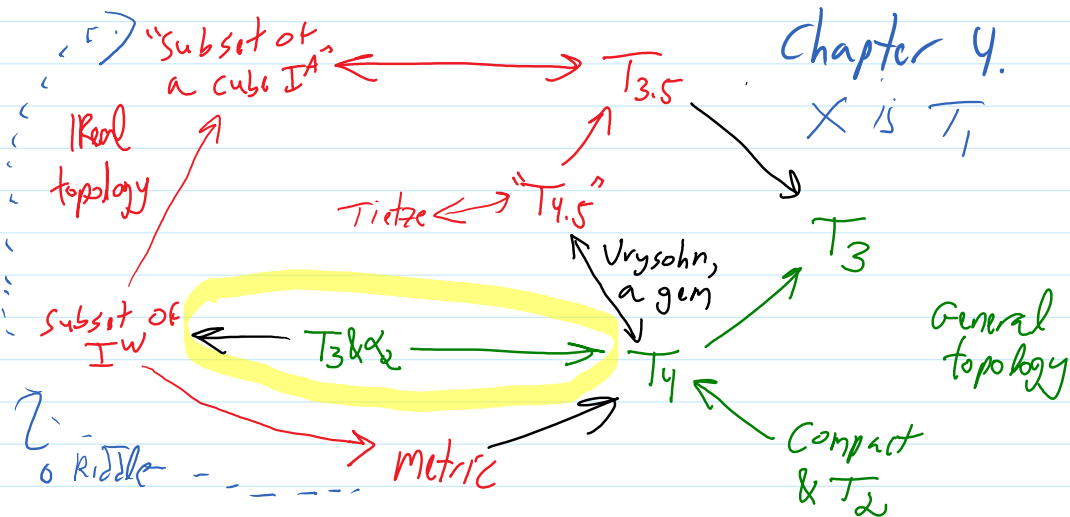


18-327 on Tuesday November 27, hour 33: Urysohn metrization

September-11-10 12:29 PM



save future generations! Do the course well!

Read Along: M30-35.

To do in 4 hours:

1. Urysohn metrization (today).
2. TFAE for compact in metric (5 parts, several new defs) } remaining 3 classes.
3. Baire Category & nowhere diffable functions [imply tutorial 😞]
4. Basic algebraic geometric topology: never had a chance.

α_2 : space has a countable basis.

NTS: 1. $\alpha_2 + T_3 \Rightarrow T_4$

2. $\alpha_2 + T_3 \Rightarrow \exists$ countable $\{F_n: X \rightarrow I\}$
s.t. $\{[F_n > 0]\}$ is a basis

A fairly unfriending story was told, but I should have made it bipartite.

$v_1, v_2, v_3, v_4, \dots$
 $v_1, v_2, v_3, v_4, \dots$

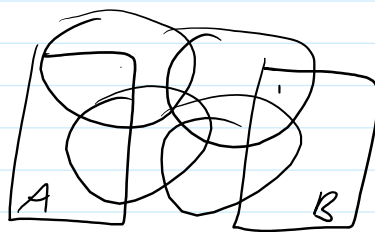
Thm $T_3 + \alpha_2 \Rightarrow T_4$ PF Given $A, B,$

Find a countable basic cover U_k of A , w/ $U_i \cap U_j = \emptyset$

Find a countable basic cover V_k of B w/ $\overline{V_i} \cap A = \emptyset$

$$\text{Set } U'_k = U_k \setminus \bigcup_{j=1}^k \overline{V_j} \quad V'_k = V_k \setminus \bigcup_{j=1}^k \overline{U_j}$$

$$U = \bigcup U'_k \quad V = \bigcup V'_k$$



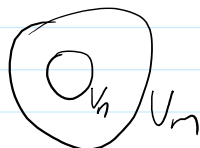
a word about the FE: You'll have to memorize all proofs, and the only way to do that is by understanding them.

Thm $T_3 + \alpha_2 \Rightarrow \exists$ a countable adequate set.

PF Let $\{U_n\}$ be a basis. If $U_n \subset U_m$, find $F_{n,m}$ w/

$$F_{n,m} \upharpoonright_{U_n} = 1, \quad F_{n,m} \upharpoonright_{U_m^c} = 0. \quad \text{Then } \{F_{n,m} > 0\} \text{ is}$$

a basis.



Corollary: The Urysohn metrization thm:

$$T_3 + \alpha_2 \Rightarrow \text{metrizable.}$$

Review the sketch of Urysohn metrization, on the map.

Start "compactness in metric spaces"!

done like

Compactness in Metric Spaces [Munkres 28, 43, 45]

Theorem. The following are equivalent for a Metric X :

1. X is compact.
2. X is "limit-point-compact".
3. X is "sequentially compact".
4. X is "totally bounded" & "satisfies Lebesgue's Lemma".
5. X is totally bounded & "complete".

Quoted phrases need to be defined!