18-327 on Tuesday September 11, hour 3: Topology, Continuity

Remember blackboard shots!

September-11-10 12:29 PM

Today's reading: Munkres: All introductions, Chapter 1 Sections 1-8, Chapter 2 section 12-13

Theorem $f:\mathbb{R}\to\mathbb{R}$ is continuous iff for every open set $U\subset\mathbb{R}$, $f^{-1}(U)$ is also open.

Properties of open sets:

1. $\emptyset, \mathbb{R}$ 2. $\bigcup$ 3 Finite $\bigcap$ } Prove!

start line

Definition 1. A topological space

2. Continuous function $f: X\to Y$.

Theorem The composition of continuous functions is continuous.

Examples 0. The std topology on $\mathbb{R}$.

1,2. The discrete and trivial topologies, continuous functions $f: X_{discrete, trivial}\to \mathbb{R}_{std}$

$f: \mathbb{R}_{std}\to X_{discrete, trivial}$.

Future warning: $f:\mathbb{R}_{std}\to\mathbb{R}_{disc}$ requires connectedness!

done line

3. The finite-complement topology $\mathcal{T}_3$.

Def Homeomorphism, homeomorphic

Examples. 1. $(-\frac{\pi}{2},\frac{\pi}{2})$ is homeomorphic to $\mathbb{R}$.

2. $(-1,1)$ is homeomorphic to $(-\frac{\pi}{2},\frac{\pi}{2})$

3. $\mathbb{R}$ is homeomorphic to $(-1,1)$.

Definition $\mathcal{T}_1\supset\mathcal{T}_2$ is "$\mathcal{T}_1$ is finer / bigger / stronger than $\mathcal{T}_2$" while "$\mathcal{T}_2$ is coarser / smaller / weaker" than $\mathcal{T}_1$.

weaker

The identity is continuous iff it goes from the finer topology to the weaker one.

Claim $Id:(X,\tau_1)\to(X,\tau_2)$ is a homeo iff $\tau_1=\tau_2$.

Def A "basis for a topology on a set X" is a collection $\mathcal{B}\subset P(X)$ s.t.

not same as linalg!

1. $\forall x\in X\ \exists B\in\mathcal{B}$ s.t. $x\in B$
2. $\forall B_{1,2}\in\mathcal{B}\ \forall x\in B_1\cap B_2\ \exists B_3\in\mathcal{B}$ s.t. $x\in B_3\subset B_1\cap B_2$

Examples 1. $\{(a,b)\}$ 2. $\{[a,b)\}$

Thm Given a basis for a topology on X,

1. There exists a unique minimal topology $\tau_{\mathcal{B}}$ containing $\mathcal{B}$.
2. $U\in\tau_{\mathcal{B}} \Longleftrightarrow \forall x\in U\ \exists B\in\mathcal{B}$ s.t. $x\in B\subset U$
3. $\tau_{\mathcal{B}}$ is the collection of all unions of elements of $\mathcal{B}$.

2010 hour 3 done line

HW (informal): For any τ_1, τ_2 considered today, find what it means to be "a continuous function $f:(\mathbb{R},\tau_1)\to(\mathbb{R},\tau_2)$".