

Catalan Numbers

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Objective

Construct a function *Poster*[*n*, *a list of methods*] to draw the poster representing the Catalan numbers.

Implementation

Backbone

The following code constructs a list of diagonals for the triangulations. We will use this code to further construct the actual triangulations, trees, and non-associative words used to construct the poster.

```
ts[n_Integer] := ts[Range[0, n + 1]];
ts[[_ , _]] = {ds[]};
ts[vs_List] := Module[{l, r, k, t1, t2, tds},
  Union@@Table[
    l = ts[Prepend[vs[[k ;;]], vs[[1]]]];
    r = ts[vs[[2 ;; k]]];
    Flatten[Table[
      tds = Join[t1, t2];
      If[k > 3, AppendTo[tds, d[vs[[2]], vs[[k]]]];
      If[k < Length[vs], AppendTo[tds, d[vs[[1]], vs[[k]]]];
      tds,
      {t1, l}, {t2, r}
    ]],
    {k, 3, Length[vs]}
  ]
];
```

Triangulations

```
createTriangulation[t_ds, n_Integer] := Module[{diags},
  diags = t /. ds[ls___] → Graphics[{ls]} /. d[i_, j_] ⇒ Line[{i, j]} /.
  j_Integer ⇒ {Cos[ $\frac{2\pi j}{n+2}$ ], Sin[ $\frac{2\pi j}{n+2}$ ]}];
  Graphics[Join[diags[[1]], {EdgeForm[Thickness[Medium]],
    Transparent, RegularPolygon[{1, 0}, n + 2]}]]
]
```

Trees

```
createTree[t_ds, n_Integer] := TreeForm[
  Last[d[0, n + 1] (Times @@ t)  $\prod_{j=0}^n$  e[j, j + 1, p] // .
  e[i_, j_, t1_] e[j_, k_, t2_] d[i_, k_]  $\Rightarrow$  e[i, k, p[t1, t2]]],
  VertexRenderingFunction  $\rightarrow$  (Inset[Text["•"], #] &)]
```

Non-associative Words

```
createSentence[t_ds, n_Integer] :=
  Last[d[0, n + 1] (Times @@ t)  $\prod_{j=0}^n$  e[j, j + 1, FromLetterNumber[j + 1]] // .
  e[i_, j_, t1_] e[j_, k_, t2_] d[i_, k_]  $\Rightarrow$  e[i, k, StringJoin["(", t1, t2, ")"]]]
```

Final Product

```
Poster[n_Integer, methods_List] := Module[{temp},
  (* methods is a list of 3 elements that indicate whether or not to include
  each method: triangulations, trees, associative words, respectively *)
  temp = {};
  If[methods[[1]] == 1,
    temp = Append[temp, Table[createTriangulation[item, n], {item, ts[n]}]];
  If[methods[[2]] == 1,
    temp = Append[temp, Table[createTree[item, n], {item, ts[n]}]];
  If[methods[[3]] == 1,
    temp = Append[temp, Table[createSentence[item, n], {item, ts[n]}]];
  GraphicsGrid[temp, Frame  $\rightarrow$  All]
]
```

Example

Poster [3, {1, 1, 1}]

| | | | |
|--------------|----------------|----------------|---------------|
| | | | |
| | | | |
| $((ab) c) d$ | $((a (bc)) d)$ | $(a ((bc) d))$ | $((ab) (cd))$ |