18-327 on Tuesday October 2, hour 12: Infinite Products

September-11-10 12:29 PM

Today: Boxes and cylinders

Read Along: Munkres 19 preread 20-22.

HW3 due Thursday.

See "Solution Sets" in website/About.html

A topology on $X=\prod_{\alpha\in I} X_\alpha = \{(x_\alpha) : x_\alpha \in X_\alpha\}$

$= \{x : I \to \bigcup X_\alpha : x(\alpha)\in X_\alpha\}$

Generalize the construction

$\mathcal{B} = \{\prod U_\alpha : U_\alpha \subset X_\alpha \text{ is open}\}$

"The box topology"

1. $\pi_\alpha : X \to X_\alpha$ cont.

2. $f: Z \to X$ s.t. $f_\alpha = \pi_\alpha \circ f$ is cont $\Rightarrow f$ is cont.

hard and worth reviewing

$\mathcal{B} = \{\pi_{\alpha_1}^{-1}(U_{\alpha_1}) \cap \dots \cap \pi_{\alpha_n}^{-1}(U_{\alpha_n})\}$

$= \{\prod U_\alpha : U_\alpha \subset X_\alpha \text{ is open For all but finitely many } \alpha\text{'s}, U_\alpha = X_\alpha\}$

"The cylinders topology"

For finite I, these two are the same.

Example $\mathbb{R} \mapsto \mathbb{R}^{\mathbb{N}}$ by $t \mapsto (t, t, \dots)$ is continuous in cyl but not in box [so box is strictly finer than cyl]

In both box & cyl:

1. The topology on $\prod A_\alpha \subset \prod X_\alpha$ as subspace is the same as ... as a product of subspaces.

only stated

done line

2. If X_α is T_2 $\forall\alpha$, then $\prod X_\alpha$ is T_2.

3. $\overline{\prod A_\alpha} = \prod \overline{A_\alpha}$

Metrics & the metric topology.

* General defs.

* Thm In a metric/metrizable space, closure = seq. closure.
* Thm $\mathbb{R}^{\mathbb{N}}_{box}$ is not metrizable. (No seq of positive seqs goes to $\bar{0}$)

* Thm $\mathbb{R}^{\mathbb{R}}_{cyl}$ is not metrizable
* A countable product of metrizable spaces is metrizable.
    * $\bar{d}(x,y) = \min(1, d(x,y))$