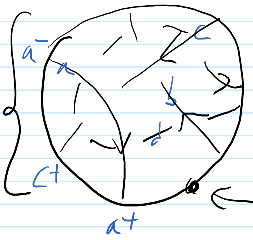
17-1350-AKT Tue Jan 24, Gentle Hours 5-6: Tangles, Meta-Monoids, and Algebraic Knot Theory } on board!

January 8, 2017 9:12 AM

Pass email contact sheet; no HW unless demanded; class photo?

Bring coffee cups for Seifert demo!

All theory today!

$\mathcal{T}^u(S) =$ { 

1. components oriented; in bijection w/ S.
2. No closed components }

← not in v/rv/bv/w

operations

1. $\sqcup: \mathcal{T}(S_1)\times\mathcal{T}(S_2)\to\mathcal{T}(S_1\sqcup S_2)$, provided $S_1\cap S_2=\emptyset$.
2. $m^{xy}_z: \mathcal{T}(S\cup\{x,y\})\to\mathcal{T}(S\cup\{z\})$, provided $x,y,z\notin S$ and provided x^+ is adjacent to y^- on ∂T
   [So better talk about $\mathcal{T}(S, \underset{\text{"frontier"}}{F})$, where F is an ordering of $S\times\{\pm\}$, but this is irrelevant in v/bv/rv/w]
3. $\eta^x: \mathcal{T}(S\cup\{x\})\to\mathcal{T}(S)$ "strand deletion".
4. $\sigma^x_y: \mathcal{T}(S\cup\{x\})\to\mathcal{T}(S\cup\{y\})$ "strand renaming"

Axioms so far: Many uninteresting, one non-trivial:

meta-associativity: $m^{xy}_u /\!/ m^{uz}_v = m^{yz}_u /\!/ m^{xu}_v$

Any thing satisfying these axioms is a "meta-monoid".

[So strictly speaking $\mathcal{T}^u$ is not a MM]

Examples: G^S, M^S, $A^{\otimes S}$, $M_{S\times S}$, $\Pi(S)=\{(G,m,l): \begin{smallmatrix}G \text{ a group}\\ m,l\in G^S\end{smallmatrix}\}$

* define Π by defining $\Pi_1: \mathcal{T}^{uf}\to\Pi$. [see [KBH]]

<u>Comment</u> with these ops, $\mathcal{T}^u/\mathcal{T}^{uf}$ are finitely presented!

More operations: [only on $\mathcal{T}^{uf}$]

5. $S^x: \mathcal{T}(S\cup\{x\})$ ↺ strand reversal.
6. $\Delta^x_{yz}: \mathcal{T}(S\cup\{x\})\to\mathcal{T}(S\cup\{y,z\})$ strand doubling.

These satisfy:

1. meta-co-associativity: $\Delta^u_{xv}/\!/\Delta^v_{yz} = \Delta^u_{vz}/\!/\Delta^v_{xy}$
2. Antipode: $\Delta^x_{yz}/\!/S^z/\!/m^{yz}_x = \eta^x$ $\quad S^z/\!/S^z = id.$

3. Compatability: $m^{xy}_z // \Delta^z_{z_1 z_2} = \Delta^x_{x_1 x_2} // \Delta^y_{y_1 y_2} // m^{x_1 y_1}_{z_1} // m^{x_2 y_2}_{z_2}$

Anything also satisfying these is a "meta-Hopf-algebra"; G^S, M^S, $\underset{\text{if } A \text{ is Hopf}}{A^{\otimes S}}$, $M_{S\times S}$, $\mathcal{T}$ are examples.

Meta-Claim A lot of knot theory is expressible in terms of $(\mathcal{T}^a, \text{ops})$ and hence can benefit from "Alg. Knot Theory".

done line

Examples In all theorems, I'll prove only the easy side.

1. Links
2. 3-manifold
3. Genus
4. ribbon knots
5. unknotting numbers

stated, not discussed.