

(\* Define Stereographic projection from the north pole of S3, as well as its inverse\*)

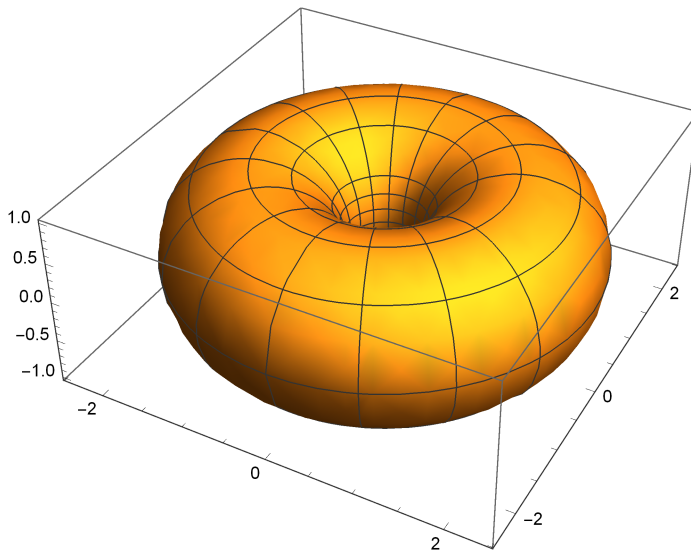
$$\text{Stereon}[x\_ ] := \left\{ \frac{x[[1]]}{1-x[[4]]}, \frac{x[[2]]}{1-x[[4]]}, \frac{x[[3]]}{1-x[[4]]} \right\};$$

$$\text{IStereon}[x\_ ] := \left\{ 2 \frac{x[[1]]}{\text{Norm}[x]^2+1}, 2 \frac{x[[2]]}{\text{Norm}[x]^2+1}, 2 \frac{x[[3]]}{\text{Norm}[x]^2+1}, \frac{\text{Norm}[x]^2-1}{\text{Norm}[x]^2+1} \right\};$$

(\*Here is our torus, stereographically projected from the north pole of S3\*)

$$\text{tor}[th1_, th2_] := \left\{ \frac{\text{Cos}[2 \text{ Pi } th1]}{\text{Sqrt}[2]}, \frac{\text{Sin}[2 \text{ Pi } th1]}{\text{Sqrt}[2]}, \frac{\text{Cos}[2 \text{ Pi } th2]}{\text{Sqrt}[2]}, \frac{\text{Sin}[2 \text{ Pi } th2]}{\text{Sqrt}[2]} \right\};$$

ParametricPlot3D[Stereon[tor[th1, th2]], {th1, 0, 1}, {th2, 0, 1}]



(\* The point {1+Sqrt[2],0,0} is on the surface of this torus. I intend to do a stereographic projection from the corresponding point in S3\*)

projpoint = IStereon[{1+Sqrt[2], 0, 0}];

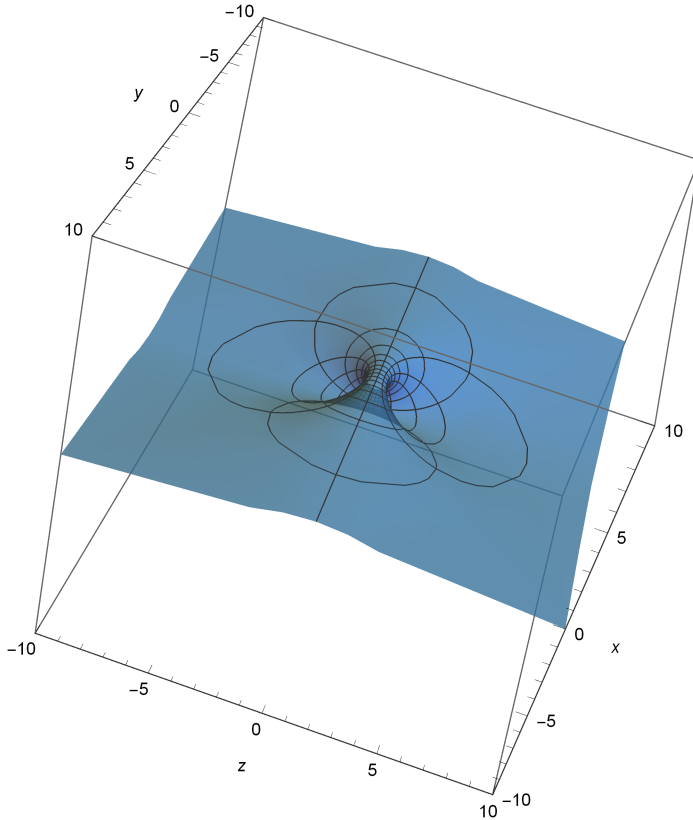
(\* Define rotation matrices for stereographic projections. R is a rotation matrix sending projpoint to the north pole of S3\*)

Rot[th\_] := {{Cos[th], 0, 0, -Sin[th]}, {0, 1, 0, 0}, {0, 0, 1, 0}, {Sin[th], 0, 0, Cos[th]}}  
 R = {Rot[Pi/2].projpoint, {0, 1, 0, 0}, {0, 0, 1, 0}, projpoint};

(\* Plot the resulting projection of the torus\*)

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ParametricPlot3D[StereoN[R.tor[th1, th2]], {th1, 0, 1}, {th2, 0, 1}, PlotRange -> {-10, 10},
PlotStyle -> {{Opacity[0.8], FaceForm[Hue[0.59, 0.73, 0.79], Hue[0.11, 0.73, 0.88]]}},
AxesLabel -> {x, y, z},
ViewPoint -> {1.721792493400969`, -0.8671937374601687`, 2.78090014768203`},
ViewVertical -> {-0.4669018501380959`, 0.2603456483843665`, -0.9471894444621316`}
    
```



(\* This is a picture of the torus going through the point at infinity. The fourth order symmetry is a rotation  $\pi/2$  radians CCW about the x axis, followed by a reflection in the xy plane.

