18-327 on Tuesday October 9, hour 15: Metrics, boxes, and cylinders

September-11-10 12:29 PM

Today: (Above)

TT info on web! HW sol'ns wanted! Note new cover page!

Read Along: M 20-21; preread 22-24.

Thm In a metric space, given $A\subset X$, $\mathrm{cl}\,A = \text{seq-cl}(A)$, namely, $x\in\bar{A} \iff \exists\, x_n\in A$ s.t. $x_n\to x$.

[Any news about our poor prisoners?]

Note. $x_n\to x \iff d(x_n,x)\to 0$.

* Thm $\mathbb{R}^{\mathbb{N}}_{box}$ is not metrizable. (No seq of positive seqs goes to $\bar{0}$)

* Thm $\mathbb{R}^{\mathbb{R}}_{cyl}$ is not metrizable

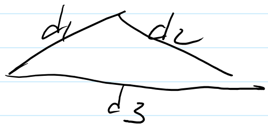
$A=\{f:\mathbb{R}\to\mathbb{R};\ f=0 \text{ except in finitely many places}\}$ then $\bar{1}\in \mathrm{cl}\,A$, $\bar{1}\notin \text{seq cl}\,A$.

* A countable product of metrizable spaces is metrizable.

done line

* $\bar{d}(x,y)=\min(1,d(x,y))$

$\bar{d}$ is a metric!



if $d_1+d_2\le 1$ easy

if $d_1+d_2\ge 1$, also easy

$\bar{d}$ defines the same topology. [Two bases $\mathcal{B},\mathcal{B}'$ define the same topology iff . . . .]

Given (X_n,d_n) w/ d_n bndd by 1.

Define

$$d(x,y)=\sup_n \frac{1}{n}d_n(x_n,y_n)$$

} Aside: The Uniform topology.

* This is a metric! $[\sup(a_n)+\sup(b_n)\ge\sup(a_n+b_n)]$

* It defines the product topology!