

**Today.** EIWL 5-8, a riddle on spheres, more on Catalan, and maybe, Etienne's project.

**Topics** (in no particular order). Whatever you may suggest; whatever comes to my mind; ~~the Fibonacci numbers~~; the Catalan numbers; ~~the Jones polynomial~~; a more efficient Jones algorithm; a riddle on spheres; Khovanov homology;  $\Gamma$ -calculus; the Hopf fibration; Hilbert's 13th problem; non-commutative Gaussian elimination; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; an order 4 torus; the Schwarz Lantern; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; sound experiments; barycentric subdivisions; a Peano curve; braid closures and Vogel's algorithm; the insolubility of the quintic; phase portraits; the Mandelbrot set; shadows of the Cantor aerogel; quilt plots; some image transformations; De Bruijn graphs; the Riemann series theorem; finite type invariants and the Willerton fish.

Pensieve header: October 2: More on the Catalan numbers.

## The Catalan Numbers

**Project Idea.** Make an easily extendible,  $n$ -dependent "Catalan objects poster", as explained on the blackboard.

```
ts[n_Integer] := ts[Range[0, n + 1]];
ts[[_ , _]] = {ds[]};
ts[vs_List] := Module[{l, r, k, t1, t2, tds},
  Union @@ Table[
    l = ts[Prepend[vs[[k ;;]], vs[[1]]]];
    r = ts[vs[[2 ;; k]]];
    Flatten[Table[
      tds = Join[t1, t2];
      If[k > 3, AppendTo[tds, d[vs[[2]], vs[[k]]]];
      If[k < Length[vs], AppendTo[tds, d[vs[[1]], vs[[k]]]];
      tds,
      {t1, l}, {t2, r}
    ]],
    {k, 3, Length[vs]}
  ]
]

ts[4]
{ds[d[0, 4], d[0, 3], d[0, 2]], ds[d[0, 4], d[1, 3], d[0, 3]],
 ds[d[1, 3], d[1, 4], d[0, 4]], ds[d[1, 4], d[1, 3], d[1, 5]], ds[d[2, 4], d[0, 4], d[0, 2]],
 ds[d[2, 4], d[1, 4], d[0, 4]], ds[d[2, 4], d[1, 4], d[1, 5]], ds[d[2, 4], d[2, 5], d[0, 2]],
 ds[d[2, 4], d[2, 5], d[1, 5]], ds[d[3, 5], d[0, 3], d[0, 2]], ds[d[3, 5], d[1, 3], d[0, 3]],
 ds[d[3, 5], d[1, 3], d[1, 5]], ds[d[3, 5], d[2, 5], d[0, 2]], ds[d[3, 5], d[2, 5], d[1, 5]]}
```