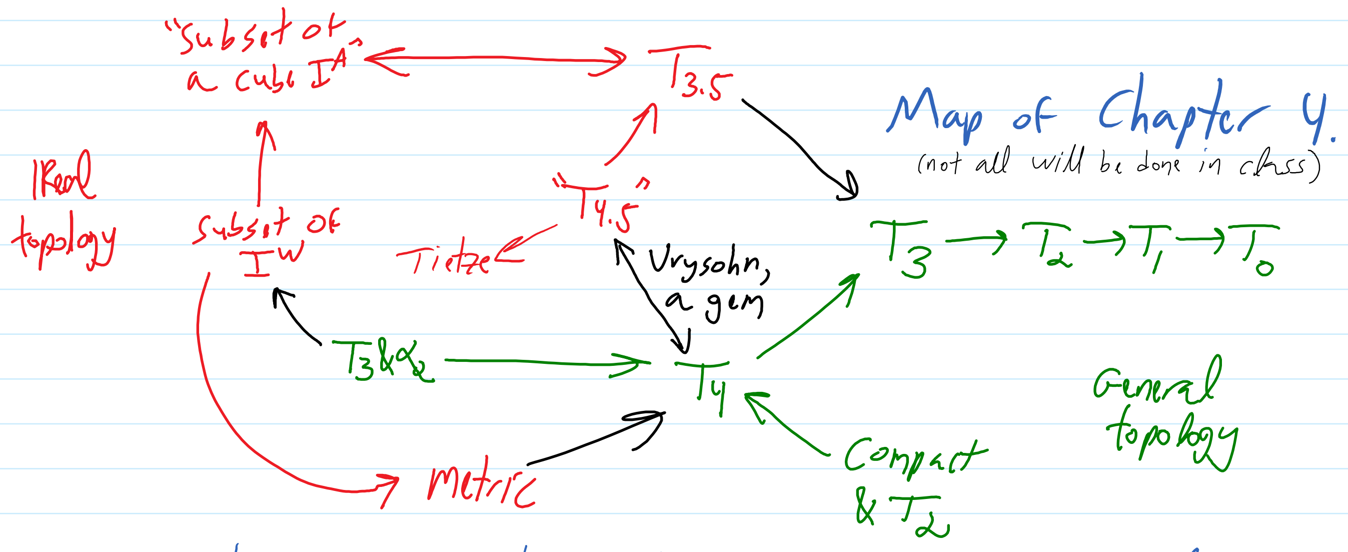
# 18-327 on Tuesday November 20, hour 30: Tietze's Theorem

September-11-10 12:29 PM




Save future generations! Do the course well!

Read Along: M35

Reminder: Normal; Urysohn.

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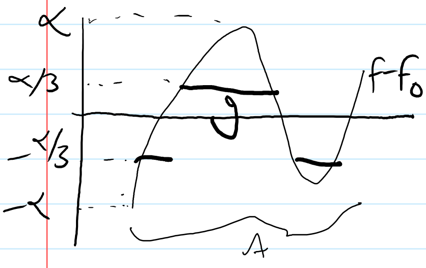
**Tietze's Theorem.** If $X$ is $T_4$, $A \subset X$ closed, and $f: A \to \mathbb{R}$ is cont., then there is a cont. extension $\tilde{f}$ of $f$ to all of $X$. (So $\tilde{f}|_A = f$)

**Remark.** Tietze $\Rightarrow$ Urysohn.

**Pf. of Tietze.** Assume first that $f$ is bdd by $M$.




$f_1 := f_0 + g$

Lemma Suppose $f_0: X \to \mathbb{R}$ is such that $|f - f_0| < \alpha$ on $A$; then $\exists\, f_1: X \to \mathbb{R}$ s.t. $|f_1 - f_0| \leq \frac{\alpha}{3}$ everywhere & $|f - f_1| \leq \frac{2\alpha}{3}$ on $A$.

I should have used $g$ & $g_0$ here.

So construct $f_0 = 0, f_1, f_2 \ldots$ s.t.

$$|f - f_n| < \left(\tfrac{2}{3}\right)^n M \text{ on } A$$

$$|f_n - f_{n+1}| < \tfrac{1}{3}\left(\tfrac{2}{3}\right)^n M \text{ on } X$$

$|f_n - f_{n+1}| < \frac{1}{3}\left(\frac{2}{3}\right)^n M$ on $X$ )

Let $\tilde{f}(x) = \lim_{n\to} f_n(x)$ (limit exists as $f_n(x)$ is cauchy)

Lemma A uniformly convergent sequence of cont. functions converges to a cont. limit.

$\Leftrightarrow \mathbb{R}^X_{\text{unif}} \supset C = \{\text{cont. fncts}\}$ is seq. closed.

Proof ....

Now do the unbounded case.....