Let' s build a set of all reachable states

```
p12[{m1_, m2_, m3_}] := {Max[m1 - (3 - m2), 0], Min[m1 + m2, 3], m3}
p13[{m1_, m2_, m3_}] := {Max[m1 - (8 - m3), 0], m2, Min[m1 + m3, 8]}
p21[{m1_, m2_, m3_}] := {Min[m2 + m1, 5], Max[m2 - (5 - m1), 0], m3}
p23[{m1_, m2_, m3_}] := {m1, Max[m2 - (8 - m3), 0], Min[m2 + m3, 8]}
p31[{m1_, m2_, m3_}] := {Min[m1 + m3, 5], m2, Max[m3 - (5 - m1), 0]}
p32[{m1_, m2_, m3_}] := {m1, Min[m2 + m3, 3], Max[m3 - (3 - m2), 0]}
```

```
it[0] := {{0, 0, 8}}
it[n_] := Union@Flatten[Table[pi[j], {j, it[n - 1]}, {i, {12, 13, 23, 21, 32, 31}}], 1]
```

```
it[4]
```

```
{{0, 0, 8}, {0, 2, 6}, {0, 3, 5}, {2, 0, 6},
 {2, 3, 3}, {3, 0, 5}, {3, 3, 2}, {5, 0, 3}, {5, 1, 2}, {5, 3, 0}}
```

```
Table[Intersection[Flatten@it[n], {4}], {n, 1, 10}]
```

```
{{}, {}, {}, {}, {}, {4}, {4}, {4}, {4}, {4}}
```

Thus, the minimal number of steps needed to have a state where one of the jugs contain 4 liters is six.

```
Union@it[6]
```

```
{{0, 0, 8}, {0, 1, 7}, {0, 2, 6}, {0, 3, 5}, {1, 0, 7}, {2, 0, 6}, {2, 3, 3},
 {3, 0, 5}, {3, 3, 2}, {4, 3, 1}, {5, 0, 3}, {5, 1, 2}, {5, 2, 1}, {5, 3, 0}}
```

It remains to find the path between {0,0,8} and {4,3,1}.

```
Quit[]
```

Let's list the edges (note that there is no need to generate the set of vertices to generate a connected graph, but they help with the construction of the sets of edges).

```
vi_[{m1_, m2_, m3_}] := {m1, m2, m3} → pi[{m1, m2, m3}]
ve[n_] := Union@Flatten@Table[vi[j], {j, it[n - 1]}, {i, {12, 13, 21, 23, 31, 32}}]
```

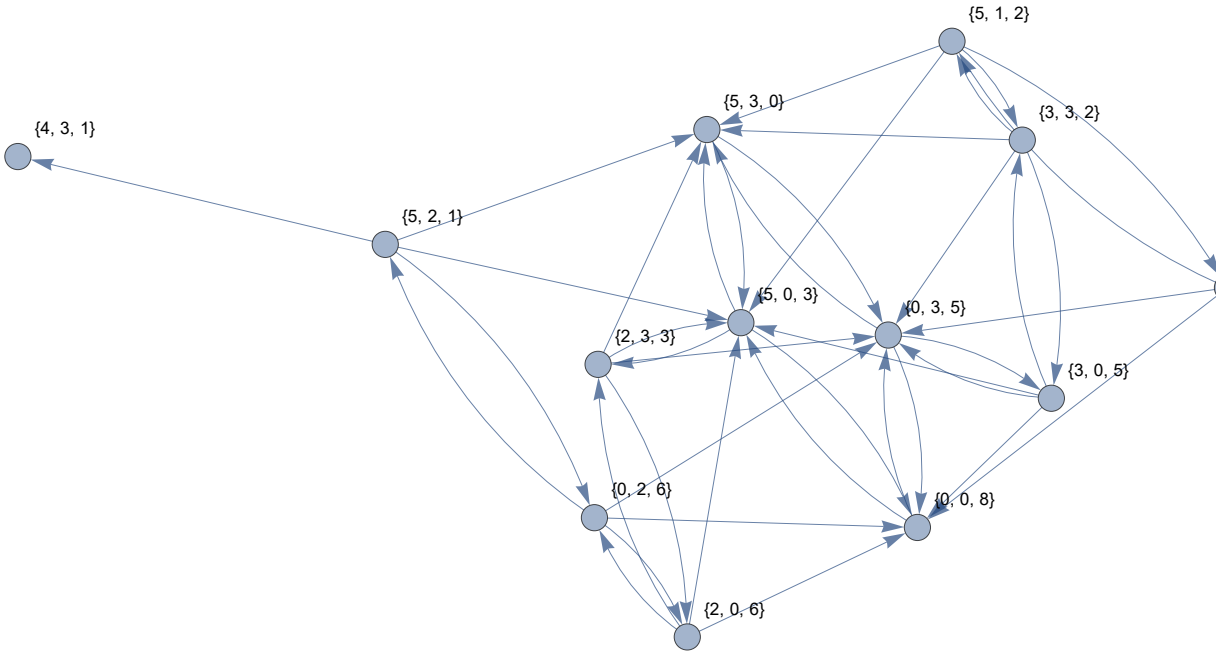
```
ve[2]
```

```
{{0, 0, 8} → {0, 0, 8}, {0, 0, 8} → {0, 3, 5}, {0, 0, 8} → {5, 0, 3},
 {0, 3, 5} → {0, 0, 8}, {0, 3, 5} → {0, 3, 5}, {0, 3, 5} → {3, 0, 5}, {0, 3, 5} → {5, 3, 0},
 {5, 0, 3} → {0, 0, 8}, {5, 0, 3} → {2, 3, 3}, {5, 0, 3} → {5, 0, 3}, {5, 0, 3} → {5, 3, 0}}
```

Let' s remove the loops.

```
clean[n_] := Table[i → i, {i, it[n]}]
```

```
g = Graph[Complement[ve[6], clean[6]], VertexLabels → "Name"]
```




```
FindShortestPath[g, {0, 0, 8}, {4, 3, 1}]
```

```
{{0, 0, 8}, {5, 0, 3}, {2, 3, 3}, {2, 0, 6}, {0, 2, 6}, {5, 2, 1}, {4, 3, 1}}
```

Let' s do this again adding a spilling jug.

```
sp12[{m1_, m2_, m3_, m4_}] := {Max[m1 - (3 - m2), 0], Min[m1 + m2, 3], m3, m4}
sp13[{m1_, m2_, m3_, m4_}] := {Max[m1 - (8 - m3), 0], m2, Min[m1 + m3, 8], m4}
sp21[{m1_, m2_, m3_, m4_}] := {Min[m2 + m1, 5], Max[m2 - (5 - m1), 0], m3, m4}
sp23[{m1_, m2_, m3_, m4_}] := {m1, Max[m2 - (8 - m3), 0], Min[m2 + m3, 8], m4}
sp31[{m1_, m2_, m3_, m4_}] := {Min[m1 + m3, 5], m2, Max[m3 - (5 - m1), 0], m4}
sp32[{m1_, m2_, m3_, m4_}] := {m1, Min[m2 + m3, 3], Max[m3 - (3 - m2), 0], m4}
sp41[{m1_, m2_, m3_, m4_}] := {Min[m4 + m1, 5], m2, m3, Max[m4 - (5 - m1), 0]}
sp42[{m1_, m2_, m3_, m4_}] := {m1, Min[m4 + m2, 3], m3, Max[m4 - (3 - m2), 0]}
sp43[{m1_, m2_, m3_, m4_}] := {m1, m2, Min[m4 + m3, 8], Max[m4 - (8 - m3), 0]}
sp14[{m1_, m2_, m3_, m4_}] := {0, m2, m3, m1 + m4}
sp24[{m1_, m2_, m3_, m4_}] := {m1, 0, m3, m2 + m4}
sp34[{m1_, m2_, m3_, m4_}] := {m1, m2, 0, m3 + m4}

sit[0] := {{0, 0, 8, 0}}
sit[n_] := Union@Flatten[
   Table[spi[j], {j, it[n - 1]}, {i, {12, 13, 23, 21, 32, 31, 41, 42, 43, 14, 24, 34}}], 1]
```

```
sit[3]
```

```
{{0, 0, 0, 8}, {0, 0, 3, 5}, {0, 0, 5, 3}, {0, 0, 8, 0}, {0, 3, 0, 5}, {0, 3, 2, 3},
 {0, 3, 3, 2}, {0, 3, 5, 0}, {2, 0, 3, 3}, {2, 0, 6, 0}, {2, 3, 0, 3}, {2, 3, 3, 0},
 {3, 0, 0, 5}, {3, 0, 5, 0}, {3, 3, 2, 0}, {5, 0, 0, 3}, {5, 0, 3, 0}, {5, 3, 0, 0}}
```

```
Table[Intersection[Flatten@sit[n], {4}], {n, 1, 6}]
```

```
{{}, {}, {}, {}, {}, {4}}
```

So allowing spilling does not make the problem any easier to solve.