

```
KU = PD[X[1, 4, 2, 3] X[2, 4, 1, 3]];
K = K31 = PD[X[3, 1, 4, 6], X[1, 5, 2, 4], X[5, 3, 6, 2]];
KB = PD[X[1, 4, 2, 5], X[3, 6, 4, 1], X[5, 2, 6, 3]];
K51 = PD[X[1, 6, 2, 7], X[3, 8, 4, 9], X[5, 10, 6, 1], X[7, 2, 8, 3], X[9, 4, 10, 5]];
K10132 = PD[X[4, 2, 5, 1], X[8, 4, 9, 3], X[5, 12, 6, 13], X[15, 18, 16, 19], X[9, 16, 10, 17],
  X[17, 10, 18, 11], X[13, 20, 14, 1], X[19, 14, 20, 15], X[11, 6, 12, 7], X[2, 8, 3, 7]];
Hp = PD[X[2, 3, 1, 4] X[4, 2, 3, 1]];
Hp1 = PD[X[2, 3, 1, 4] X[3, 2, 4, 1]];
```

Edge maps-----

```
EdgeMap[(c[i_] c[j_] -> c[k_]) S[L_]] [bv_] := {vp[i] vp[j] -> vp[k], vp[i] vm[j] -> vm[k],
  vm[i] vp[j] -> vm[k], vm[i] vm[j] -> 0, If[Head[(List @@ bv) [[1]]] === B, 0, bv]};
EdgeMap[(be_B) (c[i_] c[j_] -> c[k_]) S[L_]] [bv_] :=
  {vp[i] vp[j] -> vp[k], vp[i] vm[j] -> vm[k], vm[i] vp[j] -> vm[k], vm[i] vm[j] -> 0,
  If[Head[(List @@ bv) [[1]]] === B, bv /. b_B -> If[b != be, 0, (-1)^(Count[be, q_ /; q < L]), 0]};
EdgeMap[(be_B) cs_ (c[i_] c[j_] -> c[k_]) S[L_]] [bv_] :=
  {vp[i] vp[j] -> vp[k], vp[i] vm[j] -> vm[k], vm[i] vp[j] -> vm[k], vm[i] vm[j] -> 0,
  If[Head[(List @@ bv) [[1]]] === B, bv /. b_B -> If[b != be, 0, (-1)^(Count[be, q_ /; q < L]), 0]};
(* ***** *)
EdgeMap[(c[k_] -> c[i_] c[j_]) S[L_]] [bv_] := {vp[k] -> vp[i] vm[j] + vm[i] vp[j],
  vm[k] -> vm[i] vm[j], If[Head[(List @@ bv) [[1]]] === B, 0, bv]};
EdgeMap[be_B (c[k_] -> c[i_] c[j_]) S[L_]] [bv_] :=
  {vp[k] -> vp[i] vm[j] + vm[i] vp[j], vm[k] -> vm[i] vm[j],
  If[Head[(List @@ bv) [[1]]] === B, bv /. b_B -> If[b != be, 0, (-1)^(Count[be, q_ /; q < L]), 0]};
EdgeMap[be_B cs_ (c[k_] -> c[i_] c[j_]) S[L_]] [bv_] :=
  {vp[k] -> vp[i] vm[j] + vm[i] vp[j], vm[k] -> vm[i] vm[j],
  If[Head[(List @@ bv) [[1]]] === B, bv /. b_B -> If[b != be, 0, (-1)^(Count[be, q_ /; q < L]), 0]};
```

The generating functions from class

```

KHGroupBasis[K_PD] :=
Module[{t, GF, EDS, Vs, Es, basis, FullBasis, ComBasis, sgn, w, n = Length[K]},
SetAttributes[P, Orderless];
SetAttributes[{B, P}, Orderless]; t = 0; e := e^P /; p > 1 := 0;
GF = (Expand[
Times @@ (K /. X[i_, j_, k_, L_] :=
(++t; P[i, j] P[k, L] + B[t] P[i, L] P[j, k] + e S[t] X[i, j, k, L]))
] /. B[i_] B[j_] := B[i, j] /. P[a_, b_] := P[a, b][Min[a, b]] /.
P[a_, b_][m1_] P[b_, c_][m2_] := P[a, c][Min[m1, m2]] /.
{P[i_, i_][m_] := c[m], P[_ , _][m_] := c[m]}) /.
{X[i_, j_, k_, L_] P[i_, j_][m1_] P[k_, L_][m2_] := Rule[c[m1] c[m2], c[Min[m1, m2]]],
X[i_, j_, k_, L_] P[i_, L_][m1_] P[j_, k_][m2_] :=
Rule[c[Min[m1, m2]], c[m1] c[m2]]};
{Vs, Es} = CoefficientList[GF, e];
basis = List @@ Expand[Vs /. c[m_] := vp[m] + vm[m]];
EDS = GroupBy[
If[Head[(List @@ #)[[1]]] === B, {#, Length[#[[1]]], {#, 0}} & /@ List @@ Es, Last];
FullBasis = Merge[If[Head[(List @@ #)[[1]]] === B, <|
Rule[Length[#[[1]], # /. b_B := 1] |>, <|0 -> #|>] & /@ basis, Identity];
ComBasis = Merge[If[Head[(List @@ #)[[1]]] === B, <|Rule[Length[#[[1]], #] |>, <|
0 -> #|>] & /@ basis, Identity];
w = List @@ K /. {X[_ , 1, _ , 2 n] -> 1, X[_ , 2 n, _ , 1] -> -1,
X[_ , j_, _ , L_] := If[j > L, 1, -1]};
sgn = GroupBy[w, Sign];
{EDS, FullBasis, ComBasis, Vs, sgn}
]
BE = KHGroupBasis[K]; (*Functions below depend on this variable.*)

```

The differentials

```

d_n_[bv_] := Module[{ans, lst, rst, res, x},
If[n >= Max[Keys[BE[[2]]] ] & n < Min[Keys[BE[[2]]] ], Return[{0}]];
Union @@ Table[
ans = EdgeMap[i[[1]]][bv];
x = If[Last[ans] === 0, {0}, List @@ Last[ans]];
lst = Union[x, {Last[ans]}];
rst = Association @@ Most[ans];
res = Select[lst, MemberQ[Keys[rst], #] &];
{If[res != {}, {Last[ans]} /. res[[1]] := rst[res[[1]]]},
{i, BE[[1]][n]}
]
] // Flatten

```

Matrix of the differentials

```

M_n := Module[{v},
  If[n >= Max[Keys[BE[[2]]]] ∨ n < Min[Keys[BE[[2]]]], Return[{{0}}]];
  Table[
    v = d_n[a] // Flatten;
    Coefficient[Plus@@v, b],
    {b, BE[[2]][n+1]}, {a, BE[[3]][n]}
  ]
]

r_n := MatrixRank[M_n]; (*Rank of Matrix*)

```

Khovanov Homology

```

KHom_n := (
  If[n > Max[Keys[BE[[2]]]] ∨ n < Min[Keys[BE[[2]]]], Return[-1]];
  Length[BE[[2]][n]] - r_n - r_{n-1}
)

```

```

KHomologyTrefoilKnot = Module[{res},
  Table[
    res = KHom_i;
    If[res > 0, Times@@ConstantArray[Z, res],
      If[res === 0, {}, {0}]
    ],
    {i, 0, 2}
  ]
]
{Z^2, Z^2, Z^3}

```

Jones Polynomial From the complex

```

JP = Module[{np, nm, sgn1, nom, j},
  sgn1 = BE[[5]];
  np = If[MemberQ[Keys[sgn1], 1], Plus@@sgn1[1], 0];
  nm = If[MemberQ[Keys[sgn1], -1], Plus@@sgn1[-1], 0];
  nom = (-1)^{nm} q^{np-2nm};
  j =
    nom * Expand[If[Head[#[[1]]] === B, (-1)^{Length[First[#]]} q^{Length[First[#]]} (q + q^{-1})^{Length[Rest[#]]},
      (q + q^{-1})^{Length[#1]} & /@ BE[[4]] (q + q^{-1})^{-1};
    Simplify[Expand[j]]
  ]
]
q^2 + q^6 - q^8

```

M_0 // MatrixForm

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

 M_1 // MatrixForm

$$\begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

 M_2 // MatrixForm

$$\begin{pmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$