This program looks for solutions $(a_1,\ a_2,\ \ldots a_n)$ for which the sum $\sum_{i=1}^{n} \frac{1}{a_i} = s$ holds, where s is a rational value supplied by the user.

$$X[a, n, s] = \left\{ (a_1,\ a_2,\ \ldots a_n) \in Z^n \;\middle|\; \sum_{i=1}^{n} \frac{1}{a_i} = s,\ a \le a_1 \le a_2 \le \ldots \le a_n \right\}.$$

```
X[a_, 0, s_] = If[s == 0, {{}}, {}];
X[a_, n_, 0] = {};
X[a_, n_, s_] := Union @@ Table[
    Prepend[#, a1] & /@ X[a1, n - 1, s - 1 / a1],
   {a1, Max[⌈1 / s⌉, a], ⌊n / s⌋}
  ]
```

```
X[2, 6, 2]
```

```
{{2, 2, 2, 3, 7, 42}, {2, 2, 2, 3, 8, 24}, {2, 2, 2, 3, 9, 18}, {2, 2, 2, 3, 10, 15},
 {2, 2, 2, 3, 12, 12}, {2, 2, 2, 4, 5, 20}, {2, 2, 2, 4, 6, 12}, {2, 2, 2, 4, 8, 8},
 {2, 2, 2, 5, 5, 10}, {2, 2, 2, 6, 6, 6}, {2, 2, 3, 3, 4, 12}, {2, 2, 3, 3, 6, 6},
 {2, 2, 3, 4, 4, 6}, {2, 2, 4, 4, 4, 4}, {2, 3, 3, 3, 3, 6}, {2, 3, 3, 3, 4, 4}, {3, 3, 3, 3, 3, 3}}
```

```
X[2, 7, 2]
```

```
{{2, 2, 2, 3, 7, 43, 1806}, {2, 2, 2, 3, 7, 44, 924}, {2, 2, 2, 3, 7, 45, 630},
 {2, 2, 2, 3, 7, 46, 483}, {2, 2, 2, 3, 7, 48, 336}, {2, 2, 2, 3, 7, 49, 294},
 {2, 2, 2, 3, 7, 51, 238}, {2, 2, 2, 3, 7, 54, 189}, {2, 2, 2, 3, 7, 56, 168},
 {2, 2, 2, 3, 7, 60, 140}, {2, 2, 2, 3, 7, 63, 126}, {2, 2, 2, 3, 7, 70, 105},
 {2, 2, 2, 3, 7, 78, 91}, {2, 2, 2, 3, 7, 84, 84}, {2, 2, 2, 3, 8, 25, 600},
 {2, 2, 2, 3, 8, 26, 312}, {2, 2, 2, 3, 8, 27, 216}, {2, 2, 2, 3, 8, 28, 168},
 {2, 2, 2, 3, 8, 30, 120}, {2, 2, 2, 3, 8, 32, 96}, {2, 2, 2, 3, 8, 33, 88},
 {2, 2, 2, 3, 8, 36, 72}, {2, 2, 2, 3, 8, 40, 60}, {2, 2, 2, 3, 8, 42, 56}, {2, 2, 2, 3, 8, 48, 48},
 {2, 2, 2, 3, 9, 19, 342}, {2, 2, 2, 3, 9, 20, 180}, {2, 2, 2, 3, 9, 21, 126},
 {2, 2, 2, 3, 9, 22, 99}, {2, 2, 2, 3, 9, 24, 72}, {2, 2, 2, 3, 9, 27, 54}, {2, 2, 2, 3, 9, 30, 45},
 {2, 2, 2, 3, 9, 36, 36}, {2, 2, 2, 3, 10, 16, 240}, {2, 2, 2, 3, 10, 18, 90},
 {2, 2, 2, 3, 10, 20, 60}, {2, 2, 2, 3, 10, 24, 40}, {2, 2, 2, 3, 10, 30, 30},
 {2, 2, 2, 3, 11, 14, 231}, {2, 2, 2, 3, 11, 15, 110}, {2, 2, 2, 3, 11, 22, 33},
 {2, 2, 2, 3, 12, 13, 156}, {2, 2, 2, 3, 12, 14, 84}, {2, 2, 2, 3, 12, 15, 60},
 {2, 2, 2, 3, 12, 16, 48}, {2, 2, 2, 3, 12, 18, 36}, {2, 2, 2, 3, 12, 20, 30},
 {2, 2, 2, 3, 12, 21, 28}, {2, 2, 2, 3, 12, 24, 24}, {2, 2, 2, 3, 13, 13, 78},
 {2, 2, 2, 3, 14, 14, 42}, {2, 2, 2, 3, 14, 15, 35}, {2, 2, 2, 3, 14, 21, 21},
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 {2, 2, 2, 3, 18, 18, 18}, {2, 2, 2, 4, 5, 21, 420}, {2, 2, 2, 4, 5, 22, 220},
 {2, 2, 2, 4, 5, 24, 120}, {2, 2, 2, 4, 5, 25, 100}, {2, 2, 2, 4, 5, 28, 70},
 {2, 2, 2, 4, 5, 30, 60}, {2, 2, 2, 4, 5, 36, 45}, {2, 2, 2, 4, 5, 40, 40},
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 {2, 2, 2, 4, 6, 16, 48}, {2, 2, 2, 4, 6, 18, 36}, {2, 2, 2, 4, 6, 20, 30}, {2, 2, 2, 4, 6, 21, 28},
 {2, 2, 2, 4, 6, 24, 24}, {2, 2, 2, 4, 7, 10, 140}, {2, 2, 2, 4, 7, 12, 42},
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 {2, 2, 2, 6, 6, 12, 12}, {2, 2, 2, 6, 7, 7, 21}, {2, 2, 2, 6, 8, 8, 12}, {2, 2, 2, 6, 9, 9, 9},
 {2, 2, 2, 7, 7, 7, 14}, {2, 2, 2, 8, 8, 8, 8}, {2, 2, 3, 3, 4, 13, 156}, {2, 2, 3, 3, 4, 14, 84},
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 {2, 2, 3, 3, 6, 7, 42}, {2, 2, 3, 3, 6, 8, 24}, {2, 2, 3, 3, 6, 9, 18}, {2, 2, 3, 3, 6, 10, 15},
 {2, 2, 3, 3, 6, 12, 12}, {2, 2, 3, 3, 7, 7, 21}, {2, 2, 3, 3, 8, 8, 12}, {2, 2, 3, 3, 9, 9, 9},
 {2, 2, 3, 4, 4, 7, 42}, {2, 2, 3, 4, 4, 8, 24}, {2, 2, 3, 4, 4, 9, 18}, {2, 2, 3, 4, 4, 10, 15},
 {2, 2, 3, 4, 4, 12, 12}, {2, 2, 3, 4, 5, 5, 60}, {2, 2, 3, 4, 5, 6, 20}, {2, 2, 3, 4, 6, 6, 12},
 {2, 2, 3, 4, 6, 8, 8}, {2, 2, 3, 5, 5, 5, 15}, {2, 2, 3, 5, 5, 6, 10}, {2, 2, 3, 6, 6, 6, 6},
 {2, 2, 4, 4, 4, 5, 20}, {2, 2, 4, 4, 4, 6, 12}, {2, 2, 4, 4, 4, 8, 8}, {2, 2, 4, 4, 5, 5, 10},
 {2, 2, 4, 4, 6, 6, 6}, {2, 2, 5, 5, 5, 5, 5}, {2, 3, 3, 3, 3, 7, 42}, {2, 3, 3, 3, 3, 8, 24},
 {2, 3, 3, 3, 3, 9, 18}, {2, 3, 3, 3, 3, 10, 15}, {2, 3, 3, 3, 3, 12, 12}, {2, 3, 3, 3, 4, 5, 20},
 {2, 3, 3, 3, 4, 6, 12}, {2, 3, 3, 3, 4, 8, 8}, {2, 3, 3, 3, 5, 5, 10}, {2, 3, 3, 3, 6, 6, 6},
 {2, 3, 3, 4, 4, 4, 12}, {2, 3, 3, 4, 4, 6, 6}, {2, 3, 4, 4, 4, 4, 6}, {2, 4, 4, 4, 4, 4, 4},
 {3, 3, 3, 3, 3, 4, 12}, {3, 3, 3, 3, 3, 6, 6}, {3, 3, 3, 3, 4, 4, 6}, {3, 3, 3, 4, 4, 4, 4}}
```

Y[a, n, s] is a subset of X[a, n, s]

where divisibility applies. In this case,
the numbers are such that a_i divides a_n for all $1 \leq i \leq n-1$.

```
Y[a_, n_, s_] := Select[X[a, n, s], AllTrue[#[[-1]] / #[[1 ;; -2]], IntegerQ] &]
```

```
Y[2, 6, 2]
```

```
{{2, 2, 2, 3, 7, 42}, {2, 2, 2, 3, 8, 24}, {2, 2, 2, 3, 9, 18},
 {2, 2, 2, 3, 12, 12}, {2, 2, 2, 4, 5, 20}, {2, 2, 2, 4, 6, 12},
 {2, 2, 2, 4, 8, 8}, {2, 2, 2, 5, 5, 10}, {2, 2, 2, 6, 6, 6}, {2, 2, 3, 3, 4, 12},
 {2, 2, 3, 3, 6, 6}, {2, 2, 4, 4, 4, 4}, {2, 3, 3, 3, 3, 6}, {3, 3, 3, 3, 3, 3}}
```

```
Y[5, 8, 3] // MatrixForm
```

```
{}
```