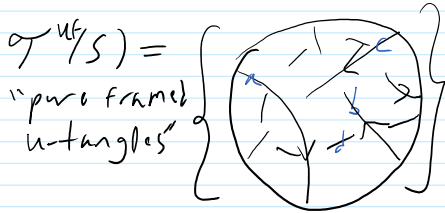


17-1350-AKT Tue Jan 31, Gentle Hours 7-8: Meta-Monoids, Algebraic Knot Theory

January 8, 2017 9:12 AM



Bring coffee cups for Seifert demo!

Finitely presented w/ the ops below!

operations

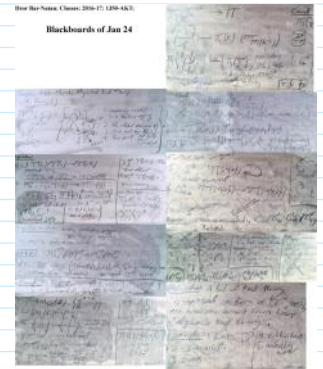
1.  $\sqcup: \mathcal{T}(S_1) \times \mathcal{T}(S_2) \rightarrow \mathcal{T}(S_1 \sqcup S_2)$  "disjoint union"
  2.  $m_z^{xy}: \mathcal{T}(S \cup \{x, y\}) \rightarrow \mathcal{T}(S \cup \{z\})$  "strand stitching" } some small print applies
  3.  $\eta^x: \mathcal{T}(S \cup \{x\}) \rightarrow \mathcal{T}(S)$  "strand deletion".
  4.  $\sigma_y^x: \mathcal{T}(S \cup \{x\}) \rightarrow \mathcal{T}(S \cup \{y\})$  "strand renaming"  
main axiom: meta-associativity:  $m_u^{xy} // m_v^{yz} = m_u^{yz} // m_v^{xu}$
  5.  $s^x: \mathcal{T}(S \cup \{x\}) \rightarrow \mathcal{T}(S \cup \{x\})$  strand reversal.
  6.  $\Delta_{yz}^x: \mathcal{T}(S \cup \{x\}) \rightarrow \mathcal{T}(S \cup \{y, z\})$  strand doubling.
- + more axioms
- meta-monoid  
meta-Hopf algebra

Meta-claim A lot of knot theory is expressible in terms of  $(\mathcal{T}^n, ops)$  and hence can benefit from "Alg. Knot Theory".

Examples In all theorems, I'll prove only the easy side.

1. Links
2. 3-manifold
3. genus
4. ribbon knots
5. unknotting numbers

all of the above on board, or use handout.



Links/closed components:

Thm  $\mathcal{T}(S \circlearrowleft^c) = \mathcal{T}(S \uparrow^c) / \left( \forall T \in \mathcal{T}(S \uparrow^c), m_c^{ab} T \sim m_c^{ba} T \right)$

$trc: \mathcal{T}(S \uparrow^c) \rightarrow \mathcal{T}(S \circlearrowleft^c); m_c^{ab} // trc = m_c^{ba} // trc$   
"meta-trace"

3-manifolds: Kirby's calculus:

$\left\{ \begin{array}{l} \text{compact connected orientable} \\ \text{3-manifold w/no bndry} \end{array} \right\} = \left\{ \begin{array}{l} \text{Framed} \\ \text{Links} \end{array} \right\} / \left\{ \begin{array}{l} \bigcirc \uparrow \downarrow \bigcirc = \emptyset \\ \bigcirc \bigcirc = \bigcirc \bigcirc \end{array} \right\}$  slide move

\* Why?

\* Slide move:  $T // \text{tray} // \text{tr}^b \sim T // D_{aa'}^a // m_b^{a'b} // \text{tr}^a // \text{tr}^b$

Then genus, ribbon, unknotting numbers

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Then go over  $\Gamma$ -calculus.