18-327 on Thursday September 6, hours 1-2: Course Introduction

September-11-10 12:29 PM

Third year Fundamental No mercy

Remember blackboard shots!

MAT 327 Introduction to Topology

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Today's reading: Munkres: All introductions, Chapter 1 Sections 1-8, Chapter 2 Section 12-13

* Discuss Three handouts: 1. index 2. about 3. first tutorial

Definition A function $f:\mathbb{R}^n\to\mathbb{R}^m$ is "continuous" if

$\forall x_0\in\mathbb{R}\ \forall\epsilon>0\ \exists\delta>0\ \forall x\in\mathbb{R}\ |x-x_0|<\delta \Rightarrow |f(x)-f(x_0)|<\epsilon$

Theorem $f:\mathbb{R}\to\mathbb{R}$ is continuous iff for every open set $U\subset\mathbb{R}$, $f^{-1}(U)$ is also open.

* Define $B_\epsilon(x_0)$ and open sets give some examples.
* A few words on "f^{-1}"
* Proof. $\Rightarrow$ Assume $f:\mathbb{R}^n\to\mathbb{R}^m$ is cont., $U\subset\mathbb{R}^m$ is open, $x_0\in f^{-1}(U)$. Then $f(x_0)\in U$ so pick $\epsilon>0$ s.t. $B_\epsilon(f(x_0))\subset U$ and δ s.t. $|x-x_0|<\delta \Rightarrow |f(x)-f(x_0)|<\epsilon$; This really means $f(B_\delta(x_0))\subset B_\epsilon(f(x_0))$ implying $B_\delta(x_0)\subset f^{-1}(f(B_\delta(x_0)))\subset f^{-1}(B_\epsilon(f(x_0)))\subset f^{-1}(U)$, So we found a ball about x_0 contained in $f^{-1}(U)$, so $f^{-1}(U)$ is open.

$\Leftarrow$ Assume that for every open $U\subset\mathbb{R}^m$, $f^{-1}(U)\subset\mathbb{R}^n$ is open, and let $x_0\in\mathbb{R}^n$ & $\epsilon>0$ be given.

Then $B_\epsilon(f(x_0))$ is open, so $f^{-1}(B_\epsilon(f(x_0)))$ is open but $x_0 \in f^{-1}(B_\epsilon(f(x_0)))$ so $\exists\, \delta > 0$ s.t. $B_\delta(x_0) \subset f^{-1}(B_\epsilon(f(x_0)))$, which means that

$$|y - x_0| < \delta \Rightarrow |f(y) - f(x_0)| < \epsilon$$

and we have proven the continuity of f at x_0.

$\square$

Properties of open sets:

1. $\emptyset, \mathbb{R}$ 2. $\bigcup$ 3 Finite $\bigcap$ } stated, not proven

2010 done line for hour 1

Definition 1. A topological space

done the

2. Continous function $f : X \to Y$. } hinted.

Theorem The composition of continuous functions is continuous.

Examples The discrete and trivial topologies, continuous functions $f : X_{\text{discrete, trivial}} \to \mathbb{R}$

$f : \mathbb{R} \to X_{\text{discrete, trivial}}$

Def Homeomorphism, homeomorphic

Examples. 1. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is homeomorphic to $\mathbb{R}$.

2. $(-1, 1)$ is homeomorphic to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

3. $\mathbb{R}$ is homeomorphic to $(-1, 1)$.

Go over "index" & "About"

Definition $\mathcal{T}_1 \supset \mathcal{T}_2$ is "$\mathcal{T}_1$ is finer / bigger / stronger than $\mathcal{T}_2$" while "$\mathcal{T}_2$ is coarser / smaller / weaker" than $\mathcal{T}_1$.

The identity is continuous iff it goes from the finer topology to the weaker one.

<u>Claim</u> $Id:(X,\mathcal{T}_1)\to(X,\mathcal{T}_2)$ is a homeo iff $\mathcal{T}_1=\mathcal{T}_2$.

<u>Def</u> A "basis for a topology on a set X" is a collection $\mathcal{B}\subset P(X)$ s.t.

not same as linalg!

1. $\forall x\in X\ \exists B\in\mathcal{B}$ s.t. $x\in B$
2. $\forall B_{1,2}\in\mathcal{B}\ \forall x\in B_1\cap B_2\ \exists B_3\in\mathcal{B}$ s.t. $x\in B_3\subset B_1\cap B_2$

Examples 1. $\{(a,b)\}$ 2. $\{[a,b)\}$

<u>Thm</u> Given a basis for a topology on X,

1. There exists a unique minimal topology $\mathcal{T}_{\mathcal{B}}$ containing $\mathcal{B}$.
2. $U\in\mathcal{T}_{\mathcal{B}}\iff\forall x\in U\ \exists B\in\mathcal{B}$ s.t. $x\in B\subset U$
3. $\mathcal{T}_{\mathcal{B}}$ is the collection of all unions of elements of $\mathcal{B}$.

2010 hour 3 done line