Pensieve header: October 30: Some further further Hochschild Homology.

Today. Some further further Hochschild homology, (beware of Wolfram the populist!), then a Peano curve, then maybe EIWL 9-12, then, if we're kidding ourselves, Patterns.

Topics (in no particular order). Whatever you may suggest; whatever comes to my mind; ~~the Fibonacci numbers~~; the Catalan numbers; ~~the Jones polynomial~~; ~~a more efficient Jones algorithm~~; ~~a riddle on spheres~~; Khovanov homology; Γ-calculus; the Hopf fibration; Hilbert's 13th problem; non-commutative Gaussian elimination; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; an order 4 torus; the Schwarz Lantern; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; sound experiments; barycentric subdivisions; a Peano curve; braid closures and Vogel's algorithm; the insolubility of the quintic; phase portraits; the Mandelbrot set; shadows of the Cantor aerogel; quilt plots; some image transformations; De Bruijn graphs; the Riemann series theorem; finite type invariants and the Willerton fish; ~~the Towers of Hanoi~~; Hochschild homology of (some) coalgebras; convolutions and image improvements.

An Image Manipulation Challenge

The image at http://drorbn.net/bbs/show?shot=17-1750-171016-111042.jpg is pathetic. Can you improve it? Whatever you do, should also work well with all other images at http://drorbn.net/bbs/show.php?prefix=17-1750.

Hochschild Homology of Polynomial Algebras

First see the image at http://drorbn.net/AcademicPensieve/Classes/17-1750-ShamelessMathematica/index.html?im=171023-HomologyBBS.png.

```
d[n_,k_][ℰ_] := ℰ /. x_i_ :> Which[i < k, x_i, i == k, x_k + x_(k+1), i > k, x_(i+1)];
d[n_][ℰ_] := Expand@Sum[(-1)^k d[n,k][ℰ], {k, 0, n + 1}];
C[0,p_] := If[p == 0, {1}, {}];
C[n_,p_] := C[n,p] = Union @@ Table[x_n^k C[n-1,p-k], {k, 0, p}];
M[n_,p_] := Transpose[Table[
    da = d[n][a];
    Table[Coefficient[da, b], {b, C[n+1,p]}],
    {a, C[n,p]}
   ]];
ρ[0,_] = 0;
ρ[n_,p_] := ρ[n,p] = MatrixRank[M[n,p]];
β[n_,p_] := Length[C[n,p]] - ρ[n,p] - ρ[n-1,p];

Table[β[n,p], {n, 1, 5}, {p, 1, 5}] // MatrixForm // Timing
```

$$\{0.390625,\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}\}$$

Hochschild Homology of the Free Associative Algebra