18-327 on Tuesday September 25, hour 9: Closed sets, T2 Spaces

September-11-10 12:29 PM

Today: closed sets, Hausdorff spaces.

Read Along: Munkres 17-18, preread 19-20.

HW2 due Thursday.

F closed $\Leftrightarrow$ $F^c = X \setminus F$ is open.

Prop. 1. $\emptyset, X$ are closed

2. Arbitrary $\cap$ of closed is closed.

3. Finite $\cup$ of closed is closed.

Prop $f: X \to Y \Leftrightarrow f^{-1}(\text{every closed set})$ is closed

"Closed" could have been used as the foundation just as well

Example: C_n: A union of closed intervals.

$C_0 = I = [0,1]$; C_n = From every interval in C_0, remove the open middle third.

C = "Cantor" $= \bigcap C_n$

Exercises 1. C has "length" 0.

2. C is uncountable, and has <u>many</u> clopen subsets.

3. $\exists f: [0,1] \to [0,1]$, cont., $f(0)=0$, $f(1)=1$, $f'(x) = 0$ for all $x \notin C$. [Picture?]

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closed in $Y \subset X$; closed in closed is closed.

interior: $\text{int}_X A = \mathring{A}$ = union of all open set contained in A = maximal open set contained in A.

closure $\text{Cl}_X A = \overline{A}$ = intersection of all closed sets containing A = minimal closed set containing A.

Claim A is open $\Leftrightarrow A = \mathring{A}$

A is closed $\Leftrightarrow A = \overline{A}$.

Exercise $(\mathring{A})^c = \overline{A^c}$

Proposition. $x \in \bar{A}$ iff every (basic) nbd of x intersects A. no proof / one line

Definition Limit pt: $x \in A' \Leftrightarrow x \in \overline{A - \{x\}}$, iff every nbd of x contains a point of A other than x itself.

Thm $\bar{A} = A \cup A'$

Hausdorff spaces. Definition.

Properties. In a T_2 space X:

1. Points are closed.
2. $x \in A'$ iff every nbd contains infinitely many points of x.
3. A sequence converges to at most one limit.
4. Products of T_2 & subspaces of T_2 are T_2.