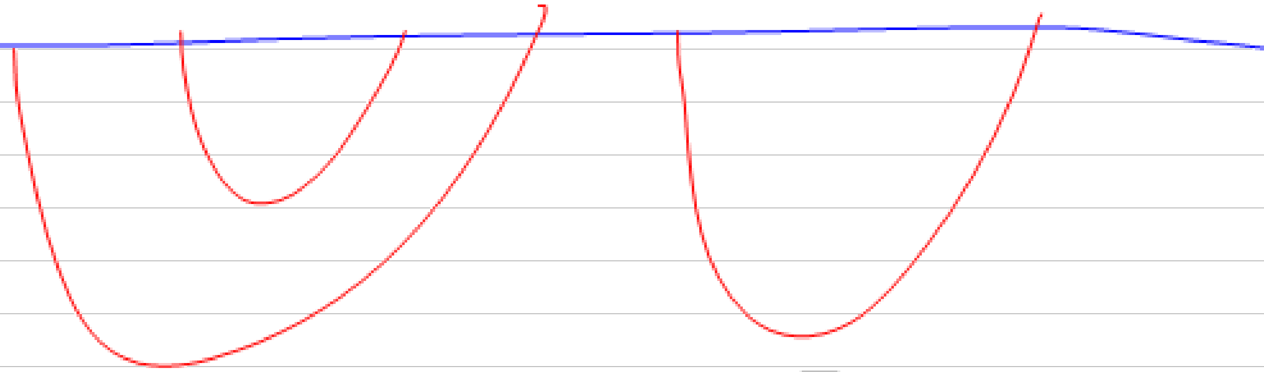
5_1 5_2

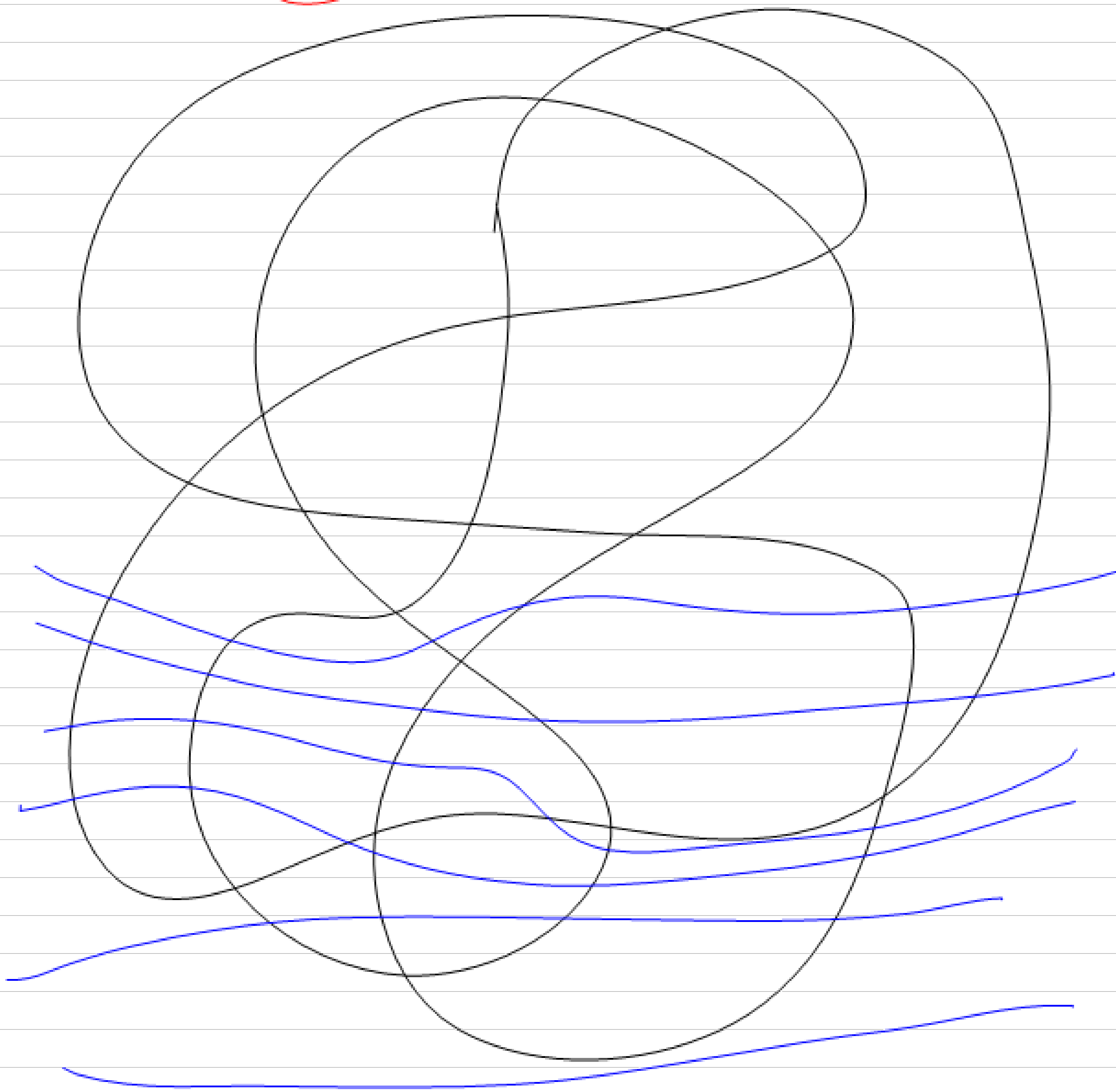


$P_{ij} P_{jK} \longrightarrow P_{iK}$

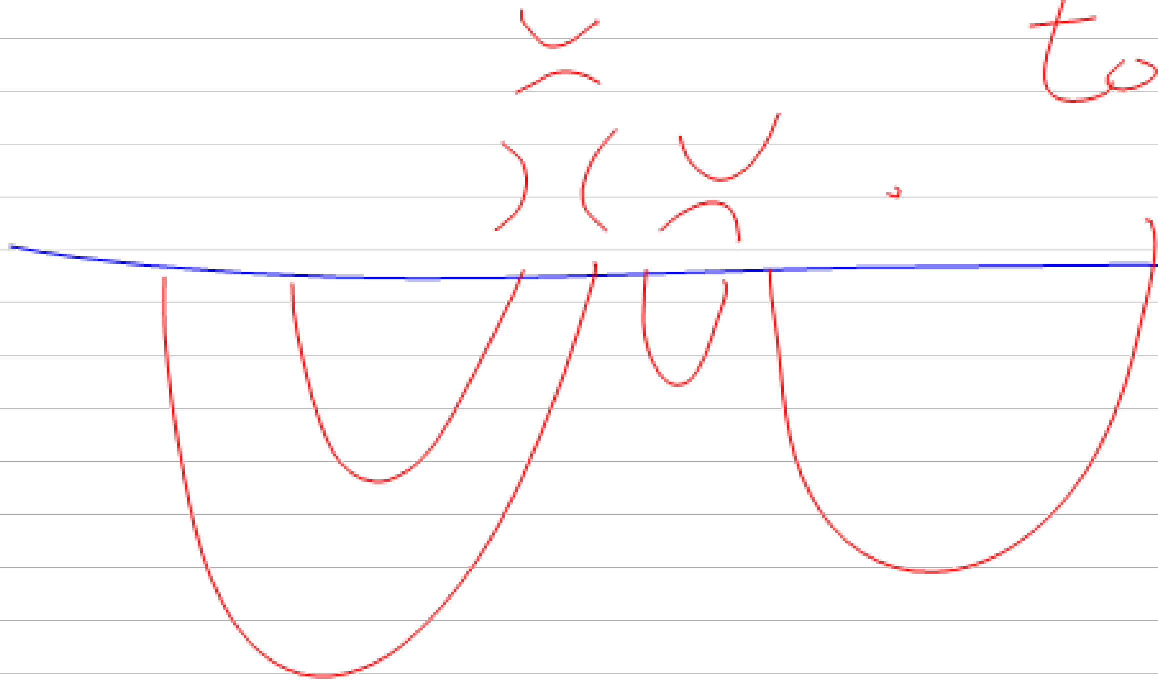

Front

$P(A)$

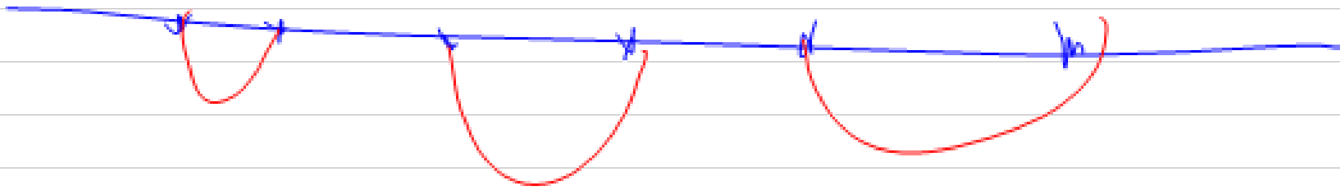


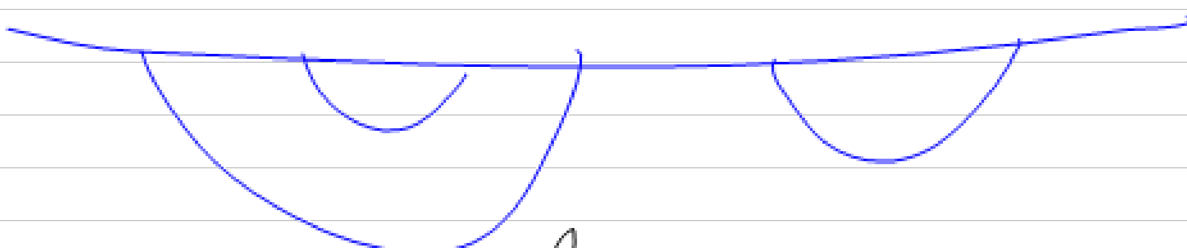


todo

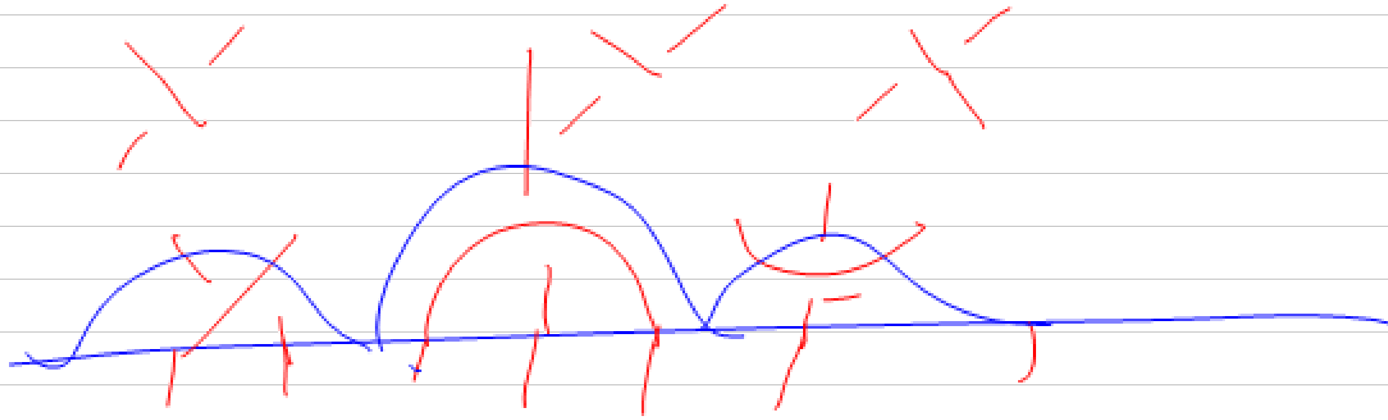


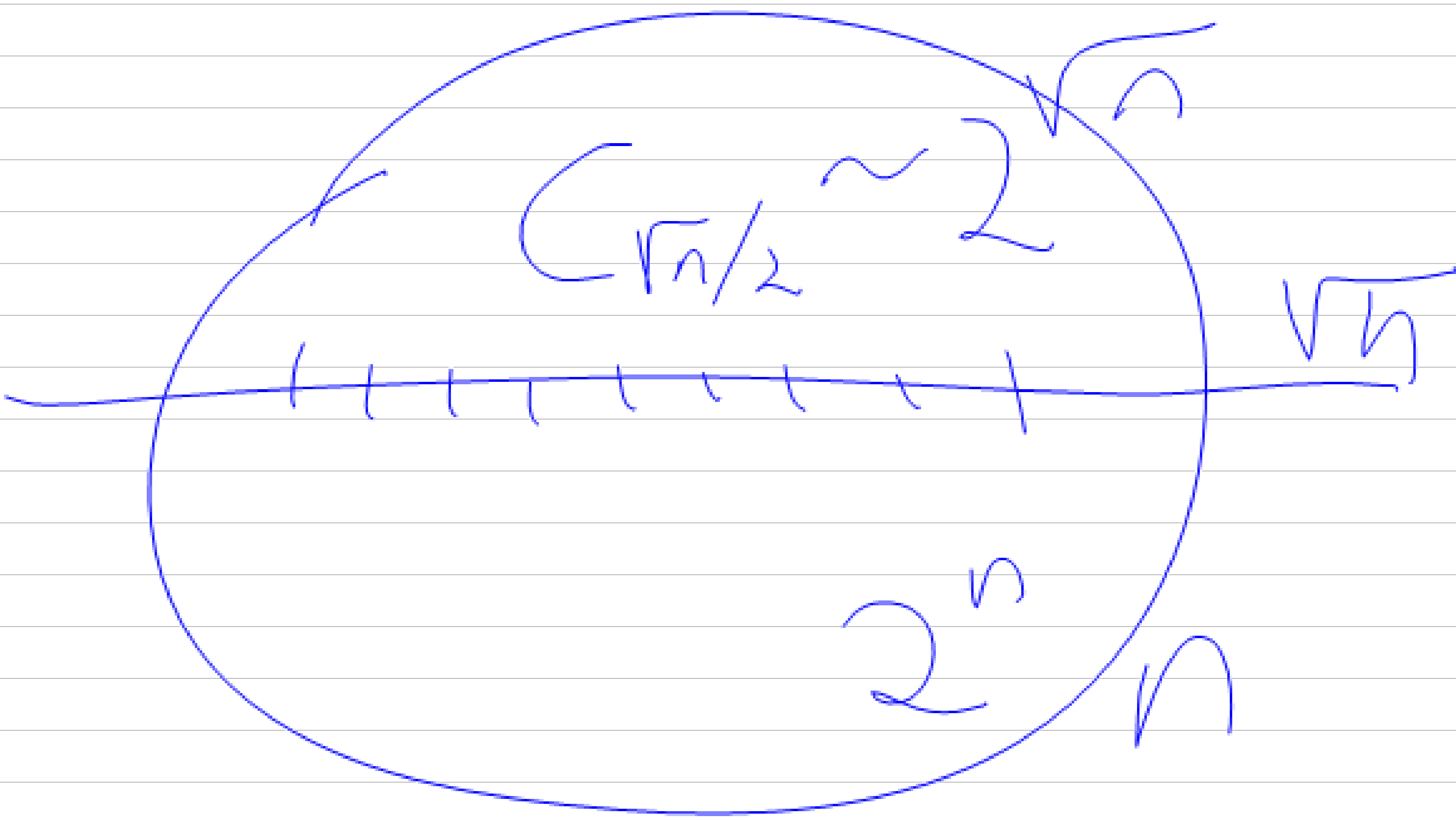
KB





Catalan $C_{n/2}$





$C_{\sqrt{n}/2} \sim 2^{\sqrt{n}}$
$\sqrt{n}$
2^n
n

Today:

Clarify Wednesday's EFKB
Talk about worms in apples
On beyond Zebra!

"The Kauffman bracket is a morphism from the planar algebra of tangles to the the Temperley-Lieb planar algebra."

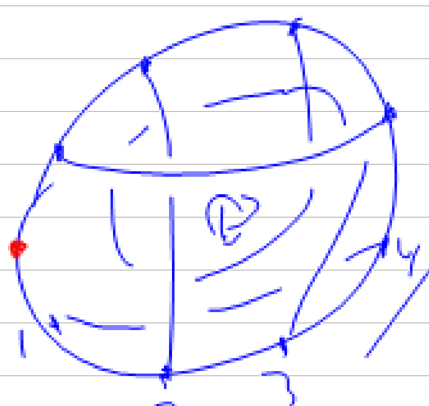
" $\mathrm{Rank}_{\mathbb{Z}[A^{\pm 1}]} TL_{2n} = C_n = \frac{1}{n+1}\binom{2n}{n} \sim \frac{4^n}{n^{3/2}\sqrt{\pi}}$

$2^{50} \ll 2^{100}$

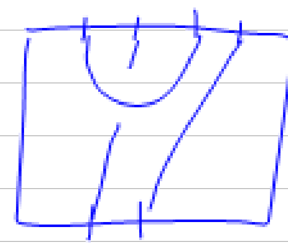
2^{50} +

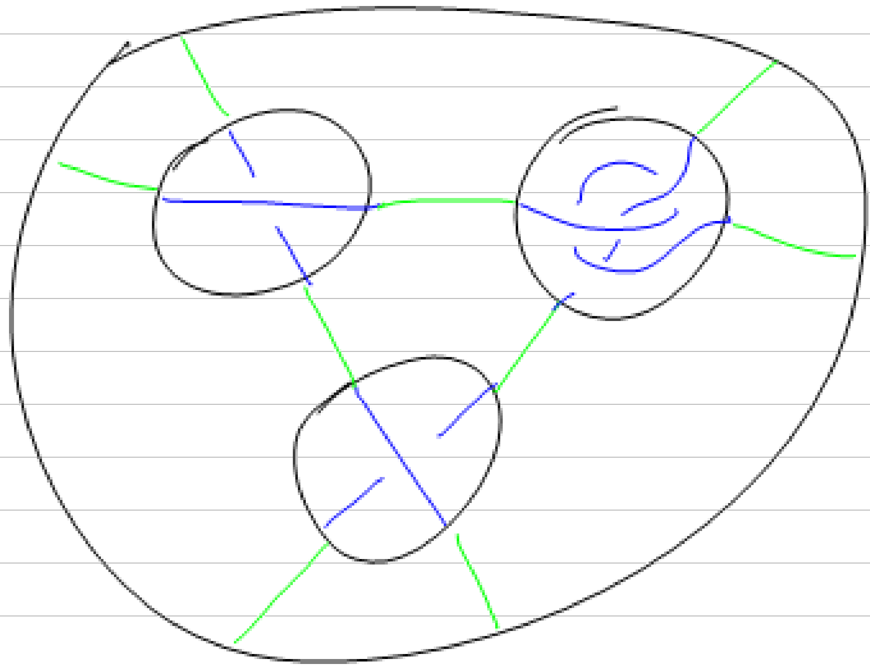
abcde . . .

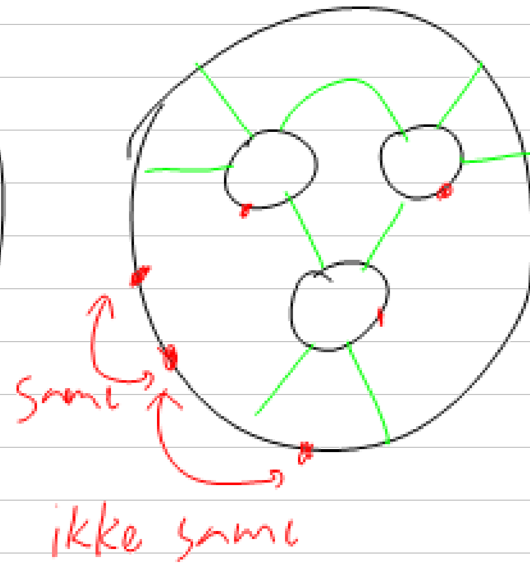
<u>Def</u> A Tangle is



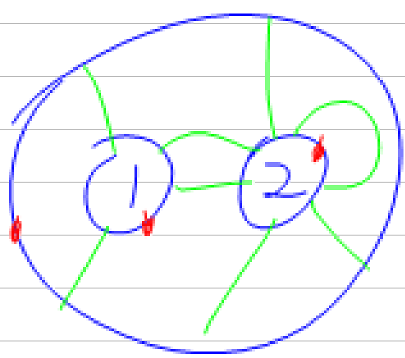
R1
R2 R3





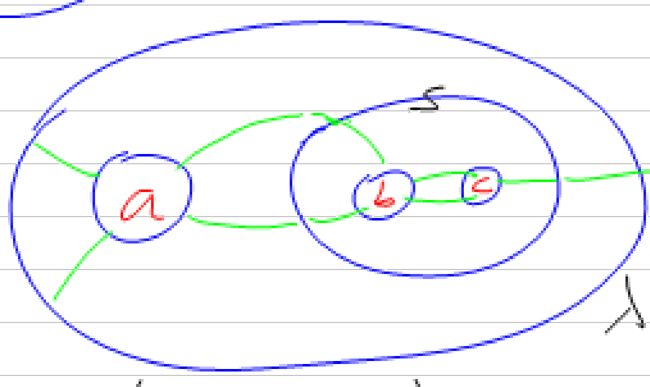


Def A planar algebra is a collection $\{P_n\}$ of sets of objects + a collection of operations labeled by "planar connection diagrams"



$: P_4 \times P_7 \longrightarrow P_5$

s.t.



Remove S:

$\varphi = \overset{a}{P_4} \times \overset{b}{P_4} \times \overset{c}{P_3} \longrightarrow P_3$

Remove outside of S

$\lambda = P_4 \times P_3 \longrightarrow P_3$

remove inside of S

$\mu = P_4 \times P_3 \longrightarrow P_3$

$$\varphi(a,b,c) = \mu(a,\lambda(b,c))$$

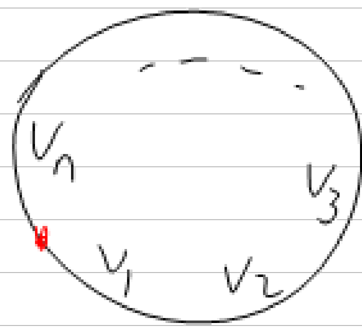
Example 1 Tangles. P_n = tangles w/ n inputs.

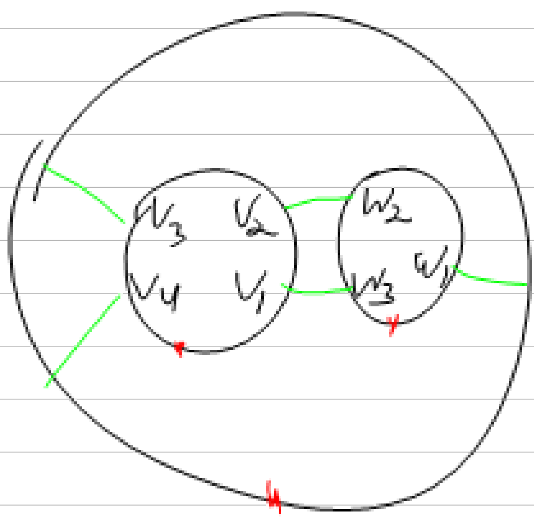
$P_{2n+1} = \emptyset$.

Ex 2 V be a f.d. V.S. w/ inner prod.

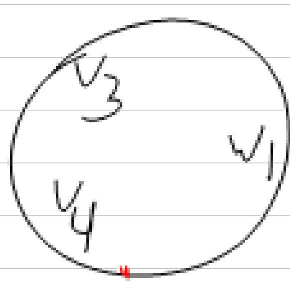
$$P_n = V^{\otimes n} = \underbrace{V \otimes V \otimes \cdots \otimes V}_{n \text{ times}}$$

$$= \langle v_1 \otimes v_2 \otimes v_3 \cdots \otimes v_n : v_i \in V \rangle$$

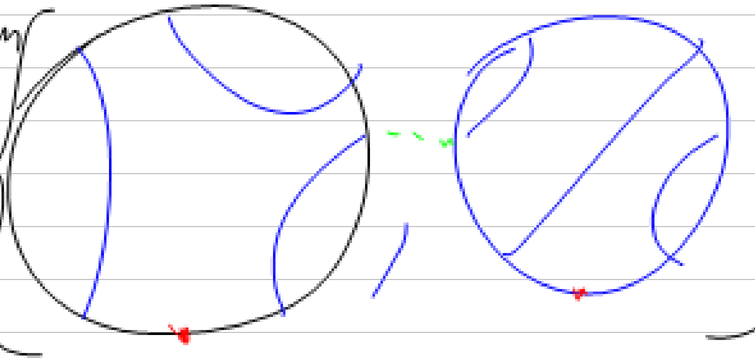




$= \langle v_2, w_2 \rangle \langle v_1, w_3 \rangle$



Example Temperley-Lieb planar alg.

$P_6 = \mathrm{Span}\Big\{$  $\Big\} \Big/ \bigcirc = d$