Pensieve header: September 29: Textbook (EIWL) chapters 5-8, evaluated.

5. Operations on Lists

```
{1, 2, 3} + 10
```

```
{11, 12, 13}
```

```
{1, 2, 3}/10
```

$$\left\{\frac{1}{10}, \frac{1}{5}, \frac{3}{10}\right\}$$

```
? List
```

$\{e_1, e_2, \ldots\}$ is a list of elements. ≫

```
?? List
```

$\{e_1, e_2, \ldots\}$ is a list of elements. ≫

```
Attributes[List] = {Locked, Protected}
```

```
? Plus
```

$x + y + z$ represents a sum of terms. ≫

```
?? Plus
```

$x + y + z$ represents a sum of terms. ≫

```
Attributes[Plus] = {Flat, Listable, NumericFunction, OneIdentity, Orderless, Protected}

Default[Plus] := 0
```

```
? Flat
```

Flat is an attribute that can be assigned to a symbol f to indicate that all expressions involving nested functions f should be flattened out. This property is accounted for in pattern matching. ≫

```
f[1, f[2, 3]]
```

```
f[1, f[2, 3]]
```

```
SetAttributes[f, Flat]
```

```
f[1, f[2, 3]]
```

```
f[1, 2, 3]
```

```
? Listable
```

Listable is an attribute that can be assigned to a symbol *f* to indicate that the function *f* should automatically be threaded over lists that appear as its arguments. ≫

```
f[1, {2, 3, 4}]
```

```
f[1, {2, 3, 4}]
```

```
SetAttributes[f, Listable]
```

```
f[1, {2, 3, 4}]
```

```
{f[1, 2], f[1, 3], f[1, 4]}
```

```
{1, 1, 2} * {1, 2, 3}
```

```
{1, 2, 6}
```

```
f[{a, b}, {x, y}]
```

```
{f[a, x], f[b, y]}
```

```
Range[10] ^2
```

```
{1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
```

```
Sort[{4, 2, 1, 3, 6}]
```

```
{1, 2, 3, 4, 6}
```

```
? SortBy
```

SortBy[*list*, *f*] sorts the elements of *list* in the order defined by applying *f* to each of them.
SortBy[*f*] represents an operator form of SortBy that can be applied to an expression. ≫

```
? RandomReal
```

RandomReal[] gives a pseudorandom real number in the range 0 to 1.
RandomReal[{x_{min}, x_{max}}] gives a pseudorandom real number in the range x_{min} to x_{max}.
RandomReal[x_{max}] gives a pseudorandom real number in the range 0 to x_{max}.
RandomReal[*range*, *n*] gives a list of *n* pseudorandom reals.
RandomReal[*range*, {n_1, n_2, …}] gives an $n_1 \times n_2 \times \ldots$ array of pseudorandom reals. ≫

```
l = RandomReal[{-1, 1}, 10]
```

```
{0.924804, 0.230363, -0.703776, -0.623061,
 0.868089, 0.0104707, 0.285499, 0.666478, 0.280836, 0.426879}
```

```
Sort[l]
```

```
{-0.703776, -0.623061, 0.0104707, 0.230363,
 0.280836, 0.285499, 0.426879, 0.666478, 0.868089, 0.924804}
```

```
SortBy[l, Abs]
```

```
{0.0104707, 0.230363, 0.280836, 0.285499,
 0.426879, -0.623061, 0.666478, -0.703776, 0.868089, 0.924804}
```

```
Length[{5, 3, 4, 5, 3, 4, 5}]
```

```
7
```

```
Dimensions[{{1, 2, 3}, {4, 5, 6}}]
```

```
{2, 3}
```

```
Total[{1, 1, 2, 2}]
```

```
6
```

```
Total[Range[10]]
```

```
55
```

```
Count[{a, b, a, a, c, b, a}, a]
```

```
4
```

```
? Count
```

Count[*list*, *pattern*] gives the number of elements in *list* that match *pattern*.
Count[*expr*, *pattern*, *levelspec*] gives the total number of subexpressions
  matching *pattern* that appear at the levels in *expr* specified by *levelspec*.
Count[*pattern*] represents an operator form of Count that can be applied to an expression. ≫

```
? Pattern
```

s : *obj* represents the pattern object *obj*, assigned the name *s*. ≫

```
Count[{a, b, a, a, c, b, a}, _^_]
```

```
0
```

Expand$\left[(a+b)^{10}\right]$

$a^{10}+10\,a^{9}\,b+45\,a^{8}\,b^{2}+120\,a^{7}\,b^{3}+210\,a^{6}\,b^{4}+252\,a^{5}\,b^{5}+210\,a^{4}\,b^{6}+120\,a^{3}\,b^{7}+45\,a^{2}\,b^{8}+10\,a\,b^{9}+b^{10}$

List @@ Expand$\left[(a+b)^{10}\right]$

$\left\{a^{10},\ 10\,a^{9}\,b,\ 45\,a^{8}\,b^{2},\ 120\,a^{7}\,b^{3},\ 210\,a^{6}\,b^{4},\ 252\,a^{5}\,b^{5},\ 210\,a^{4}\,b^{6},\ 120\,a^{3}\,b^{7},\ 45\,a^{2}\,b^{8},\ 10\,a\,b^{9},\ b^{10}\right\}$

Count$\left[\text{List @@ Expand}\left[(a+b)^{10}\right],\ _\hat{}_\right]$

```
2
```

Count$\left[\text{List @@ Expand}\left[(a+b)^{10}\right],\ _ * (_\hat{}_)\right]$

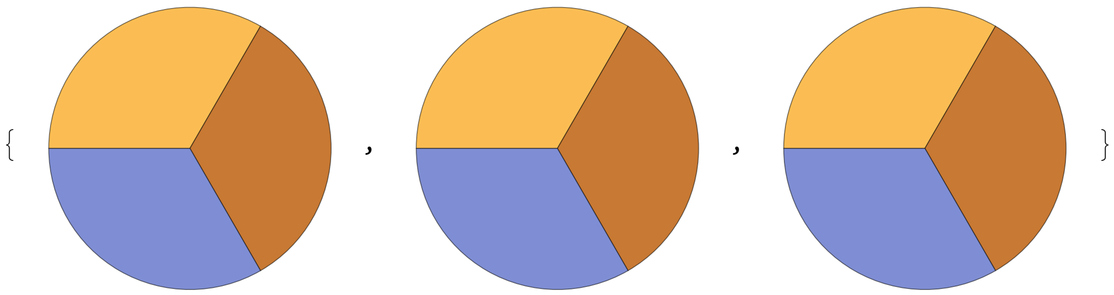
```
0
```

```
Count[List @@ Expand[(a + b)^10], __ * (_^_)]

9

g1[x_] := x^2

g1[2]

4

g1[2, 3]

g1[2, 3]

g2[x__] := {x}

g2[2]

{2}

g2[2, 3]

{2, 3}

g3[x___] := {x}

g3[]

{}

First[{7, 6, 5}]

7

Last[{7, 6, 5}]

5

Part[{7, 6, 5}, 2]

6

First[Sort[{6, 7, 1, 2, 4, 5}]]

1

Min[{6, 7, 1, 2, 4, 5}]

1

IntegerDigits[1988]

{1, 9, 8, 8}

Last[IntegerDigits[1988]]

8

Take[{101, 203, 401, 602, 332, 412}, 3]

{101, 203, 401}
```

```
Take[IntegerDigits[2^100], 10]
{1, 2, 6, 7, 6, 5, 0, 6, 0, 0}

Drop[{101, 203, 401, 602, 332, 412}, 3]
{602, 332, 412}
```

6. Making Tables

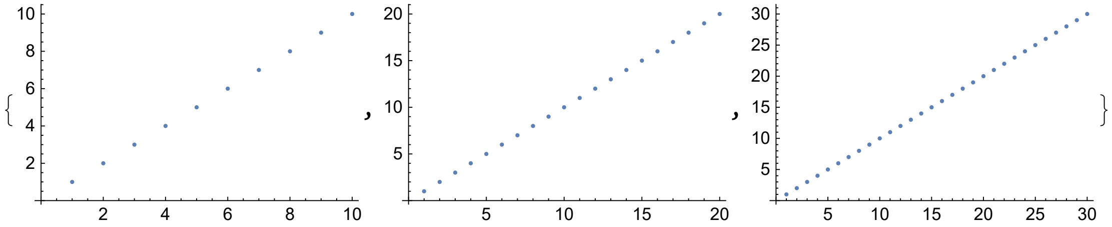
```
Table[5, 10]
{5, 5, 5, 5, 5, 5, 5, 5, 5, 5}

Table[x, 10]
{x, x, x, x, x, x, x, x, x, x}

Table[{1, 2}, 10]
{{1, 2}, {1, 2}, {1, 2}, {1, 2}, {1, 2}, {1, 2}, {1, 2}, {1, 2}, {1, 2}, {1, 2}}

Table[{1, 2}, 10] // Dimensions
{10, 2}

Table[{1, 2}, 10, 3] // Dimensions
{10, 3, 2}

Table[PieChart[{1, 1, 1}], 3]
```



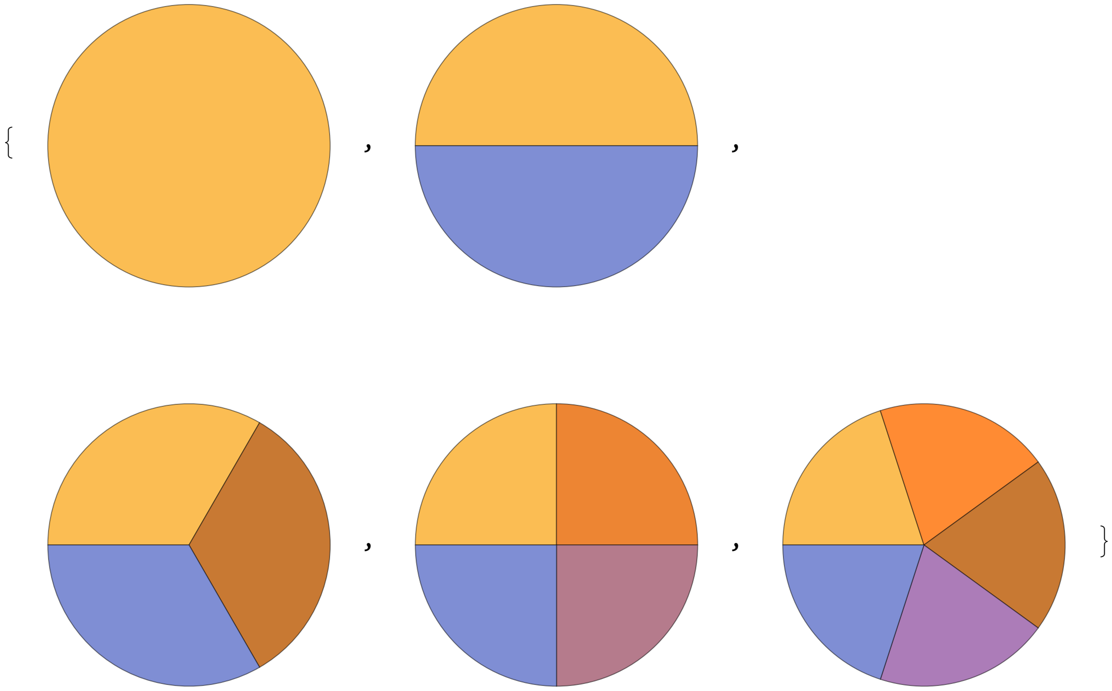
```
Table[a[n], {n, 5}]
{a[1], a[2], a[3], a[4], a[5]}

Table[n + 1, {n, 10}]
{2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

Table[n^2, {n, 10}]
{1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
```

```
tab = Table[n + Range[n]^2, {n, 5}]

{{2}, {3, 6}, {4, 7, 12}, {5, 8, 13, 20}, {6, 9, 14, 21, 30}}

tab〚5, 2〛

9

tab〚2, 5〛

Part: Part 5 of {3, 6} does not exist.

{{2}, {3, 6}, {4, 7, 12}, {5, 8, 13, 20}, {6, 9, 14, 21, 30}}〚2, 5〛

tab〚All, 1〛

{2, 3, 4, 5, 6}

tab〚3, All〛

{4, 7, 12}

tab〚3〛

{4, 7, 12}

tab〚All, 2〛

Part: Part 2 of {2} does not exist.

{{2}, {3, 6}, {4, 7, 12}, {5, 8, 13, 20}, {6, 9, 14, 21, 30}}〚All, 2〛

tab〚2 ;;, 2〛

{6, 7, 8, 9}

tab // Length

5

tab // Dimensions

{5}

Table[Column[Range[n]], {n, 8}]

                              1
                          1
                      1       2
                  1       2
              1       2       3
          1       2       3
      1       2       3       4
{ 1 ,   , 2 ,   , 3 ,   , 4 ,   }
      2       3       4       5
          3       4       5
              4       5       6
                  5       6
                      6       7
                          7
                              8
```

```
Table[ListPlot[Range[10 * n]], {n, 3}]
```



```
Table[PieChart[Table[1, n]], {n, 5}]
```



```
Table[2^expt, {expt, 10}]
```

```
{2, 4, 8, 16, 32, 64, 128, 256, 512, 1024}
```

```
Table[{x, x + 1, x^2}, {x, 5}]
```

```
{{1, 2, 1}, {2, 3, 4}, {3, 4, 9}, {4, 5, 16}, {5, 6, 25}}
```

```
Table[f[n], {n, 10}]
```

```
{f[1], f[2], f[3], f[4], f[5], f[6], f[7], f[8], f[9], f[10]}
```

```
Table[f[n], {n, 4, 10}]
```

```
{f[4], f[5], f[6], f[7], f[8], f[9], f[10]}
```

```
Table[f[n], {n, 4, 10, 2}]
```

```
{f[4], f[6], f[8], f[10]}
```

```
Range[4, 10]
```

```
Range[4, 10, 2]
```

```
f /@ Range[4, 10, 2]
```

```
Range[0, 1, 0.1]
```

```
ListPlot[Table[x - x^2, {x, 0, 1, .02}]]
```

```
ListPlot[Range[0, 1, .02] - Range[0, 1, .02]^2]
```

```
Table[RandomInteger[10], 20]
```

```
RandomInteger[10, 20]
```

7. Colors and Styles

```
Red
```

🟥

```
{Red, Green, Blue, Purple, Orange, Black}
```

{🟥, 🟩, 🟦, 🟪, 🟧, ⬛}

```
ColorNegate[Yellow]
```

🟦

```
Blend[{Yellow, Pink, Green}]
```

🟩

```
RGBColor[1, 0, 0]
```

🟥

```
RGBColor[1, 1, 0]
```

🟨

```
Table[RGBColor[1, g, 0], {g, 0, 1, 0.05}]
```

{🟥, 🟥, 🟥, 🟥, 🟥, 🟧, 🟧, 🟧, 🟧, 🟧, 🟧, 🟧, 🟧, 🟨, 🟨, 🟨, 🟨, 🟨, 🟨, 🟨, 🟨}

```
Hue[0.5]
```

🟦

```
Table[Hue[u], {u, 0, 1, 0.05}]
```

{🟥, 🟧, 🟧, 🟨, 🟨, 🟩, 🟩, 🟩, 🟩, 🟩, 🟦, 🟦, 🟦, 🟦, 🟪, 🟪, 🟪, 🟪, 🟥, 🟥, 🟥}

```
RandomColor[]
```

🟫

```
Table[RandomColor[], 30]
```

```
{■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■,
 ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■}
```

```
Blend[Table[RandomColor[], 20]]
```

```
Style[1000, Red]
```

```
1000
```

```
Table[Style[RandomInteger[1000], RandomColor[]], 30]
```

```
{666, 837, 806, 331, 268, 61, 903, 176, 564, 659, 640, 157, 156, 983,
 820, 2, 311, 568, 801, 719, 306, 264, 584, 343, 530, 637, 569, 815, 220, 148}
```

```
Style[x, 30]
```

```
x
```

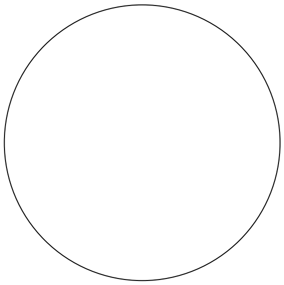
```
Table[Style[100, n], {n, 30}]
```

```
Table[Style[x, RandomColor[], RandomInteger[30]], 25]
```

```
{x, , , x, x, x, x, x, x, x, x, x, x, x, x, x, , x, x, x, x, x, x, x, x}
```

8. Basic Graphics Objects

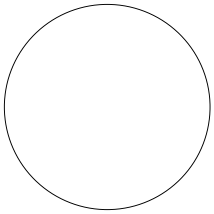
```
Graphics[Circle[]]
```



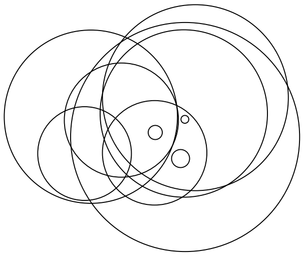
```
?Circle
```

Circle[{x, y}, r] represents a circle of radius r centered at {x, y}.
Circle[{x, y}] gives a circle of radius 1.
Circle[{x, y}, {r_x, r_y}] gives an axis-aligned ellipse with semiaxes lengths r_x and r_y.
Circle[{x, y}, …, {θ_1, θ_2}] gives a circular or ellipse arc from angle θ_1 to θ_2. ≫

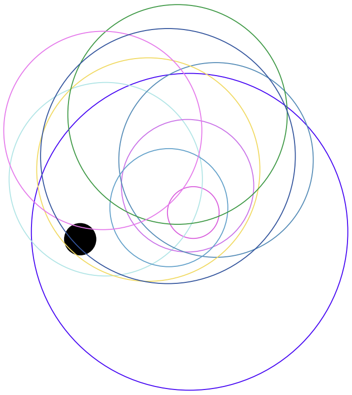
```
Graphics[Circle[{1, 2}, 4]]
```



```
Graphics[Table[Circle[{RandomReal[], RandomReal[]}, RandomReal[]], 10]]
```

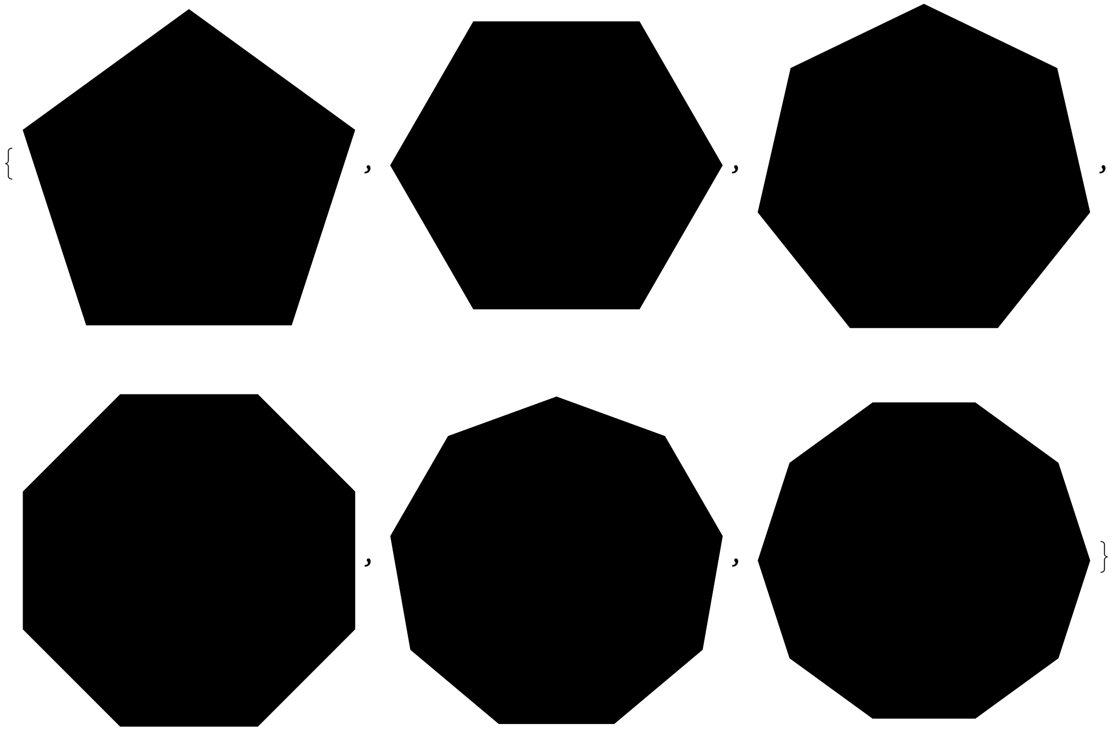


```
Graphics[{Disk[{0, 0}, 0.1],
  Table[
   {RandomColor[], Circle[{RandomReal[], RandomReal[]}, RandomReal[]]},
   10]
 }]
```



```
Graphics[Disk[]]

Graphics[RegularPolygon[5]]
```

```
Table[Graphics[RegularPolygon[n]], {n, 5, 10}]
```



```
? Line
```

```
Line[{p1, p2, ...}] represents the line segments joining a sequence for points pi.
Line[{{p11, p12, ...}, {p21, ...}, ...}] represents a collection of lines.  >>
```

```
? Arrow
```

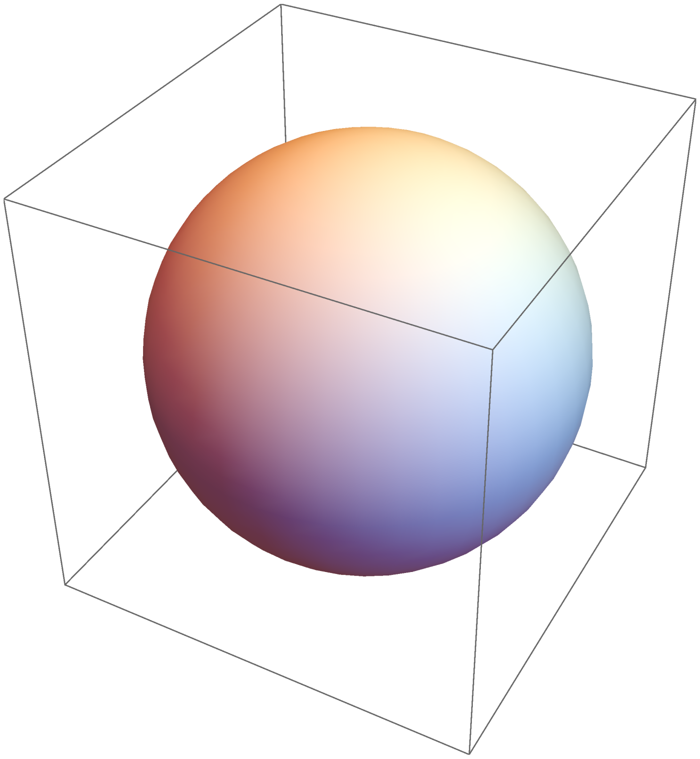
```
Arrow[{pt1, pt2}] is a graphics primitive that represents an arrow from pt1 to pt2.
Arrow[{pt1, pt2}, s] represents an arrow with its ends set back from pt1 and pt2 by a distance s.
Arrow[{pt1, pt2}, {s1, s2}] sets back by s1 from pt1 and s2 from pt2.
Arrow[curve, ...] represents an arrow following the specified curve.  >>
```

```
? Polygon
```

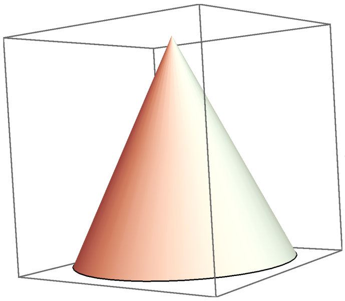
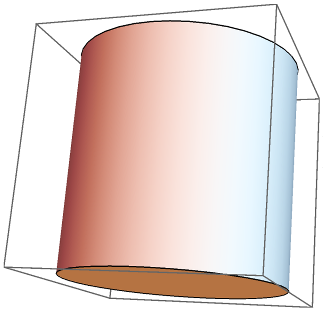
```
Polygon[{p1, ..., pn}] represents a filled polygon with points pi.
Polygon[{{p11, ...}, {p21, ...}, ...}] represents a collection of polygons.  >>
```

```
Graphics[{Orange, RegularPolygon[5]}]
```

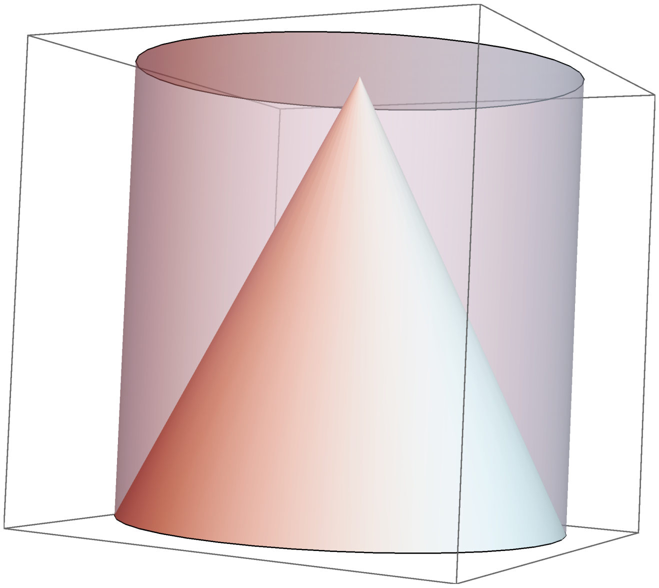
```
Graphics3D[Sphere[]]
```



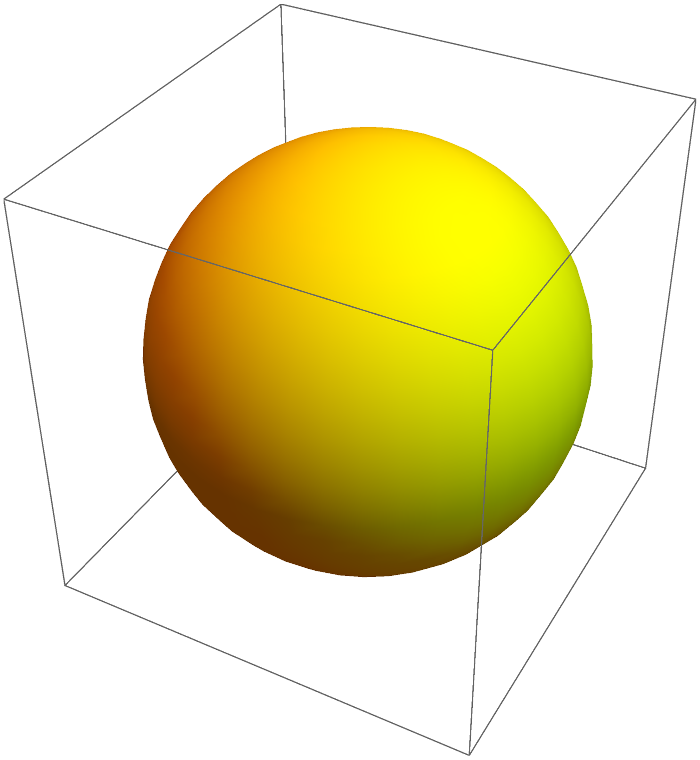
```
{Graphics3D[Cone[]], Graphics3D[Cylinder[]]}
```

{  ,  }

```
Graphics3D[{Cone[], Opacity[0.5], Cylinder[]}]
```



```
Graphics3D[{Yellow, Sphere[]}]
```



```
Graphics3D[Style[Sphere[], Yellow]]
```

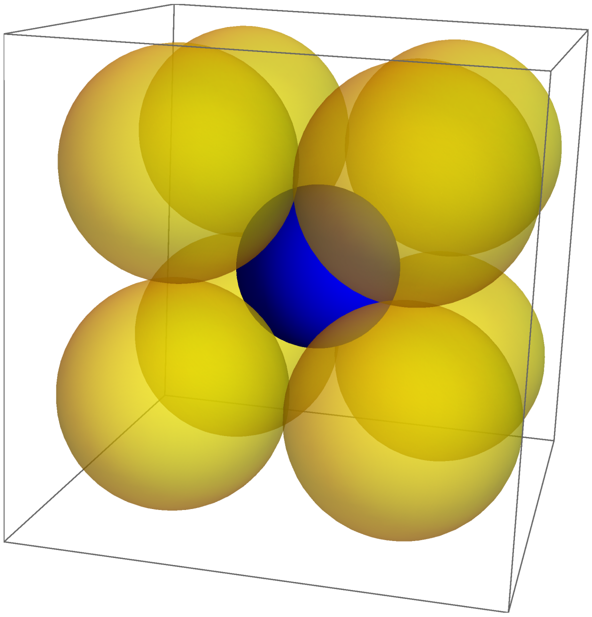
```
? Tuples
```

Tuples[*list*, *n*] generates a list of all possible n-tuples of elements from *list*.
Tuples[{$list_1$, $list_2$, …}] generates a
list of all possible tuples whose i^{th} element is from $list_i$. >>

```
Tuples[{1, -1}, 3]
```

```
{{1, 1, 1}, {1, 1, -1}, {1, -1, 1}, {1, -1, -1},
 {-1, 1, 1}, {-1, 1, -1}, {-1, -1, 1}, {-1, -1, -1}}
```

```
Graphics3D[{
  Yellow, Opacity[0.5], Table[Sphere[c, 1], {c, Tuples[{1, -1}, 3]}],
  Blue, Opacity[1], Sphere[{0, 0, 0}, Sqrt[3] - 1]
 }]
```



volume of an n dimensional ball

Result

$$\frac{2\,\pi^{n/2}\,r^n}{n\,\Gamma\left(\frac{n}{2}\right)} \approx \frac{2 \times 3.14159^{0.5\,n}\,r^n}{n\,\Gamma(0.5\,n)}$$

(assuming radius *r*)