

Pensieve header: October 13: A Faster Jones Program.

Today. A faster Jones, then whatever you may suggest, then EIWL 9-12, then, if time, Patterns.

Topics (in no particular order). Whatever you may suggest; whatever comes to my mind; ~~the Fibonacci numbers;~~ the Catalan numbers; ~~the Jones polynomial;~~ a more efficient Jones algorithm; ~~a riddle on spheres;~~ Khovanov homology; Γ -calculus; the Hopf fibration; Hilbert's 13th problem; non-commutative Gaussian elimination; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; an order 4 torus; the Schwarz Lantern; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; sound experiments; barycentric subdivisions; a Peano curve; braid closures and Vogel's algorithm; the insolubility of the quintic; phase portraits; the Mandelbrot set; shadows of the Cantor aerogel; quilt plots; some image transformations; De Bruijn graphs; the Riemann series theorem; finite type invariants and the Willerton fish; the Towers of Hanoi.

```
<< KnotTheory`
```

```
PD[Knot[3, 1]]
```

```
Jones[PD[Knot[3, 1]]][q]
```

```
AllKnots[{3, 10}] // Length
```

```
SetAttributes[P, Orderless];
```

```
JP[K_Times, opts___Rule] := Module[{verb, n, b1, b2, b3, b4, b5, w, J},
```

```
  verb = Verbose /. {opts} /. Verbose → False;
```

```
  n = Length[K];
```

```
  If[verb, Print["K has ", n, " crossings."]];
```

```
  b1 = K //. X[i_, j_, k_, L_] => AP[i, j] P[k, L] + BP[j, k] P[i, L];
```

```
  b2 = Expand[b1];
```

```
  b3 = b2 //. P[i_, j_] P[j_, k_] => P[i, k];
```

```
  b4 = b3 //. {P[i_, j_]² → d, P[i_, i_] → d};
```

```
  b5 = Expand[b4 //. {B → 1/A, d → -A² - 1/A²}];
```

```
  If[verb, Print["The Kauffman bracket is "]];
```

```
  If[verb, Print[b5]];
```

```
  w = K /. {Times → Plus, X[_ , 1, _ , 2 n] → 1,
```

```
    X[_ , 2 n, _ , 1] → -1, X[_ , j_, _ , L_] => If[j > L, 1, -1]};
```

```
  If[verb, Print["The writhe is "]];
```

```
  If[verb, Print[w]];
```

```
  If[verb, Print["The Jones Polynomial is "]];
```

```
  J = Expand@Cancel[
$$\frac{(-A^3)^{-w} b5}{-A^2 - 1/A^2} /. A \rightarrow q^{-1/4}$$

```

```
];
```

```
JP[K_PD, opts___] := JP[Times@@K, opts];
```

```
JP[K_Knot, opts___] := JP[PD@K, opts];
```

```
JP[Knot[3, 1], Verbose → True]
```

```
Timing[tab1 = Table[JP[K], {K, AllKnots[{3, 10}]}];]
```

```
tab2 = Table[Jones[K][q], {K, AllKnots[{3, 10}]}];
```

```
tab1 == tab2
```

```
Union[tab1] // Length
```

A 48-crossing knot

```
Import["http://drorbn.net/AcademicPensieve/2016-09/GST48-Marked.png"]
```

```
GST48 = PD[
  X[01, 15, 02, 14], X[29, 02, 30, 03],
  X[40, 04, 41, 03], X[04, 44, 05, 43], X[05, 26, 06, 27],
  X[95, 07, 96, 06], X[07, 01, 08, 96], X[08, 14, 09, 13],
  X[28, 09, 29, 10], X[41, 11, 42, 10],
  X[11, 43, 12, 42], X[12, 27, 13, 28], X[15, 31, 16, 30],
  X[61, 16, 62, 17], X[72, 17, 73, 18],
  X[83, 18, 84, 19], X[34, 20, 35, 19], X[20, 89, 21, 90],
  X[92, 21, 93, 22], X[22, 79, 23, 80],
  X[23, 68, 24, 69], X[24, 57, 25, 58], X[56, 25, 57, 26],
  X[31, 63, 32, 62], X[32, 74, 33, 73],
  X[33, 85, 34, 84], X[35, 50, 36, 51], X[81, 37, 82, 36],
  X[70, 38, 71, 37], X[59, 39, 60, 38],
  X[54, 39, 55, 40], X[55, 45, 56, 44], X[45, 59, 46, 58],
  X[46, 70, 47, 69], X[47, 81, 48, 80],
  X[91, 49, 92, 48], X[49, 91, 50, 90], X[82, 52, 83, 51],
  X[71, 53, 72, 52], X[60, 54, 61, 53],
  X[74, 63, 75, 64], X[85, 64, 86, 65], X[65, 76, 66, 77],
  X[66, 87, 67, 88], X[94, 67, 95, 68],
  X[86, 75, 87, 76], X[77, 88, 78, 89], X[93, 78, 94, 79] ];
```