

<< KnotTheory`

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.

Read more at <http://katlas.org/wiki/KnotTheory>.

```
CG[K_PD] := Module[{k, kk, A, AA, B, BB, a, aa, b, bb, n = Length[K]},
  k = K /. X[i_, j_, k_, L_] => If[TrueQ[Mod[i, 2] == 0], X[i, j, k, L], X[L, i, j, k]];
  A = k /. X[i_, j_, k_, L_] => P[i, j] P[k, L];
  B = k /. X[i_, j_, k_, L_] => P[L, i] P[j, k];
  a = (Times @@ A) /. P[i_, j_] P[j_, k_] => P[i, k];
  b = (Times @@ B) /. P[i_, j_] P[j_, k_] => P[i, k];

  kk = K /. X[i_, j_, k_, L_] => If[TrueQ[Mod[i, 2] == 0], X[L, i, j, k], X[i, j, k, L]];
  AA = kk /. X[i_, j_, k_, L_] => P[j, i] P[L, k];
  BB = kk /. X[i_, j_, k_, L_] => P[i, L] P[k, j];
  aa = (Times @@ AA) /. P[i_, j_] P[j_, k_] => P[i, k];
  bb = (Times @@ BB) /. P[i_, j_] P[j_, k_] => P[i, k];
  Cases[
    { $\frac{1}{2} (n - \text{Length}[a] - \text{Length}[b] + 2)$ ,  $\frac{1}{2} (n - \text{Length}[aa] - \text{Length}[bb] + 2)$ }, _Integer][[1]]
  ]
```

This now uses oriented paths.

Same program to convert Gauss codes.

```
GCToPD[K_] := Module[{j, l = Length@Characters[K] / 3},
  i[j_] :=
    (StringPosition[K, StringReplace["On", "n" -> TextString[j]]][[1]][[1]] - 1) / 3 + 1;
  p[j_] := If[StringContainsQ[K, StringReplace["On+", "n" -> TextString[j]]],
    i[j], n[j] + 1];
  n[j_] := If[StringContainsQ[K, StringReplace["On-", "n" -> TextString[j]]],
    i[j], p[j] + 1];
  PD @@ Table[X[(t - 1) / 3 + 1, Mod[n[Characters[K]][[t + 1]], l, 1], Mod[(t - 1) / 3 + 2, l, 1],
    Mod[p[Characters[K]][[t + 1]], l, 1]], {t, Union@Flatten@StringPosition[K, "U"]}]
  ]
```

Program that computes the genus in general.

```
genus[K_PD] := Module[{k, kk, n = Length[K]},
  k = K /. X[i_, j_, k_, m_] => P[-i, Mod[j - m, n, -1] m]
    P[Mod[j - m, n, -1] j, i] P[k, -Mod[j - m, n, -1] j] P[-Mod[j - m, n, -1] m, -k];
  kk = (Times @@ k) /. P[i_, j_] P[j_, k_] => P[i, k];
   $\frac{1}{2} (n - \text{Length}[kk] + 2)$ 
  ]
genus[K_String] := genus@GCToPD[K]
```

A knot can be represented as a knotoid if it contains a cycle that goes around both sides of an edge (that is, a P that has some n and -n), and it has genus 1, or if it has genus 0.

```
cycles[K_PD] := Module[{k, n = Length[K]},
  k = K /. X[i_, j_, k_, m_] => P[-i, Mod[j - m, n, -1] m]
    P[Mod[j - m, n, -1] j, i] P[k, -Mod[j - m, n, -1] j] P[-Mod[j - m, n, -1] m, -k];
  (Times @@ k) /. P[i____, j_] P[j_, k____] => P[i, j, k]
  ]
```

**k416** = PD[X[2, 8, 3, 1], X[4, 1, 5, 2], X[6, 3, 7, 4], X[5, 8, 6, 7]]

**cycles**[**k416**]

P[-1, 4, -7, -6, -4, -2, -1] P[-5, 7, 3, 8, 5, 1, -3, 6, -8, 2, -5]

**k444** = PD[X[2, 8, 3, 1], X[5, 1, 6, 2], X[7, 3, 8, 4], X[4, 7, 5, 6]]

PD[X[2, 8, 3, 1], X[5, 1, 6, 2], X[7, 3, 8, 4], X[4, 7, 5, 6]]

**cycles**[**k444**]

P[3, 8, 3] P[6, 1, -3, 7, 4, -8, 2, -6, -5, -2, -1, 5, -7, -4, 6]