The Milk Jug Problem

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```
A = {{8, 0, 0}};
Lines = {};

For[i = 1, i ≤ Length[A], i++,
 If[(0 <= A[[i]][[1]] - 5 + A[[i]][[2]] ≤ 8) ∧ (A[[i]][[1]] + A[[i]][[2]] + A[[i]][[3]] == 8),
  Lines = AppendTo[Lines, A[[i]] -> {A[[i]][[1]] - 5 + A[[i]][[2]], 5, A[[i]][[3]]}];
  If[Count[A, {A[[i]][[1]] - 5 + A[[i]][[2]], 5, A[[i]][[3]]}] == 0,
   A = AppendTo[A, {A[[i]][[1]] - 5 + A[[i]][[2]], 5, A[[i]][[3]]}] ]];
 If[(0 ≤ A[[i]][[1]] - 3 + A[[i]][[3]] ≤ 8) ∧ (A[[i]][[1]] + A[[i]][[2]] + A[[i]][[3]] == 8),
  Lines = AppendTo[Lines, A[[i]] -> {A[[i]][[1]] - 3 + A[[i]][[3]], A[[i]][[2]], 3}];
  If[ Count[A, {A[[i]][[1]] - 3 + A[[i]][[3]], A[[i]][[2]], 3}] == 0,
   A = AppendTo[A, {A[[i]][[1]] - 3 + A[[i]][[3]], A[[i]][[2]], 3}]]];
 If[(0 ≤ A[[i]][[2]] - 8 + A[[i]][[1]] ≤ 5) ∧ (A[[i]][[1]] + A[[i]][[2]] + A[[i]][[3]] == 8),
  Lines = AppendTo[Lines, A[[i]] -> {8, A[[i]][[2]] - 8 + A[[i]][[1]], A[[i]][[3]]}];
  If[Count[A, {8, A[[i]][[2]] - 8 + A[[i]][[1]], A[[i]][[3]]}] == 0, A = AppendTo[A, {8, A[[i]][[2]] - 8 + A[[i]][[
        1]], A[[i]][[3]]}]]]; If[(0 <= A[[i]][[1]] + A[[i]][[2]] ≤ 8) ∧ (A[[i]][[1]] + A[[i]][[2]] + A[[i]][[3]] == 8),
  Lines = AppendTo[Lines, A[[i]] -> {A[[i]][[1]] + A[[i]][[2]], 0, A[[i]][[3]]}];
  If[Count[A, {A[[i]][[1]] + A[[i]][[2]], 0, A[[i]][[3]]}] == 0,
   A = AppendTo[A, {A[[i]][[1]] + A[[i]][[2]], 0, A[[i]][[3]]}]]];
 If[(0 ≤ (A[[i]][[2]] - 3 + A[[i]][[3]]) ≤ 5),
  Lines = AppendTo[Lines, A[[i]] -> {A[[i]][[1]], A[[i]][[2]] - 3 + A[[i]][[3]], 3}] ;
  If[ Count[A, {A[[i]][[1]], A[[i]][[2]] - 3 + A[[i]][[3]], 3}] == 0,
   AppendTo[A, {A[[i]][[1]], A[[i]][[2]] - 3 + A[[i]][[3]], 3}]]];
 If[(0 ≤ A[[i]][[2]] + A[[i]][[3]] ≤ 3), Lines =
   AppendTo[Lines, A[[i]] -> {A[[i]][[1]], 0, A[[i]][[2]] + A[[i]][[3]]}];
  If[Count[A, {A[[i]][[1]], 0, A[[i]][[2]] + A[[i]][[3]]}] == 0,
   AppendTo[A, {A[[i]][[1]], 0, A[[i]][[2]] + A[[i]][[3]]}] ]];
 If[(0 ≤ A[[i]][[2]] + A[[i]][[3]] ≤ 5), Lines =
   AppendTo[Lines, A[[i]] -> {A[[i]][[1]], A[[i]][[2]] + A[[i]][[3]], 0}];
  If[Count[A, {A[[i]][[1]], A[[i]][[2]] + A[[i]][[3]], 0}] == 0,
   AppendTo[A, {A[[i]][[1]], A[[i]][[2]] + A[[i]][[3]], 0}]]];
 If[(0 ≤ A[[i]][[3]] - 5 - A[[i]][[2]] ≤ 3),
  Lines = AppendTo[Lines, A[[i]] -> {A[[i]][[1]], 5, A[[i]][[3]] - 5 + A[[i]][[2]]}];
  If[Count[A, {A[[i]][[1]], 5, A[[i]][[3]] - 5 - A[[i]][[2]]}] == 0,
   AppendTo[A, {A[[i]][[1]], 5, A[[i]][[3]] - 5 + A[[i]][[2]]}]]];
 If[(0 ≤ A[[i]][[1]] + A[[i]][[3]] ≤ 3), Lines =
   AppendTo[Lines, A[[i]] -> {0, A[[i]][[2]], A[[i]][[1]] + A[[i]][[3]]}];
  If[ Count[A, {0, A[[i]][[2]], A[[i]][[1]] + A[[i]][[3]]}] == 0 ,
   AppendTo[A, {0, A[[i]][[2]], A[[i]][[1]] + A[[i]][[3]]}]]];
 If[(0 ≤ A[[i]][[1]] + A[[i]][[3]] ≤ 8),
  Lines = AppendTo[Lines, A[[i]] -> { A[[i]][[1]] + A[[i]][[3]], A[[i]][[2]], 0}];
  If[ Count[A, { A[[i]][[1]] + A[[i]][[3]], A[[i]][[2]], 0}] == 0,
   AppendTo[A, { A[[i]][[1]] + A[[i]][[3]], A[[i]][[2]], 0}]]];
 If[(0 ≤ A[[i]][[3]] - 8 + A[[i]][[1]] ≤ 3), Lines =
   AppendTo[Lines, A[[i]] -> {8, A[[i]][[2]], A[[i]][[3]] - 8 + A[[i]][[1]]}];
  If[Count[A, {8, A[[i]][[2]], A[[i]][[3]] - 8 + A[[i]][[1]]}] == 0,
   AppendTo[A, {8, A[[i]][[2]], A[[i]][[3]] - 8 + A[[i]][[1]]}]]]]
A
```

```
{{8, 0, 0}, {3, 5, 0}, {5, 0, 3}, {0, 5, 3}, {3, 2, 3}, {5, 3, 0}, {6, 2, 0}, {2, 3, 3},
 {6, 0, 2}, {1, 5, 2}, {1, 4, 3}, {4, 4, 0}, {4, 1, 3}, {7, 1, 0}, {7, 0, 1}, {2, 5, 1}}
```

Lines

```
{{8, 0, 0} → {3, 5, 0}, {8, 0, 0} → {5, 0, 3}, {8, 0, 0} → {8, 0, 0},
 {8, 0, 0} → {8, 0, 0}, {8, 0, 0} → {8, 0, 0}, {8, 0, 0} → {8, 0, 0}, {8, 0, 0} → {8, 0, 0},
 {8, 0, 0} → {8, 0, 0}, {3, 5, 0} → {3, 5, 0}, {3, 5, 0} → {0, 5, 3}, {3, 5, 0} → {8, 0, 0},
 {3, 5, 0} → {8, 0, 0}, {3, 5, 0} → {3, 2, 3}, {3, 5, 0} → {3, 5, 0}, {3, 5, 0} → {0, 5, 3},
 {3, 5, 0} → {3, 5, 0}, {5, 0, 3} → {0, 5, 3}, {5, 0, 3} → {5, 0, 3}, {5, 0, 3} → {5, 0, 3},
 {5, 0, 3} → {5, 0, 3}, {5, 0, 3} → {5, 0, 3}, {5, 0, 3} → {5, 3, 0}, {5, 0, 3} → {8, 0, 0},
 {5, 0, 3} → {8, 0, 0}, {0, 5, 3} → {0, 5, 3}, {0, 5, 3} → {0, 5, 3}, {0, 5, 3} → {5, 0, 3},
 {0, 5, 3} → {0, 5, 3}, {0, 5, 3} → {0, 5, 3}, {0, 5, 3} → {3, 5, 0}, {3, 2, 3} → {0, 5, 3},
 {3, 2, 3} → {3, 2, 3}, {3, 2, 3} → {5, 0, 3}, {3, 2, 3} → {3, 2, 3}, {3, 2, 3} → {3, 5, 0},
 {3, 2, 3} → {6, 2, 0}, {5, 3, 0} → {3, 5, 0}, {5, 3, 0} → {2, 3, 3}, {5, 3, 0} → {8, 0, 0},
 {5, 3, 0} → {8, 0, 0}, {5, 3, 0} → {5, 0, 3}, {5, 3, 0} → {5, 0, 3}, {5, 3, 0} → {5, 3, 0},
 {5, 3, 0} → {5, 3, 0}, {6, 2, 0} → {3, 5, 0}, {6, 2, 0} → {3, 2, 3}, {6, 2, 0} → {8, 0, 0},
 {6, 2, 0} → {8, 0, 0}, {6, 2, 0} → {6, 0, 2}, {6, 2, 0} → {6, 2, 0}, {6, 2, 0} → {6, 2, 0},
 {2, 3, 3} → {0, 5, 3}, {2, 3, 3} → {2, 3, 3}, {2, 3, 3} → {5, 0, 3}, {2, 3, 3} → {2, 3, 3},
 {2, 3, 3} → {5, 3, 0}, {6, 0, 2} → {1, 5, 2}, {6, 0, 2} → {5, 0, 3}, {6, 0, 2} → {6, 0, 2},
 {6, 0, 2} → {6, 0, 2}, {6, 0, 2} → {6, 2, 0}, {6, 0, 2} → {8, 0, 0}, {6, 0, 2} → {8, 0, 0},
 {1, 5, 2} → {1, 5, 2}, {1, 5, 2} → {0, 5, 3}, {1, 5, 2} → {6, 0, 2}, {1, 5, 2} → {1, 4, 3},
 {1, 5, 2} → {0, 5, 3}, {1, 5, 2} → {3, 5, 0}, {1, 4, 3} → {0, 5, 3}, {1, 4, 3} → {1, 4, 3},
 {1, 4, 3} → {5, 0, 3}, {1, 4, 3} → {1, 4, 3}, {1, 4, 3} → {4, 4, 0}, {4, 4, 0} → {3, 5, 0},
 {4, 4, 0} → {1, 4, 3}, {4, 4, 0} → {8, 0, 0}, {4, 4, 0} → {8, 0, 0}, {4, 4, 0} → {4, 1, 3},
 {4, 4, 0} → {4, 4, 0}, {4, 4, 0} → {4, 4, 0}, {4, 1, 3} → {0, 5, 3}, {4, 1, 3} → {4, 1, 3},
 {4, 1, 3} → {5, 0, 3}, {4, 1, 3} → {4, 1, 3}, {4, 1, 3} → {4, 4, 0}, {4, 1, 3} → {7, 1, 0},
 {7, 1, 0} → {3, 5, 0}, {7, 1, 0} → {4, 1, 3}, {7, 1, 0} → {8, 0, 0}, {7, 1, 0} → {8, 0, 0},
 {7, 1, 0} → {7, 0, 1}, {7, 1, 0} → {7, 1, 0}, {7, 1, 0} → {7, 1, 0}, {7, 0, 1} → {2, 5, 1},
 {7, 0, 1} → {5, 0, 3}, {7, 0, 1} → {7, 0, 1}, {7, 0, 1} → {7, 0, 1}, {7, 0, 1} → {7, 1, 0},
 {7, 0, 1} → {8, 0, 0}, {7, 0, 1} → {8, 0, 0}, {2, 5, 1} → {2, 5, 1}, {2, 5, 1} → {0, 5, 3},
 {2, 5, 1} → {7, 0, 1}, {2, 5, 1} → {2, 3, 3}, {2, 5, 1} → {0, 5, 3}, {2, 5, 1} → {3, 5, 0}}
```

```
flag = 0;
```

```
counter = 0;
```

```
For[a = 1, a ≤ Length[Lines], a++,
 If[Lines[[a]][[1]] === Lines[[a]][[2]], Lines = Delete[Lines, a]; flag = 1] ×
  If[flag == 1, a = a - 1;
   flag = 0;
   counter++]]
```

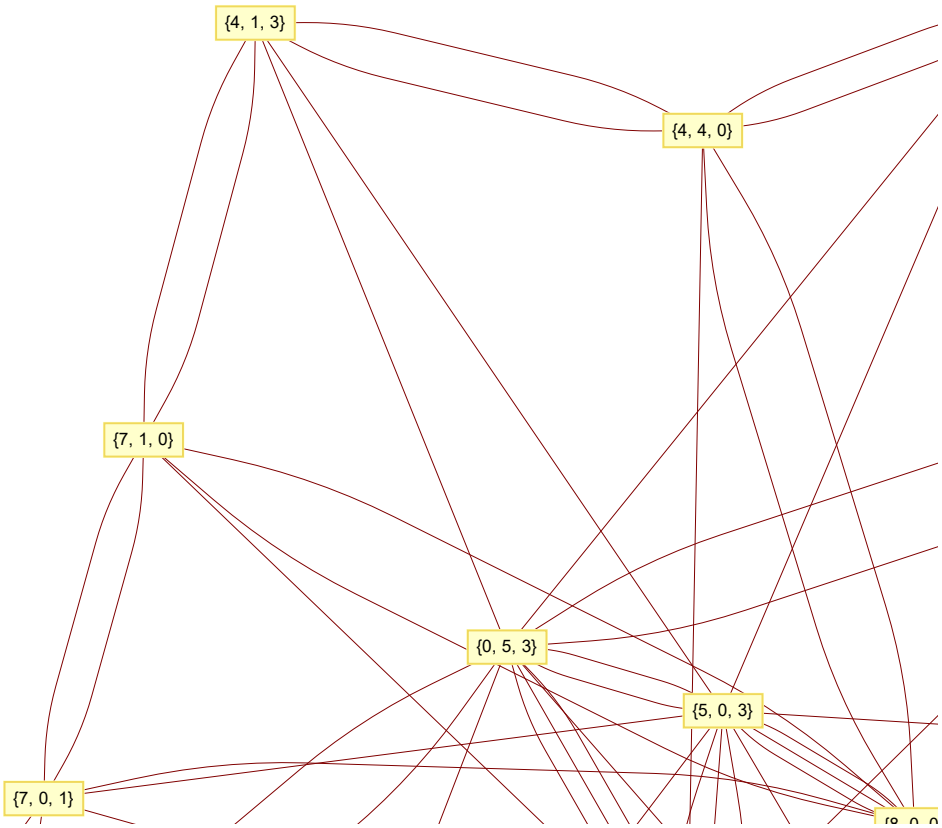
```
counter
```

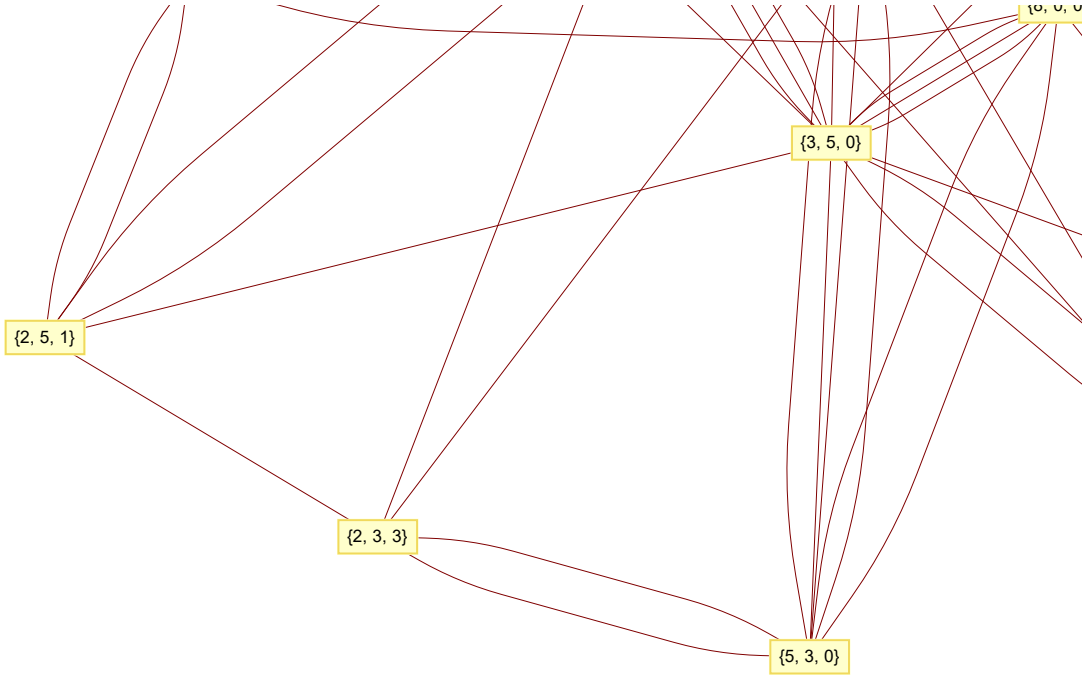
```
39
```

Lines

```
{{8, 0, 0} → {3, 5, 0}, {8, 0, 0} → {5, 0, 3}, {3, 5, 0} → {0, 5, 3}, {3, 5, 0} → {8, 0, 0},
 {3, 5, 0} → {8, 0, 0}, {3, 5, 0} → {3, 2, 3}, {3, 5, 0} → {0, 5, 3}, {5, 0, 3} → {0, 5, 3},
 {5, 0, 3} → {5, 3, 0}, {5, 0, 3} → {8, 0, 0}, {5, 0, 3} → {8, 0, 0}, {0, 5, 3} → {5, 0, 3},
 {0, 5, 3} → {3, 5, 0}, {3, 2, 3} → {0, 5, 3}, {3, 2, 3} → {5, 0, 3}, {3, 2, 3} → {3, 5, 0},
 {3, 2, 3} → {6, 2, 0}, {5, 3, 0} → {3, 5, 0}, {5, 3, 0} → {2, 3, 3}, {5, 3, 0} → {8, 0, 0},
 {5, 3, 0} → {8, 0, 0}, {5, 3, 0} → {5, 0, 3}, {5, 3, 0} → {5, 0, 3}, {6, 2, 0} → {3, 5, 0},
 {6, 2, 0} → {3, 2, 3}, {6, 2, 0} → {8, 0, 0}, {6, 2, 0} → {8, 0, 0}, {6, 2, 0} → {6, 0, 2},
 {2, 3, 3} → {0, 5, 3}, {2, 3, 3} → {5, 0, 3}, {2, 3, 3} → {5, 3, 0}, {6, 0, 2} → {1, 5, 2},
 {6, 0, 2} → {5, 0, 3}, {6, 0, 2} → {6, 2, 0}, {6, 0, 2} → {8, 0, 0}, {6, 0, 2} → {8, 0, 0},
 {1, 5, 2} → {0, 5, 3}, {1, 5, 2} → {6, 0, 2}, {1, 5, 2} → {1, 4, 3}, {1, 5, 2} → {0, 5, 3},
 {1, 5, 2} → {3, 5, 0}, {1, 4, 3} → {0, 5, 3}, {1, 4, 3} → {5, 0, 3}, {1, 4, 3} → {4, 4, 0},
 {4, 4, 0} → {3, 5, 0}, {4, 4, 0} → {1, 4, 3}, {4, 4, 0} → {8, 0, 0}, {4, 4, 0} → {8, 0, 0},
 {4, 4, 0} → {4, 1, 3}, {4, 1, 3} → {0, 5, 3}, {4, 1, 3} → {5, 0, 3}, {4, 1, 3} → {4, 4, 0},
 {4, 1, 3} → {7, 1, 0}, {7, 1, 0} → {3, 5, 0}, {7, 1, 0} → {4, 1, 3}, {7, 1, 0} → {8, 0, 0},
 {7, 1, 0} → {8, 0, 0}, {7, 1, 0} → {7, 0, 1}, {7, 0, 1} → {2, 5, 1}, {7, 0, 1} → {5, 0, 3},
 {7, 0, 1} → {7, 1, 0}, {7, 0, 1} → {8, 0, 0}, {7, 0, 1} → {8, 0, 0}, {2, 5, 1} → {0, 5, 3},
 {2, 5, 1} → {7, 0, 1}, {2, 5, 1} → {2, 3, 3}, {2, 5, 1} → {0, 5, 3}, {2, 5, 1} → {3, 5, 0}}
```

```
GraphPlot[Lines, VertexLabeling → True]
```






```
graph = Graph[A, Lines]
```

```
For[j = 1, j ≤ Length[A], j++,
 If[A[[j]][[1]] == 4 ∨ A[[j]][[2]] == 4 ∨ A[[j]][[3]] == 4, AppendTo[fours, A[[j]]]]]
```

```
fours = DeleteDuplicates[fours]
```

```
{{1, 4, 3}, {4, 4, 0}, {4, 1, 3}}
```

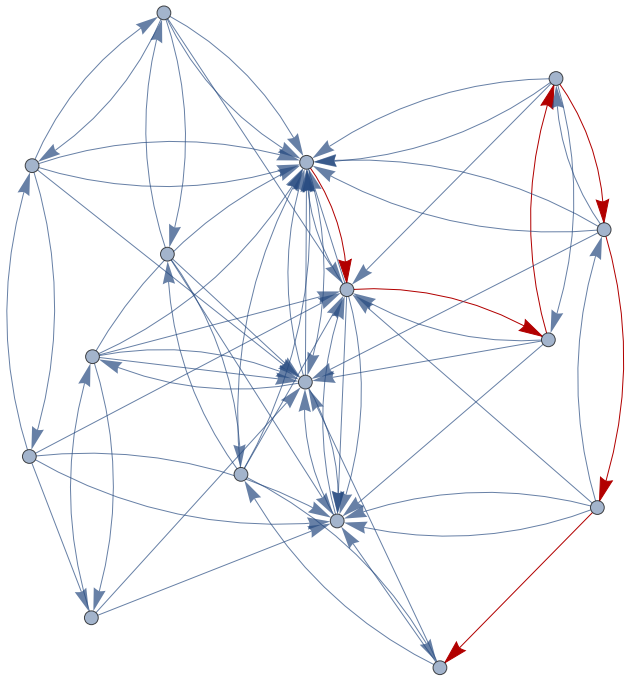
```
For[k = 1, k ≤ Length[fours], k++,
 Print[FindShortestPath[Graph[A, Lines], {8, 0, 0}, fours[[k]]]]
]
```

```
{{8, 0, 0}, {3, 5, 0}, {3, 2, 3}, {6, 2, 0}, {6, 0, 2}, {1, 5, 2}, {1, 4, 3}}
```

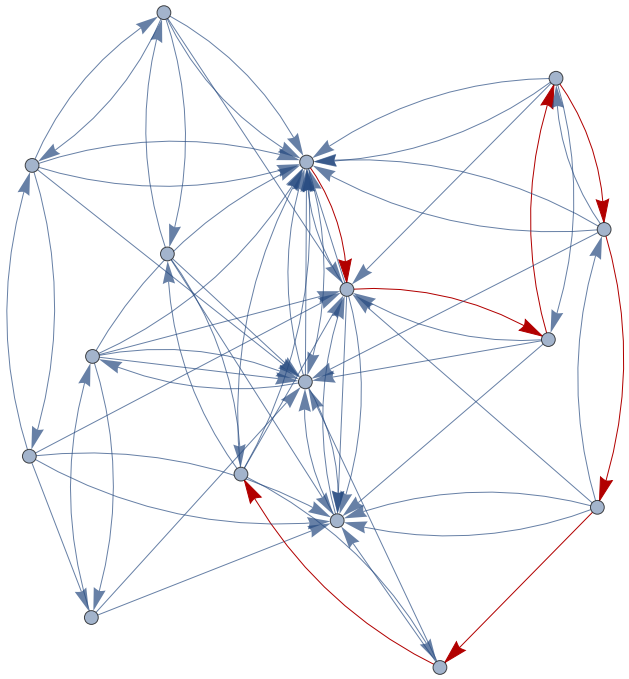
```
{{8, 0, 0}, {3, 5, 0}, {3, 2, 3}, {6, 2, 0}, {6, 0, 2}, {1, 5, 2}, {1, 4, 3}, {4, 4, 0}}
```

```
{{8, 0, 0}, {3, 5, 0}, {3, 2, 3}, {6, 2, 0}, {6, 0, 2}, {1, 5, 2}, {1, 4, 3}, {4, 4, 0}, {4, 1, 3}}
```

```
HighlightGraph[Graph[A, Lines],
 PathGraph[FindShortestPath[Graph[A, Lines], {8, 0, 0}, fours[[1]]], DirectedEdges → True]]
```



```
HighlightGraph[Graph[A, Lines],
 PathGraph[FindShortestPath[Graph[A, Lines], {8, 0, 0}, fours[[2]]], DirectedEdges → True]]
```



```
HighlightGraph[Graph[A, Lines],
 PathGraph[FindShortestPath[Graph[A, Lines], {8, 0, 0}, fours[[3]]], DirectedEdges → True]]
```