18-327 on Tuesday October 23, hour 21: path-connectedness, compactness

September-11-10 12:29 PM

Today: (Above)

TT: Need sol'n! [Also skim through web page]

Read Along: 23-24, 26-27.

Connected: $X \neq \emptyset$ & $\nexists\, X = A \dot\cup B$, A, B non-empty clopen.

path-connected: $X \neq \emptyset$ & $\forall a, b \in X\ \exists$ cont. $\gamma: [0,1] \to X$ s.t. $\gamma(0) = a$, $\gamma(1) = b$.

Thm path-connected $\Rightarrow$ connected.

Example/apology: $\mathbb{R}^{\mathbb{N}}_{unif}$ is not connected, as $B = \{\text{bndd seqs}\}$ is clopen.

Yet $L_0 = \{(x_n) : x_n \to 0\}$ is not clopen

Aside. B is path connected!

The topologist's sine curve.

Riddle: Find connected subsets A & B of $[0,1]^2$ s.t. $(0,0), (1,1) \in A$ & $(0,1), (1,0) \in B$.

Hard: Can't do it with path connected!

A product of path connected is path connected.

Def. Cover, open cover, Compact.

done line

Thm. A continuous function $\overset{\text{to } \mathbb{R}}{\text{on}}$ a compact set is bounded.

PF1 Local to global.

PF2 Sneaky - $X = \bigcup_{n=1}^{\infty} f^{-1}(-n, n)$.

Example. A finite set is compact.

Thm. $[0,1]$ is compact.

Proof. Let $\mathcal{U}$ be an open cover of $[0,1]$.

Let $G = \left\{ g \in [0,1] : \begin{array}{l} \text{a finite subset} \\ \text{of } \mathcal{U} \text{ covers } [0,g] \end{array} \right\}$ Wish: $1 \in G$.

G is non-empty and bounded, so $g_0 = \sup(G)$ exists.

step 1. $g_0 > 0$. step 2. $g_0 = 1$. step 3. $1 \in G$.