18-327 on Thursday October 11, hours 16-17: Metrizability of countable products, quotients, connectedness

September-11-10 12:29 PM

Today: (Above)

TT info on web! HW sol'ns wanted! Note new cover page!

HW3/4 returned/due. Send back excellent sol'ns!

Read/preread: $\emptyset$, but study!

Thm 1. $\mathbb{R}^{\mathbb{N}}_{box}$ is not metrizable.

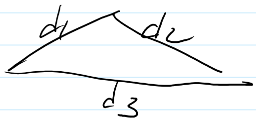
Thm 2. $\mathbb{R}^{\mathbb{R}}_{cyl}$ —//—

[Any news about our poor prisoners?]

Thm 3. A countable product of metrizable spaces is too.

$*$ $\bar{d}(x,y) = \min(1, d(x,y))$

$\bar{d}$ is a metric!



if $d_1 + d_2 \leq 1$ easy
if $d_1 + d_2 \geq 1$, also easy

$\bar{d}$ defines the same topology. [Two bases $\mathcal{B}, \mathcal{B}'$ define the same topology iff ...]

Given (X_n, d_n) w/ d_n bnd'd by 1.

Define

$$d(x,y) = \sup_n \frac{1}{n} d_n(x_n, y_n)$$

} Aside: The Uniform topology.

$*$ This is a metric! [$\sup(a_n) + \sup(b_n) \geq \sup(a_n + b_n)$]

$*$ It defines the product topology!

The quotient topology. Given a surjection $\pi : X \to Y$ or a "quotient map" $\pi : X \to X/\sim$, where X is a topological space, $\exists!$ a topology τ_Y on Y s.t.

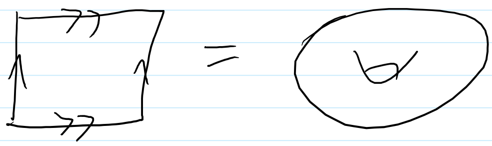
1. $\pi : X \to Y$ is cont.
2. Given $f : Y \to Z$, if $f \circ \pi$ is cont., then so is f

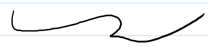
In fact, $\tau_Y = \{V \in Y : \pi^{-1}(V) \in \tau_X\}$

don. Ohe.

In fact, $\mathcal{T}_Y = \{V \in Y : \pi^{-1}(V) \in \mathcal{T}_X\}$. done. Ohe.

Examples 1. Good: $S^1 = [0,1]/_{0\sim 1} = \mathbb{R}/\mathbb{Z}$

2. 

3. Yet $M_{2\times 2}(\mathbb{C})/_{\text{conjugation}} =$  is not T_2!

Connectedness. Separation, connectedness, clopen sets.

The I.V.T. If X is connected, $f: X \to \mathbb{R}$ cont.,
$f(x_0) < 0$, $f(x_1) > 0 \Rightarrow \exists x$ s.t. $f(x) = 0$.

Theorem. A continuous image of a connected set is connected.

Theorem $I = [0,1]$ is connected.

Proof. Assume $0 \in A \subset I$ is clopen. Let $G = \{x : [0,x] \subset A\}$ $g = \sup G$
1. $g > 0$ 2. $g \not< 1$ 3. $1 \in G$.

Theorem. If $A_\alpha \subset X$ are connected, $\bigcap A_\alpha \neq \emptyset$, then $\bigcup A_\alpha$ is connected.

Theorem. $A \subset \mathbb{R}$ is connected iff it is an interval, or a ray, or the whole thing. [I.e., if it is "convex"]

Theorem. If A is connected & $A \subset B \subset \bar{A}$, B is too.

Pf Assume C is clopen in B, $C \cap A \neq \emptyset$. Then $C \supset A$ so $\mathrm{cl}_X C \supset \bar{A} \supset B$,
so $\mathrm{cl}_X C \cap B = B$, so $\mathrm{cl}_B C = B$, so $C = B$.

Theorem. If $\forall \alpha$ X_α is connected, then $\prod X_\alpha$ is connected.

Example. $\mathbb{R}^\omega = \left\{\begin{matrix}\text{bndd}\\ \text{seqs}\end{matrix}\right\} \cup \left\{\begin{matrix}\text{unbndd}\\ \text{seqs.}\end{matrix}\right\}$ is a box-separation.